

Superposability of the unsteady aerodynamic loads on bridge deck sections

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ABSTRACT: The 2-dimensional unsteady aerodynamic forces are investigated from a point of view of whether these forces conform to the principle of linear superposition. It is shown that the basic preconditions, which lead to the linear superposability in cases of thin airfoil sections, are no longer valid for a bluff section. However, theoretical models in bridge aerodynamics, such as the one of flutter-buffeting analysis and those concerning aerodynamic admittance (AA) functions, always necessitate this superposability. The contradiction revealed in this literature may throw light on the perplexing problem of AA functions pertaining to the wind loads on bridge decks. Some theoretical AA models, which were employed rather extensively and derived from flutter derivatives according to interrelations resulted from thin airfoil theories, are demonstrated to a great extent to be illogical. Finally, with understanding of the linear superposability of the unsteady aerodynamic forces, suggestions in connection with experiment-based AA functions are presented in the last section.

KEYWORDS: bridge; unsteady; aerodynamic; superposition; buffeting; flutter

1 INTRODUCTION

Aerodynamic forces acting on bridge decks have generally been classified into steady and unsteady, according to whether or not the loads are time-varying. Unsteady loads in this context are distinguished from those of quasi-steady in whether they are dependent uniquely on the current state of motions (both structure and wind fluctuation) or on the whole motion histories [1]. Strictly speaking, both the motion-related and the turbulence induced aerodynamic forces are, to some extent, unsteady. The determination of aerodynamic response of a long flexible bridge involves the description of the mean wind loads, the motion induced loads (self-excited), and the turbulence induced loads. These three parts of aerodynamic loads are generally described with aerostatic coefficients, flutter derivatives and aerodynamic admittance (AA) functions respectively, and then, are linearly superposed according to the principle of linear superposition. However, it will be shown in the paper that a question arises as to whether these loads can be linearly superposed or, in other words, as to whether there exist a group of AA functions independent neither on the turbulent gust structure nor on the structural motions. That is the main concern of this literature.

2 FUNDAMENTALS OF BRIDGE AERODYNAMICS

The stochastic response of an elongated and flexible bridge structure to turbulent wind flow can be determined roughly by two classes of dynamic loads: (i) those due to wind-structure interactions, namely aeroelastic effects; (ii) those due to the wind speed fluctuations, including those inherent in the oncoming flow and those initiated by the body section itself.

Scanlan and Tomko [2] put forward a method to describe the self-excited loads for bluff sections put in smooth flow, which necessitates six experiment-based flutter-derivatives, functions of a dimensionless frequency. Later, Scanlan and Jones [3] expanded this method and expressed the lift and torque per unit span with eight flutter-derivatives, allowing for inclusion of the displacement terms, as below:

$$L_{ae} = \frac{1}{2} \rho U^2 B [KH_1^* \frac{\dot{h}}{U} + KH_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B}] \quad (1)$$

$$M_{ae} = \frac{1}{2} \rho U^2 B^2 [KA_1^* \frac{\dot{h}}{U} + KA_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B}] \quad (2)$$

where ρ is the air density, U the steady wind velocity, $K=B\omega/U$ the dimensionless frequency, ω the oscillation frequency, B the reference width of the deck section, H_i^* ($i=1\sim 4$) and A_i^* ($i=1\sim 4$) flutter derivatives, h the vertical motion, α the section rotation, $\dot{h}$ and $\dot{\alpha}$ the derivatives of h and α to real time t , respectively.

Though the part of aerodynamic load due to wind fluctuations is, strictly speaking, unsteady both for well-streamlined and bluff body sections, it is always given in quasi-steady form, as

$$D_b(t) = \frac{1}{2} \rho U^2 B \left(2C_D \chi_{Du} \frac{u(t)}{U} + C'_D \chi_{Dw} \frac{w(t)}{U} \right) \quad (3)$$

$$L_b(t) = \frac{1}{2} \rho U^2 B \left(2C_L \chi_{Lu} \frac{u(t)}{U} + (C'_L + C_D) \chi_{Lw} \frac{w(t)}{U} \right) \quad (4)$$

$$M_b(t) = \frac{1}{2} \rho U^2 B^2 \left(2C_M \chi_{Mu} \frac{u(t)}{U} + C'_M \chi_{Mw} \frac{w(t)}{U} \right) \quad (5)$$

Where $u(t)$ and $w(t)$ are horizontal and vertical wind fluctuations, respectively; C_D , C_L , C_M are aerostatic drag, lift and torque coefficients; C'_D , C'_L , C'_M are the derivatives of C_D , C_L , C_M with respect to wind attack angle α ; χ_{Du} , χ_{Dw} , χ_{Lu} , χ_{Lw} , χ_{Mu} , χ_{Mw} are the so-called aerodynamic admittances, functions of dimensionless frequency K , playing the role of connecting the wind fluctuations with the buffeting loads.

The most important assumption implied implicitly in Eqs. (3) to (5) is that the AA functions wherein are dependent uniquely upon the reduced frequency and therefore would not be affected by the turbulent gust structures, or, shapes of the curves of wind spectrum.

3 SUPERPOSABILITY OF WAKE EFFECTS

3.1 Basic theory

A general form describing the periodic motion of an airfoil with circular frequency ω is [5, 6]

$$w(\theta) = Ue^{i\omega t} \left\{ A_0 + 2 \sum_{n=1}^{\infty} A_n \cos n\theta \right\}, \quad (6)$$

where w is the vertical velocity distribution relative to the airflow along the chord of the airfoil, and the relationship between x and θ is $x = \cos \theta$.

The multiplier $e^{i\omega t}$ on the right side of Eq. (6) describes the periodic variation of the velocity with respect to time; the infinite series in the bracket expresses the modes of velocity distribution along the chord. The first five modes ($x = -1$ to 1) are plotted in figure 1. Especially, the term of A_0 denotes the mode of translatory oscillation where a uniform velocity distributed along the whole chord width; the term of A_1 denotes the mode of rotational oscillation where the velocity is proportional to x , the distance aft of the midpoint. The first two terms describe the rigid motion and the other higher-order terms allow the deformative motions.

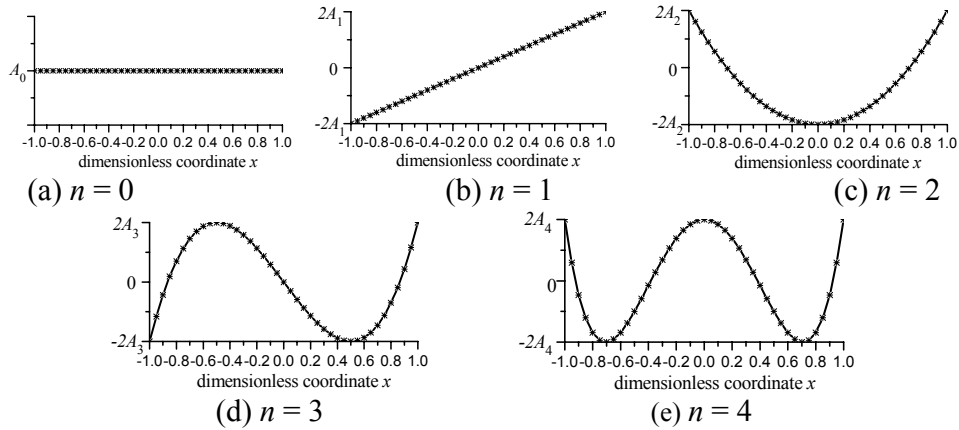


Fig. 1 Fundamental vertical distributions of the first five terms in Eq. (6)

It is worthy of mention that the resulted lift depends only on the first three coefficients in Eq. (6) and the moment on the first four, which means, for any given velocity distributions, the aerodynamic effects are embodied in these four terms. Sears [6] summarized the total lift and moment (about the midpoint) acting on a airfoil with chord length of B , corresponding to the general form of velocity expressed by Eq. (6), as

$$L = \pi \rho B U^2 e^{i\omega t} \{ (A_0 + A_1) C(k) + (A_0 - A_2) \cdot ik/2 \}, \quad (7)$$

$$M = (\pi/4) \rho B^2 U^2 e^{i\omega t} \{ A_0 C(k) - A_1 [1 - C(k)] - (A_1 - A_3) \cdot ik/4 - A_2 \}. \quad (8)$$

where $k = B\omega/(2U) = K/2$, and $C(k) = K_1(ik) / [K_0(ik) + K_1(ik)]$ is in accord with the Theodorsen circular function that was expressed in the original paper as [10]

$$C(k) = \frac{\int_1^\infty \frac{x_0}{\sqrt{x_0^2 - 1}} e^{-ikx_0} dx_0}{\int_1^\infty \frac{x_0 + 1}{\sqrt{x_0^2 - 1}} e^{-ikx_0} dx_0} = \frac{K_1}{K_0 + K_1}. \quad (9)$$

The Theodorsen circular function is commonly expressed in complex form as

$$C(k) = F_T(k) + iG_T(k) \quad (10)$$

where $F_T(k)$ and $G_T(k)$ are real functions of the reduced frequency k .

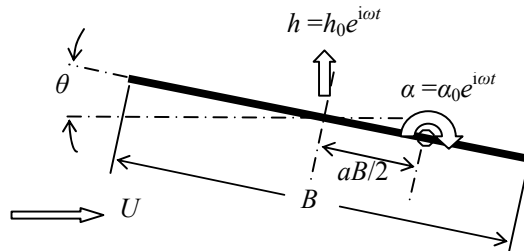


Fig. 2 Diagram show the rigid motion of the airfoil

3.2 Rigid motion induced loads

Considering a type of periodic rigid motion shown in figure 2, where the vertical motion is $h = h_0 e^{i\omega t}$, the torsional motion is $\alpha = \alpha_0 e^{i\omega t}$, $aB/2$ is the distance from the airfoil mid-chord to the oscillatory rotation point ($a > 0$ denote the location at the right half chord), the relative vertical velocity distribution of the mode shown in Fig. 1(b) in this case is

Reservation of A_0 and A_1 in Eq. (7) and (8) could result in the lift and moment immediately, as

$$L = -\pi\rho UBC(k) \left[U\alpha + \dot{h} + \frac{B}{2} \left(\frac{1}{2} - a \right) \dot{\alpha} \right] - \frac{\pi\rho B^2}{4} \left(U\dot{\alpha} + \ddot{h} - \frac{B}{2} a\ddot{\alpha} \right), \quad (11)$$

$$M_{1/2} = -\frac{\pi\rho UB^2}{4} C(k) \left[U\alpha + \dot{h} + \frac{B}{2} \left(\frac{1}{2} - a \right) \dot{\alpha} \right] + \frac{\pi\rho B^3}{16} \left[\dot{\alpha}U + \frac{B}{8} \ddot{\alpha} \right]. \quad (12)$$

It should be noted that the moment expressed by (12) is about the mid-chord point of the airfoil. The moment about the rotation point may be calculated by

$$M_a = M_{1/2} + L \cdot \frac{aB}{2} = -\frac{\pi\rho UB^2}{2} \left(a + \frac{1}{2} \right) C(k) \left[U\alpha + \dot{h} + \frac{B}{2} \left(\frac{1}{2} - a \right) \dot{\alpha} \right] - \frac{\pi\rho B^2}{4} \left[\frac{aB}{2} \ddot{h} - \frac{B^2}{4} \left(a^2 + \frac{1}{8} \right) \ddot{\alpha} - \frac{BU}{4} \dot{\alpha} + \frac{BUa}{2} \alpha \right]. \quad (13)$$

The lift and moment, i.e., Eqs. (11) and (13), are in accord with those reported in other literatures [7,9].

3.3 Wind gusts induced loads

A vertical gust that changes harmonically both in time and space could, if the coordinate axes are fixed to the airfoil, be described as a form of relative velocity distribution of the airfoil, as

$$w = We^{i\omega(t-x/U)} = We^{i\omega t} e^{\frac{i\omega x}{U}}, \quad (14)$$

the expressions of lift and moment can be simplified to

$$L = \pi\rho BUWe^{i\omega t} \{ [J_0(k) - iJ_1(k)]C(k) + iJ_1(k) \}, \quad (15)$$

$$M_{1/2} = L \cdot \frac{B}{4}, \quad (16)$$

where the Bessel functions $J_0(k)$ and $J_1(k)$ have the forms as bellow

$$J_0(k) = \frac{2}{\pi} \int_1^\infty \frac{\sin kx}{\sqrt{x^2 - 1}} dx, \quad J_1(k) = -\frac{2}{\pi} \int_1^\infty \frac{x \cos kx}{\sqrt{x^2 - 1}} dx. \quad (17)$$

Expression (15) can be rewritten in the notation of complex AA as

$$L = \pi\rho BUWe^{i\omega t} \chi_s(k), \quad (18)$$

where

$$\chi_s(k) = F_s(k) + iG_s(k) = [J_0(k) - iJ_1(k)]C(k) + iJ_1(k) \quad (19)$$

is the so-called complex Sears aerodynamic admittance function.

In the case of when the flow turbulence of broad-spectrum and steady stochastic, instead of a gust with a pure circular frequency ω , is encountered, the contributions of each gust component to the unsteady aerodynamic forces could be superposed linearly due to the potential flow around the section induced by gust components fulfill the principle of superposition. Thus, the power spectral density of L in this case can be obtained according to (19), as

$$S_L(k) = \left(\frac{1}{2} \rho UBC'_L \right)^2 |\chi_s|^2 S_w(k), \quad (20)$$

where S_w is the power spectral density of the vertical gust, $C'_L = dC_L/d\alpha = 2\pi$ is the differential of the lift coefficient with respect to wind attack angle α , $|\chi_s|^2 = F_s^2(k) + G_s^2(k)$ is known as the Sears admittance function which serves to link, for an airfoil, the spectrum of vertical gusting

velocity to that of associated lift [4].

In the case of when both the rigid motions and wind fluctuations are involved, the unsteady aerodynamic lift or torque acting on an airfoil section can, theoretically, be determined by the linear superposition of those determined by (11) and (15), or those by (12) and (16), respectively.

3.4 Applications of superposability via indicial functions

Another application of the superposability of unsteady aerodynamic loads in 2-D thin airfoil theory is that the resulted aerodynamic forces acting on a section in question due to arbitrary rigid motions or wind gusts could be expressed as convolution integrals over the time histories, resorting to a kind of so-called indicial functions. The general mathematic expression of the indicial function takes the form of

$$\varphi(s) = 1 - \sum_1^i a_i e^{-d_i s}, \quad (21)$$

Where $s=Ut/(2B)$ is dimensionless time, a_i, d_i are constants to be identified and $d_i > 0$.

Applying the following Fourier transform

$$\bar{f}(k) = \int_{-\infty}^{\infty} f(s) e^{-iks} ds, \quad (22)$$

to $\varphi'(s)$, the derivative of $\varphi(s)$ with respect to s , and then plus $\varphi(0)$ would yield

$$\varphi(0) + \bar{\varphi}' = 1 - \sum_1^i a_i + \sum_1^i \frac{a_i d_i^2}{d_i^2 + k^2} - ik \sum_1^i \frac{a_i d_i}{d_i^2 + k^2}, \quad (23)$$

which will be employed for power spectrum analysis later in this paper.

3.4.1 Wagner function

One of the typical transient phenomena is, the lift acting on a flat-plate section due to an abrupt step-function change of the attitude of the section relative to the oncoming flow, α_0 , would undergo a transient evolution described by

$$L(s) = \frac{1}{2} \rho U^2 B C_L' \alpha_0 \varphi(s), \quad (24)$$

where $C_L' = dC_L / d\alpha$, $\varphi(s)$ is an indicial lift-growth function.

If an arbitrary small rotation $\alpha(s)$ is encountered, the resulted lift would, according to the principle of superposition, be obtained with a convolution integral as

$$L(s) = \frac{1}{2} \rho U^2 B C_L' \int_{-\infty}^s \varphi(s-\sigma) \alpha'(\sigma) d\sigma, \quad (25)$$

where $\alpha'(\sigma) = d\alpha / d\sigma$.

A change of variables and integration by parts in Eq. (25) would lead to

$$L(s) = \frac{1}{2} \rho U^2 B C_L' \left[\varphi(0) \alpha(s) + \int_0^{\infty} \varphi'(\sigma) \alpha(s-\sigma) d\sigma \right]. \quad (26)$$

3.4.2 Küssner function

Another typical transient phenomenon is associated with a flat-plate section entering a sharp-edged vertical gust. The lift evolution in this situation is described by

$$L(s) = \frac{1}{2} \rho U^2 B C_L' \alpha_0 \psi(s) \quad (27)$$

Where $\psi(s)$, different from the Wagner function, is the so-called Küssner function, of which an

excellent approximation was also offered by Jones [8], as

$$\psi(s) = 1 - 0.5e^{-0.13s} - 0.5e^{-s} \quad (28)$$

Now consider the section penetrating a gust of any form $w(s)$ of variable vertical velocity. In a similar way as that leading to equation (26), the lift in this case can be expressed as

$$L(s) = \frac{1}{2} \rho U B C'_L \left[\psi(0)w(s) + \int_0^\infty \psi'(\sigma)w(s-\sigma)d\sigma \right]. \quad (29)$$

4 PRINCIPLES FROM AIRFOIL THEORY TO BRIDGE AERODYNAMICS

4.1 “Equivalent” indicial function

Scanlan put forward a method trying to acquire the AA functions for bluff bridge deck sections from tested flutter derivatives [3, 9, 10]. This method is outlined in the following paragraphs.

Taking the lift for an example and considering the part due to vertical motion only, a replacement of α in eq. (26) by $\dot{h}/U$ and ϕ by ϕ_{Lh} , may lead to

$$L(s) = \frac{1}{2} \rho U^2 B C'_L \left\{ \phi_{Lh}(0) \frac{h'(s)}{B} + \int_0^\infty \phi'_{Lh}(\sigma) \frac{h'(s-\sigma)}{B} d\sigma \right\}, \quad (30)$$

where $h' = dh/ds$ and $\phi' = d\phi/ds$.

Applying the Fourier transform of (22) to Eq. (1) (Neglecting the terms of α and $\dot{\alpha}$) and to (30) would yield

$$4k(H_1^* - iH_4^*) = C'_L [\phi_{Lh}(0) + \bar{\phi}'_{Lh}], \quad (31)$$

Other relations associated with rotation could be obtained in a similar way.

On the other hand, the connection between Küssner function and AA function, for example, the complex Sears AA function, may be resulted from the Fourier transform, as

$$\bar{L}(k) = \frac{1}{2} \rho U B C'_L [\psi(0) + \bar{\psi}'] \bar{w} = \frac{1}{2} \rho U B C'_L \cdot \chi_{L'} \cdot \bar{w}, \quad (32)$$

where $\chi_{L'}$ is also a complex aerodynamic admittance function.

Scanlan assumed directly the equivalence between the Küssner function ψ and the Wagner function ϕ . Thus, according to (31) and (32), one obtain immediately [3]

$$\bar{L}(k) = 2\rho U B k [H_1^* - iH_4^*] \bar{w} = \frac{1}{2} \rho U B C'_L \cdot \chi_{L'} \cdot \bar{w}. \quad (33)$$

Multiplying (33) by its complex conjugate yields the relation between the auto-PSD of the lift and that of the vertical gust:

$$S_{LL}(k) = (2\rho U B k)^2 \cdot \left[(H_1^*)^2 + (H_4^*)^2 \right] \cdot S_{ww} = \left(\frac{1}{2} \rho U B C'_L \right)^2 \cdot |\chi_{L'}|^2 \cdot S_{ww}. \quad (34)$$

This yields immediately a bearing between the aerodynamic admittance and the flutter derivatives, as

$$|\chi_{L'}|^2 = 16k^2 \left[(H_1^*)^2 + (H_4^*)^2 \right] / (C'_L)^2 = 4K^2 \left[(H_1^*)^2 + (H_4^*)^2 \right] / (C'_L)^2. \quad (35)$$

Other AA functions associated with torsional motion or with resulted moment could be derived in a similar way as that of (35).

The vital question in such a methodology is whether or not $\dot{h} = e^{-i\omega t}$ equals to $w = e^{i\omega t} e^{-i\omega x/U}$ as to the resulted aerodynamic effects. The answer is obviously negative.

We can see from what follows that the AA function so obtained would result in incorrect

power spectrum of the associated aerodynamic force component.

Considering only vertical translatory velocity in Eq. (11) and ignoring the apparent mass and quasi-steady terms would result in

$$L = -\pi\rho UBC(k)\dot{h} = -\frac{1}{2}\rho UBC'_L \cdot C(k) \cdot \dot{h} \quad (36)$$

The interrelation between the Theodorsen function and the indicial function can be resulted from a comparison of the Fourier transform of Eq. (36) to that of Eq. (30) for $\alpha = \dot{h}/U$, as [11]

$$C(k) = \varphi(0) + \bar{\varphi}', \quad (37)$$

Thus, the following relationships could be established according to Eq. (10) and Eq. (23):

$$F_T(k) = 1 - a_1 - a_2 + \frac{a_1 d_1^2}{d_1^2 + k^2} + \frac{a_2 d_2^2}{d_2^2 + k^2} = 0.5 + \frac{0.0003416}{0.00207 + k^2} + \frac{0.03015}{0.09 + k^2}, \quad (38)$$

$$G_T(k) = -k \left[\frac{a_1 d_1}{d_1^2 + k^2} + \frac{a_2 d_2}{d_2^2 + k^2} \right] = -k \left[\frac{0.0075075}{0.00207 + k^2} + \frac{0.1005}{0.09 + k^2} \right], \quad (39)$$

Where $F_T(k)$ and $G_T(k)$ are the real and imaginary parts of the Theodorsen function. These lead immediately to

$$\lim_{k \rightarrow \infty} |\chi_T|^2 = \lim_{k \rightarrow \infty} [F_T^2(k) + G_T^2(k)] = (1 - \sum a_i)^2 = 0.25. \quad (40)$$

where $|\chi_T|^2 = F_T^2(k) + G_T^2(k)$ is recognized as Theodorsen aerodynamic admittance function [12].

In a similar way, the limit value of Sears AA function can be obtained as

$$\lim_{k \rightarrow \infty} |\chi_s|^2 = \lim_{k \rightarrow \infty} [F_s^2(k) + G_s^2(k)] = (1 - \sum a_i)^2 = 0 \quad (41)$$

The comparison of the Theodorsen and Sears AA functions is plotted in figure 3, where it can be seen that the relative discrepancy between the two curves increases rapidly with k .

Thus, the methodology of deriving AA functions by “equivalent” indicial functions would lead to a limit value of the AA functions determined by $(1 - \sum a_i)^2$, which is dependent on the identified parameters and therefore not necessary a zero value.

The essence of this incorrect can be ascribed to the introduction of an assumption that doesn't even hold in aeronautics to bluff sections (the equivalence between Küssner function and Wagner's indicial function). This is logically unacceptable.

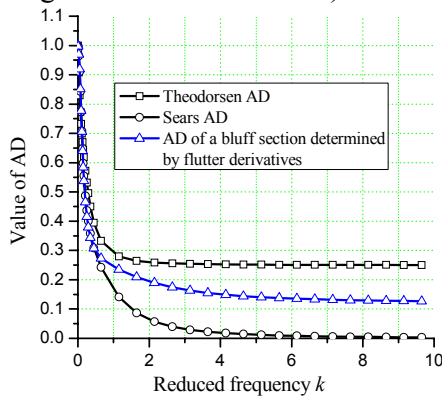


Fig. 3 Thoedorsen and Sears aerodynamic admittance

It is this nonzero limit characteristic of the above AA model that leads to a modification factor

to yield correct limit AA values [9,12], as

$$|\chi_{L'}|^2 = \frac{k^2}{(C_L')^2} \left[(H_1^*)^2 + (H_4^*)^2 \right] \cdot R(c), \quad (42)$$

Where

$$R(c) = \int_0^B \int_0^B \exp \left[-c_0 \frac{\sigma_w}{U} n |x_1 - x_2| \right] dx_1 dx_2 \quad (43)$$

is the introduced coefficient determined by the along-wind coherence of vertical gusting over the whole chord width, where B is the chord width, c_0 a constant reflecting the level of coherence, σ_w/U the turbulence intensity and n the frequency of turbulent component. Note that in this situation $R(c) \rightarrow 0$ when $n \rightarrow \infty$.

While the introduction of $R(c)$ provided the AA function with a correct limit value when k (or n) approaches to an infinite value, there still seems to be no way to provide a quantitative validation for such a model resulted from the combination of an illogical one and a quasi-steady factor based upon the wind coherence.

4.2 “Equivalent” Theodorsen function

A number of researchers also tried to determine the AA functions via flutter derivatives, as that proposed by Scanlan but in another point of view of “equivalent” Theodorsen function [13-16]. The main idea of this method is supposing Eq. (19), the bearing between Theodorsen circulation function $C(k)$ and Sears function $\chi_s(k)$, is applicable in bluff deck sections. Thus, according to (31), an “equivalent” indicial function can be identified through the experimentally obtained flutter derivatives, then a “equivalent” $C_{eq}(k)$ may be determined by Garrick’s interrelation $C(k) = \varphi(0) + \overline{\varphi'}$, finally the “equivalent” Sears admittance function can be written according to (19) as

$$\chi_{eq,s}(k) = [J_0(k) - iJ_1(k)]C_{eq}(k) + iJ_1(k). \quad (44)$$

This is logically incorrect either. As a matter of fact, according to Theodorsen’s original paper [10], one may obtain

$$C(k) = F + iG = \frac{-J_0(k) + iY_1(k)}{-[J_0(k) + Y_0(k)] + i[Y_1(k) - J_0(k)]}, \quad (45)$$

where the integration of J_0 and J_1 are given out in Eq. (17), Y_0 and Y_1 were expressed in the original paper as

$$Y_0(k) = -\frac{2}{\pi} \int_1^\infty \frac{\cos kx}{\sqrt{x^2 - 1}} dx, \quad Y_1(k) = -\frac{2}{\pi} \int_1^\infty \frac{x \sin kx}{\sqrt{x^2 - 1}} dx. \quad (46)$$

Obviously, J_0, J_1, Y_0, Y_1 all are component functions included in the Theodorsen function $C(k)$. Thus, if one tries to use an “equivalent” form of $C(k)$ to describe aerodynamic characteristics of bluff sections, it warrants justification for component functions J_0 and J_1 remaining the original forms applied in aeronautics instead of “equivalent J_{eq0} ” and “equivalent J_{eq1} ”.

5 EXPERIMENT-BASED AA FUNCTIONS

A number of researchers have experienced the method of determining the AA functions of bluff bridge deck sections through model tests in wind tunnel with certain turbulence characteristics [17-20]. This seems to be the only possible approach to date to deal with bluff sections, and the AA functions so-obtained are expected to be applicable to the buffeting assessment of the full

bridge immersed in various turbulent fields, as that of a thin airfoil. Such a methodology entails the AA functions being univariate functions of the reduced frequency k and, particularly being independent on the structural motion and the turbulent gust structure. This is true for an airfoil due to the assumption of potential flow around it always works, and also due to un-stationary aerodynamic forces resulted from rigid translation, rigid rotation, harmonic gusts, or an arbitrary combination of them can be treated separately and superposed linearly.

Thus, for a bluff body section, the critical problem is not whether the linear superposition principle still holds, but, for a given configuration, to what extent this principle can be considered to hold approximately. Some researches indicate that the AA functions of bridge deck do show dependence on the wind field characteristics (Matsuda, et al, 1999; Larose and Mann, 1998). But they failed to provide an explanation. Therefore, if experiments are designed in order to identify AA functions of a bridge deck section, the following two problems should be considered cautiously:

6 CONCLUSIONS

Theoretical investigation presented in this paper indicated that the linear superposability of the unsteady aerodynamic forces acting on a 2-dimensional section, which is applicable for a thin airfoil where wake effects are linear superposable, would not be valid for bluff bridge deck sections due to a series of hypotheses, such as the approximate linear distribution of the wake vortices, incompressible and inviscid flow, as well as potential flow around, etc., are no longer applicable. This directly throws doubt on some basic concepts underlying the current bridge aerodynamics, especially on, for a given deck configuration, the existence of AA functions dependent uniquely on reduced frequency. Some theoretical AA models, which have been obtained in essence according to the linear superposition principle and employed now rather extensively in bridge aerodynamics, are discussed in this literature and turn out to be, to a great extent, illogical. Thus, the superposability of the unsteady aerodynamic forces, or the level of this superposability could arrive at, should probably be a root factor relating to the issue of AA functions as well as to the flutter-buffeting theoretical model in bridge aerodynamics. Future work relating to this subject, the superposability of unsteady aerodynamic forces, will be experiment-based investigations aiming at deck configurations extensively employed in long-span bridges.

7 ACKNOWLEDGEMENTS

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8 REFERENCES

- 1 Dowell EH. A modern course in aeroelasticity, 4th Edition. Boston: Kluwer academic publishers; 2004.
- 2 Scanlan RH, Tomko JJ. Airfoil and bridge deck flutter derivatives. J Eng Mech, ASCE 1971; 97(EM6):1717-1737.
- 3 Scanlan RH, Jones NP. A form of aerodynamic admittance for use in bridge aeroelastic analysis. J Fluid Struct 1999; 13(7-8): 1017-1027.
- 4 Scanlan RH. Problematics in formulation of wind-force models for bridge decks. J Eng Mech, ASCE 1993; 119(7):1353-1375.
- 5 Karman Th V, Sears WR. Airfoil Theory for Non-Uniform Motion. J Aeronaut Sci 1938; 5(10):5-15.
- 6 Sears WR. Some aspects of non-stationary airfoil theory and its practical application. J Aeronaut Sci 1941; 8(3): 104-108.

- 7 Theodorsen T. General theory of aerodynamic instability and the mechanism of flutter. NACA Report 496, US Advisory Committee for Aeronautics, Langley, VA, USA. 1935.
- 8 Jones RT. The unsteady lift on a wing of finite aspect ratio. NACA report 681, US Advisory Committee for Aeronautics, Langley, VA, USA. 1940.
- 9 Scanlan RH. Motion-related body force functions in two-dimensional low-speed flow. *J Fluid Struct* 2000; 14(1): 49-63.
- 10 Scanlan RH. Bridge deck aeroelastic admittance revisited. *J Bridge Eng*, ASCE 2000; 5(1):1-7.
- 11 Garrick IE. On some reciprocal relations in the theory of nonstationary flows. NACA Report 629, US Advisory Committee for Aeronautics, Langley, VA, USA. 1938
- 12 Scanlan RH. Reexamination of sectional aerodynamic force functions for bridges. *J Wind Eng Ind Aerodyn* 2001; 89(14-15):1257-1266.
- 13 Hatanaka A, Tanaka H. New estimation method of aerodynamic admittance function. *J Wind Eng Ind Aerodyn* 2002; 90(12-15):2073-2086.
- 14 Hatanaka A, Tanaka H. Aerodynamic admittance functions of rectangular cylinders. *J Wind Eng Ind Aerodyn* 2008; 96(6-7):945-953.
- 15 Costa C. Aerodynamic admittance functions and buffeting forces for bridges via indicial functions. *J Fluid Struct* 2007; 23(3):413-428.
- 16 Costa C, Borri C, Olivier Flamand, O, Grillaud G. Time-domain buffeting simulations for wind-bridge interaction. *J Wind Eng Ind Aerodyn* 2007; 95(9-11):991-1006.
- 17 Larose GL, Tanaka H, Gimsing NJ, Dyrbye C. Direct measurements of buffeting wind forces on bridge decks. *J Wind Eng Ind Aerodyn* 1998; 74-76:809-818.
- 18 Larose GL, Mann J. Gust loading on streamlined bridge decks. *J Fluid Struct* 1998; 12(5):511-536.
- 19 Diana G, Bruni S, Cigada A, Zappa E. Complex aerodynamic admittance function role in buffeting response of a bridge deck. *J Wind Eng Ind Aerodyn* 2002; 90(12-15):2057-2072.
- 20 Matsuda M, Hikami Y, Fujiwara T, Moriyama A. Aerodynamic admittance and the 'strip theory' for horizontal buffeting forces on a bridge deck. *J Wind Eng Ind Aerodyn* 1999; 83(1-3):337-346.