

Along-wind self-excited forces of two-dimensional cables under extreme wind speeds

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ABSTRACT: Self-excited characteristics of cables under extreme wind speeds in along-wind direction currently is not clear, but may be of great interests for cable-supported bridge when dealing with flutter critical conditions. The flutter derivative with respect to along-wind aerodynamic damping is first derived according to the quasi-steady theory. For reduced wind speeds ranged from 53~1050, the flow fields around the along-wind forced vibration cable are simulated at Reynolds number of 5.18×10^5 , followed by identifying flutter derivatives of P^*_1 . The present results are in good agreement with the values from the quasi-steady theory, which suggests that the quasi-steady theory can be applied to obtaining flutter derivatives of P^*_1 at extreme wind condition. It is found that for cables under such condition, the vortex-induced forces dominate the variation of aerodynamic forces, and the self-excited forces can be negligible. Since the cable oscillation does not change the vortex-shedding frequency, it seems that the vortex-shedding pattern become very stable under extremely high wind speeds.

KEYWORDS: cable; along-wind vibration; self-excited forces; CFD; extreme wind; flutter derivatives.

1 INTRODUCTION

For cable-supported bridges with super-long spans, flutter instability at extreme wind condition may be a great concern. Normally, when carrying out flutter analysis, self-excited forces acting on bridge girders are always included, which can be determined through wind tunnel tests. However, it is found that the self-excited forces on cables are rarely considered among available references (Ge, 2008). But for super-long span bridges at flutter critical condition, the wind-induced forces on the cable may become significant since the increasing length and lower damping ratio of the cable. Based on this consideration, self-excited characteristics of cables in along wind under such kind of extreme wind condition should be first addressed.

It is well known that flow around circular cylinders show Reynolds-dependent characteristics, including vortex mode, separation point, mean and fluctuating drag and lift coefficients and so on. For flow around cables, such kind of characteristics always requires a full Reynolds simulation, which may help to provide meaningful results against the reality. This is also the case when dealing with cable aeroelasticity. If one considers a cable with diameter of 0.12m under extreme wind speeds, the corresponding Re number may be above 5×10^5 , and if the cable oscillates at very low frequency, such as 0.1Hz, the resultant reduced wind speeds may be over several hundreds or even above thousand, which is far above the normal reduced wind speed relative to bridge girders elasticity. Meanwhile, under extreme wind speeds, the mean wind forces on the cable may be significantly large compared to the self-excited forces. Such kind of conditions almost prevents any test set and application dealing with identification of flutter derivatives of cables in wind tunnel.

The striking advantages of CFD in application to identification of flutter derivatives is that, neither test set nor model inertia forces are involved (Zhu, 2007). This helps to overcome many difficulties in wind tunnel test, and allows one to carry out self-excited simulations at the real Re

number under extreme wind speeds. In the following, one cable is picked up from a real bridge engineering, and is forced to undergo along-wind oscillation under high wind speeds. The flutter derivative of the cable corresponding to the along-wind degree of freedom is extracted, and compared to values from the quasi-steady theory.

2 FLUTTER DERIVATIVES OF CABLES IN QUASI-STEADY FORM

Under extreme wind speeds, the cables in cable-supported bridges are assumed to undergo small-amplitude vibration with a low frequency, which means a small velocity compared to the incoming wind. Therefore, the resultant wind angle of attack relative to cables will be small.

Consider a two-dimensional cable immersed in the incoming wind with velocity of U_∞ , and oscillated both in along-wind direction and cross-wind direction, as shown in Figure 1. The synthesized wind velocity to the cable can be expressed as,

$$U_r = \sqrt{(U_\infty - \dot{p})^2 + \dot{h}^2} \approx U_\infty - \dot{p} \quad (1)$$

where $\dot{p}$ and $\dot{h}$ are the vibration velocities in along-wind and cross-wind directions on the coordinate system oxy with the x axis along the incoming wind, respectively. Obviously, Equation 1 recognizes the assumption that both $\dot{p}$ and $\dot{h}$ are small enough compared to incoming flow.

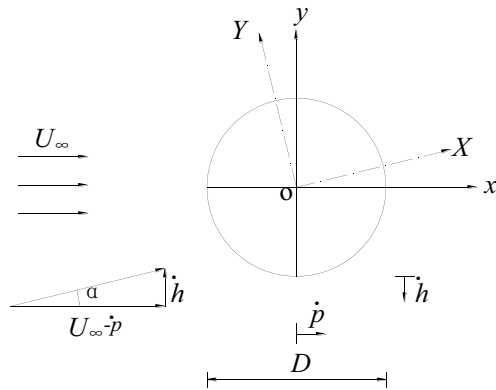


Figure 1 Cable in motion in incoming wind

The unsteady drag and lift forces acting on the cable can be rewritten as,

$$F_D = 0.5\rho U_r^2 D C_D(t) \quad (2a)$$

$$F_L = 0.5\rho U_r^2 D C_L(t) \quad (2b)$$

in which ρ denotes the air density; D the cable diameter; $C_L(t)$ and $C_D(t)$ are the unsteady lift and drag coefficients, and is depended on the relative wind angle of attack.

Under extreme wind speeds, the angle between the velocity of the cable and resultant wind velocity can be expressed as,

$$\alpha(t) = \tan^{-1}\left(\frac{\dot{h}}{U_\infty - \dot{p}}\right) \approx \frac{\dot{h}}{U_\infty - \dot{p}} \approx \frac{\dot{h}}{U_\infty} \quad (3)$$

with $\sin \alpha(t) \approx \alpha(t)$ and $\cos \alpha(t) \approx 1$ because of very small value of angle.

If one transfers Equation 2 onto the structural coordinate system OXY located at the center of the cable, with X axis along the synthesized wind direction, Equation 4 can be obtained as,

$$F_V = F_D \sin \alpha(t) + F_L \cos \alpha(t) \quad (4a)$$

$$F_H = F_D \cos \alpha(t) - F_L \sin \alpha(t) \quad (4b)$$

where F_V and F_H represent the wind forces normal to and along the resultant wind velocity, respectively.

This paper limits the study only on the along-wind oscillation and hence the drag force of the cable. Since the cable oscillates around the reference position with a small amplitude, Equation 4 can be expanded at the zero angle of attack based on Taylor's theory. If high-order terms are ignored, Equation 4b can be derived as,

$$\begin{aligned} F_H &= F_D - F_L \alpha(t) \\ &= 0.5 \rho (U_\infty - \dot{p})^2 D [C_D(t) - C_L(t) \alpha(t)] \\ &= 0.5 \rho (U_\infty - \dot{p})^2 D [C_{D0} + C'_{D0} \alpha(t) - (C_{L0} + C'_{L0} \alpha(t)) \alpha(t)] \\ &= 0.5 \rho (U_\infty - \dot{p})^2 D C_{D0} = 0.5 \rho D (U_\infty^2 C_{D0} - 2 U_\infty \dot{p} C_{D0}) \\ &= 0.5 \rho D U_\infty^2 (C_{D0} - 2 \frac{\dot{p}}{U_\infty} C_{D0}) \end{aligned} \quad (5)$$

in which C_{D0} and C'_{D0} ; C_{L0} and C'_{L0} are mean drag and lift coefficient, and their slopes at zero angle of attack, respectively.

It can be seen that the first term in Equation 5 is the drag force at steady state, while the second term represents the self-excited force induced by along-wind oscillation, which corresponds to aerodynamic damping as shown in the following.

In along-wind direction, if comparing the unsteady forces acting on the cables with the well-known Scanlan formulation (1971), the flutter derivative in along-wind direction can be obtained as,

$$P_1^* = -2 C_{D0} / K \quad (6)$$

where $K = \omega D / U_\infty$ is the reduced frequency; $\omega = 2\pi f$ the circular frequency; f the cable vibration frequency.

3 NUMERICAL SIMULATION STRATEGIES

Under extreme wind speeds, the Re number is very high. If one considers the spanwise correlation of the flow and then performs full three-dimensional simulations, significantly high requirement will be put forward on computer resources. So, this paper limits the research only on the two-dimensional situation.

3.1 The governing equations

The two-dimensional incompressible flow around the circular cable cross section can be described by the unsteady Reynolds-averaged Navier-Stokes equations as follows,

$$\frac{\partial}{\partial x_i} u_i = 0 \quad (7)$$

$$\rho \frac{\partial}{\partial t} u_i + \rho \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} \right) + \rho \frac{\partial}{\partial x_j} \left(-\overline{u_i' u_j'} \right) \quad (8)$$

where μ denotes air viscosity; p and t are the pressure and time; u_i and u_i' ; u_j and u_j' ($i=1, 2$; $j=1, 2$) represent the mean and fluctuating velocities along the axis in Cartesian coordinate system, respectively; $-\rho \overline{u_i' u_j'}$ the Reynolds stress.

If Eddy Viscosity Model is employed to close the equations, the Reynolds stress can be expressed as,

$$-\rho \overline{u_i' u_j'} = \mu_t \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \quad (9)$$

where $\mu_t = \rho C_\mu k^2 / \varepsilon$ is the turbulent viscosity; C_μ the empirical constant; k the turbulent kinetic energy and ε the dissipation rate of turbulent kinetic energy.

The standard $k-\varepsilon$ model employs wall functions in near wall region in order to avoid fine mesh, which results in significant saving on computer power. It has been widely used in engineering practice due to its robust formation and lower computational overhead. However, such kind of treatment is not sufficient to resolve complex flow accurately. The model often overpredicts the turbulence near stagnation regions, and presents the length scale which is too large in adverse pressure gradient flow. In many engineering applications, it fails to represent flow field due to the presentation of swirling flows and curved boundary layers flows.

By defining the turbulent viscosity in form of $\mu_t = k / \omega$, and the dissipation rate per unit kinetic energy $\omega = \varepsilon / k$, the $k-\omega$ turbulence model, which have been developed to improve the performance of the standard $k-\varepsilon$ (Menter, 1994). It is observed that the standard $k-\varepsilon$ model performed well in the shear layer flow while the $k-\omega$ model is excellent near to the wall. This leads to the development of the Shear Stress Transport (SST) model combining the advantage of both models, in which the $k-\omega$ model is employed near the wall and the standard $k-\varepsilon$ model is used near the boundary layer edge. To achieve this, the $k-\omega$ model is multiplied by a blending function F_l and the $k-\varepsilon$ model by $(1-F_l)$ such that F_l has a value of one near wall region and switches to zero at the boundary layer where $k-\varepsilon$ model is recovered. It is found that the SST $k-\omega$ model can predict better flow separation compared to both the $k-\varepsilon$ and $k-\omega$ models (Liaw, 2005).

3.2 Numerical strategy

The cable with a diameter of $D=0.12\text{m}$, is located near the center of the computational domain, with all of outer boundaries extended over $25D$, as shown in Figure 2. Such domain arrangement

can largely avoid numerical interference coming from outside boundaries. Since the cable is forced to undergo imposed vibration in CFD simulation, dynamic mesh system will be activated in Fluent which requires a suitable grid arrangement. In order to ensure good mesh quality, the whole computational domain is divided into four zones. The Zone Z1 around the cable is covered with body-fitted structured quadrilateral grids, which will oscillate with the cable but the grids topological structure will not be changed all the time. In this zone, the mesh orthogonality is always kept, as shown in Figure3, which helps to improve the accuracy and capture flow features. The Zone Z2 is paved with unstructured triangle grids, and will be re-meshed after each time step as to reflect the displacement of cable. Re-mesh quality can be ensured based on suitable selection of spring constant in spring-based smoothing method. Zone Z3 is also covered with structured orthogonal quadrilateral grids, but Zone Z4 is paved with unstructured quadrilateral grids. The grids in Z3 and Z4 are fixed all the time although the cable is undergoing along-wind oscillation.

Three different grid systems are created, with the closest grid space near the cable wall of $h_0=0.000005\text{m}$, 0.0001m and 0.0002m , respectively, as shown in Table 1. For all grid systems, the adjacent cell expansion ratios are always controlled under 1.1, which provides a smooth grid dimension over the whole computational domain. The total number of cells is shown in Tab.1. Numerical simulations show that the maximum Y^+ values on the wall are less than one in all grid systems, which satisfy grid resolution requirement for the SST $k-\omega$ turbulent model. Figure 4 shows the Y^+ distribution around the cable for G1.

Temporal discretization of the governing equations is performed with a second-order implicit scheme, while the convective terms are approximated with a second-order upwind scheme. The SIMPLEC algorithm is used to decouple the pressure and velocities field.

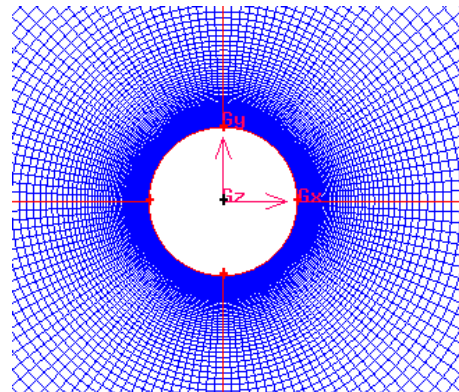
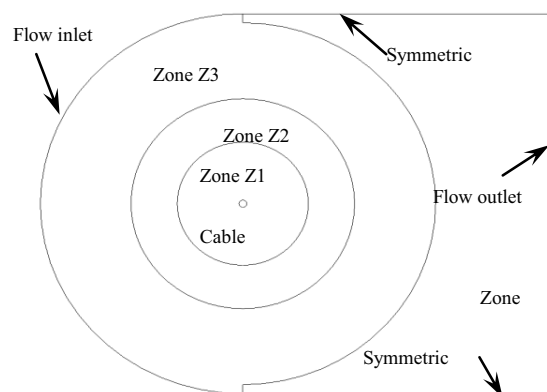


Figure 2 Partition and boundary conditions in computational domain

Figure 3 Grids arrangement around cable

Table 1. Grid parameters

Grid	h_0/m	Nodes	Y^+_{max}
G1	0.000005	26950	0.095
G2	0.00001	25270	0.22
G3	0.00002	23730	0.42

2.3 Cable motion and boundary conditions

In order to represent the body motion-induced forces acting on the cable and identify flutter derivatives, during CFD

simulations, the cable will undergo imposed along-wind sinusoidal vibration, i.e.,

$$p(t) = p_0 \sin(2\pi ft) \quad (12)$$

where p_0 is the vibration amplitude with a value of $0.025D$, which is very small as to ensure small disturbance on flow field; f the vibration frequency. Since all simulations under different reduced wind speeds are performed at the same Re number, f needs to be changed as to obtain the desired reduced wind speeds.

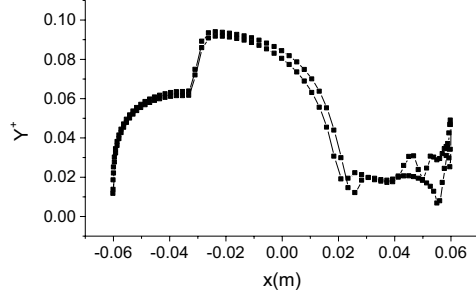


Figure 4 Y^+ values on cable surface of G1

In order to reflect surface roughness on the cable, a roughness height is assumed to be of $3 \times 10^{-5}D$ (Poulin, 2007). Suitable boundary conditions are imposed on the computational domain, as shown in Figure 2. The flow inlet boundary is defined as velocity inlet with a value of 63m/s in horizontal direction (corresponds to the Reynolds number of 5.18×10^5), while flow outlet boundary is defined as the outflow boundary condition. The symmetric boundary condition is imposed on the top and bottom boundaries. As for the cable surface, no-slip condition is applied on it, which means the flow on the wall shares the same velocity as the cable when the cable undergoing imposed motion.

2.4 Grid-independence check

Simulation tests show that, even under extreme wind speeds, vortex shedding is still a dominating feature in flow around the cable. As to effectively capture vortex shedding, non-dimensional time step size is set to 0.01.

Define unsteady drag and lift coefficients on the cable as,

$$C_D = F_D / (0.5\rho U_\infty^2 D), \quad C_L = F_L / (0.5\rho U_\infty^2 D) \quad (13)$$

where F_D and F_L are drag and lift, which are positive in downstream and upside direction, respectively.

Grid check is performed for the fixed cable. Except the cable motion, boundary conditions and time step size will be the same. Data will be collected after 0.1s simulations as to avoid any influence from initial computation and thereafter the flow attains stable stage. As shown in Figure 5, time histories before 0.3s are force coefficients when the cable is fixed. It indicates that the flow around the cable experience a steady vortex shedding, with the St number indicated in Figure 4, which are slightly high compared to other reports (Norberg, 2003).

The results obtained on three different grid systems, including the mean drag and lift coefficients, RMS values of drag and lift coefficients, as well as the strouhal number are displayed in Figure 6. One can see that the results agree well with each other, especially the G1 and G2, which

means the simulation results will not be influenced by grid refinement. In this paper, G1 is selected for the further study on the moving cable.

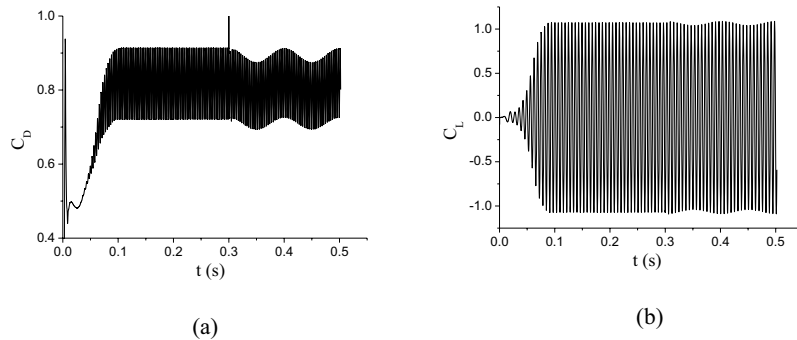


Figure 5 Aerodynamic force time histories on cable

The mean drag coefficient from G1 is of 0.81, which matches well with the suggested value under extreme wind speeds (Poulin, 2007). This value will be used for the quasi-steady theory in the following.

Figure 7 is the vortex shedding plot around the fixed cable at four key states. It shows the vortex generation, elongation, shedding and drifting with the flow in the wake.

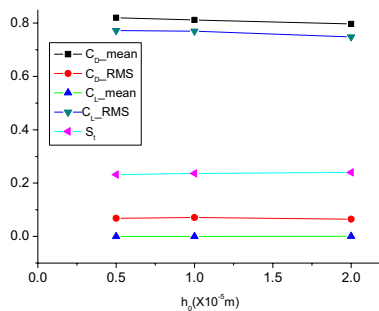


Figure 6 Aerodynamic coefficients and Strouhal number for flow around fixed cable

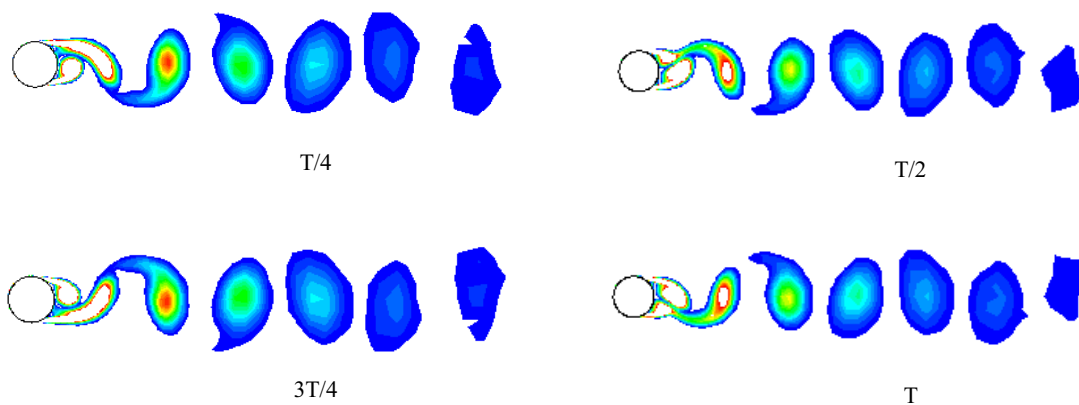


Figure 7 Vortex contour plot in wake of the fixed cable during one cycle of vortex shedding period

4 SIMULATIONS ON MOVING CABLE AND RESULTS

Considering simulations on the moving cable, enough time step simulations are first carried out for the fixed cable in order to establish a steady vortex shedding flow configuration, as shown in Figure 5. Then forced vibration is imposed on the cable, and force time histories are recorded on each time step. As shown in Figure 5 and more detailed in Figure 8, two cycles of data are plotted under the reduced wind speed $v_r=53$. Figure 9 also shows the total aerodynamic force time histories at a extreme high reduced wind speed of $v_r=1050$.

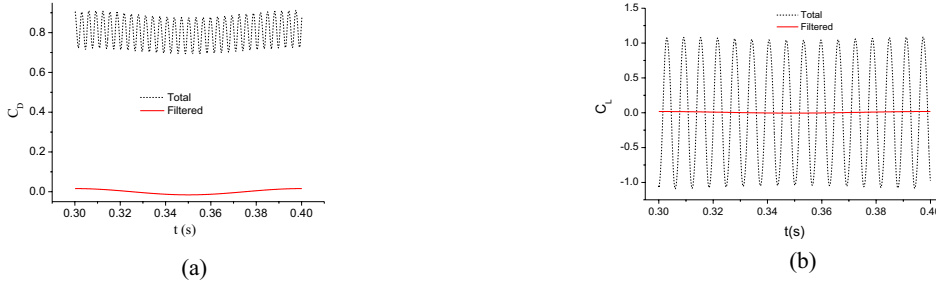


Figure 8 Total and self-excited aerodynamic force time histories on forced vibration cables ($V_r=53$)



Figure 9 Total and self-excited aerodynamic force time histories on forced vibration cables ($V_r=1050$)

Figures.8 and 9 appears that both of the total force time histories are formed by superposition of a high frequency wave on a low frequency oscillation. Frequency analysis indicates that two frequencies are included in the time histories, with the dominating one equals to the vortex shedding frequency, while the low one equals to the forced vibration frequency. It is worth noting that under such extreme wind speed, the oscillated cable still keeps its vortex shedding frequency as in the fixed situation. It seems that the vortex shedding mechanism will not be changed even the cable is under oscillation. Hence the vortex shedding mode is very stable under extreme wind speeds.

Based on assumption of linear superposition of aerodynamic forces, the self-excited forces can be separated from the total forces. Since the vortex-shedding frequency is significantly higher than the imposed motion frequency, the low-pass filtering technology can be used to obtain the self-excited forces, as shown in Figures.8 and 9. One can see that the self-excited mean and RMS value of drag is much smaller than the total one, while the RMS value of lift is significantly smaller than the total lift, especially at extreme high reduced wind speeds. In other words, the self-excited forces on the vibration cable are very small compared to the vortex-shedding forces. If comparing the self-excited drag coefficient with the mean total drag coefficient at the reduced

wind speed of 1050, one can conclude that the self-excited component may be negligible at extreme high wind speeds.

With the obtained self-excited drag time history, the flutter derivative of P_1^* can be identified based on the least square method (Zhu, 2009). The present results are plotted against the quasi-steady ones from Equation 6 with $C_d=0.81$, as shown in Figure 10. One can see that the CFD results agree well with the quasi-stead theory. Hence if the self-excited drag force is considered under extreme wind speeds, the value of P_1^* can be obtained straightforward from the quasi-stead theory.

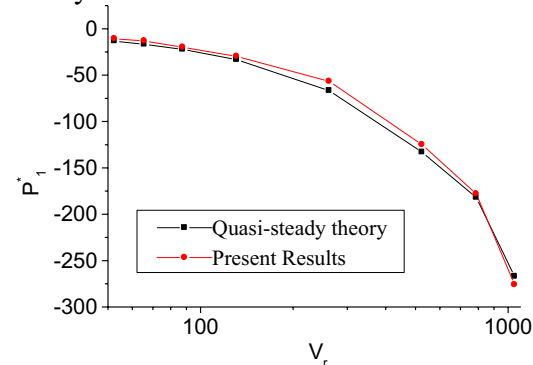


Figure 10 Identified flutter derivatives P_1^* in comparison with quasi-steady theory

Table 2 also shows the value of KP_1^* , where $K=2\pi/v_r$ is the reduced frequency. If $C_{D0} = 0.81$, the quasi-steady theory will give the value of $KP_1^* = -1.62$, which matches well with the present results at extreme high reduced wind speeds.

Table 2 CFD simulation cases and results

Vibration frequency/Hz	0.5	0.75	1	2	4	6	8	10
V_r	1050	788	525	263	131	88	66	53
K	0.006	0.009	0.012	0.024	0.048	0.072	0.096	0.12
KP_1^*	-1.65	-1.58	-1.5	-1.36	-1.42	-1.43	-1.25	-1.29

One striking feature in Figure 10 is that the P_1^* become more negative with the increase of reduced wind speed, which means the aerodynamic damping relative to along-wind oscillation is always positive. It can be deduced that the one degree-of-freedom cable under along-wind oscillation is always stable.

Figure 11 shows the vortex-shedding patterns of the cable during one cycle of forced vibration, where T is the forced vibration period and denotes the cable coming back to the reference position. Since the vortex-shedding mechanism dominates the flow around the cable, the vortex street is apparent in the wake of vibration cable.

4 CONCLUSIONS AND REMARKS

The along-wind vibration of a cable is simulated by CFD method in this paper, with the following conclusions and remarks,

- 1) The identified flutter derivatives P_1^* are in good agreement with the quasi-steady theory, which demonstrates the feasibility of CFD method to address the aeroelastic features of cables under extreme high wind speeds.
- 2) The quasi-steady theory, based on drag coefficient of 0.8, can provide a reasonable estimation

of the along-wind flutter derivatives of the cables at extremely high wind speeds.

- 3) It is found that under extreme wind speeds, the vortex shedding pattern will not be influenced by the cable vibration, and the vortex shedding still dominates the fluctuation on aerodynamic forces. It should be highlighted that the self-excited forces induced by along-wind vibration is negligible compared to the vortex-induced forces, i.e., the self-excited component may be negligible at extreme high wind speeds.
- 4) From aerodynamic point of view, the one degree-of-freedom cable oscillating in the along-wind direction is always stable, even under extreme high wind speeds.

If more computer resources are available, three-dimensional simulations may be carried out as to get further insight and extend the study to cross-wind vibration.

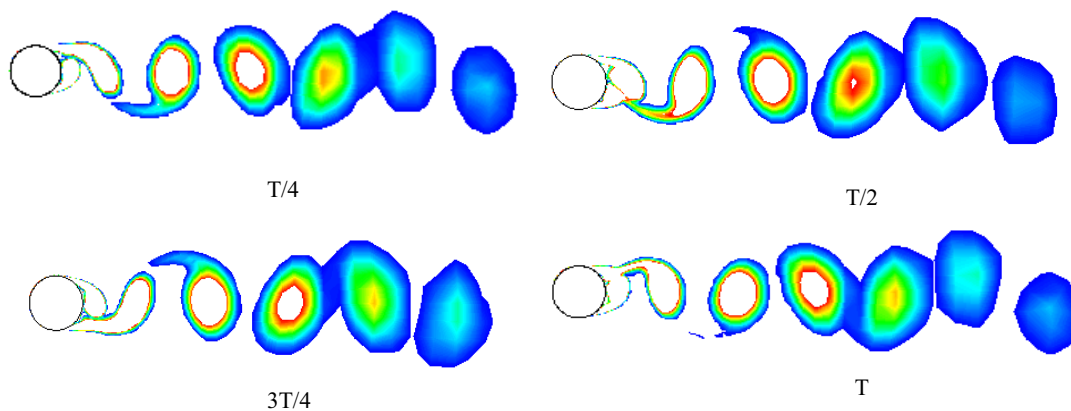


Figure 11 Snap shooting of vortex shedding of cables at four key states during one cycle of forced vibration

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