

# Flow past two tandem circular cylinders using Spectral element method

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**ABSTRACT:** In this paper, flow past around two circular cylinders which are tandem arranged at a Reynolds number of 200 is numerically investigated by spectral element method. To validate the numerical method for incompressible Navier–Stokes equations, the benchmark problem of flow over a single circular cylinder is employed, with the numerical results in a good agreement with the available literatures. Then, by changing the spacing ratios  $L/D$  from 1.2 to 10.0, the flow characteristics, including the flow patterns, force statistical parameters such as the drag and lift coefficients as well as wake oscillation frequencies (Strouhal numbers) are investigated. Numerical results show that there are around three wake flow patterns which are determined by the spacing ratio. In addition, the force parameters are highly affected by the flow patterns.

**KEYWORDS:** spectral element method; laminar flow; spacing ratio; Navier–Stokes equations; two tandem cylinders.

## 1 INTRODUCTION

The problem of flows past cylinders is very common in the fields of civil engineering, ocean engineering and hydraulic engineering, such as in high-rise buildings, long span cable-stayed or suspension bridges and chimneys experiencing wind loadings, as well as oil pipelines immersed in ocean current. Thus it has attracted, as an important aspect of computational fluid dynamics, much attention in scientific research. Among these problems, the flow past a circular cylinder plays an important part for its broad engineering applications and for being the foundation of flows past other bluff bodies. So far, wind tunnel experiments are still regarded as the most reliable means of studying this flow. In the last decades, however, the numerical simulation has been developed as the potential technique for its low cost, short time period, high efficiency and parameter adjustability.

Concerning the case of flow over two tandem circular cylinders, many important phenomena of fluid flow have been found through both experimental investigation and numerical simulation. For example, there is a critical spacing ratio,  $L/D \approx 3.6$  ( $L$  is the center-to-center distance between the two cylinders,  $D$  is the cylinder diameter), at Reynolds number ( $Re$ ) of 200, and the vortex shedding behind the upstream cylinder actually disappears when the spacing ratio is smaller than the critical one. Also, if the spacing ratio is over 3.6, the periodic vortex shedding will occur from both cylinders, which significantly changes the fluctuating lift force, drag force and Strouhal number ( $St$ ).

More recently, some research studies have provided important insights into the numerical analysis of flow past two tandem circular cylinders. Using the Collocated Unstructured Computational Fluid Dynamic Code (CUCFDC), Sharman *et al.*<sup>[1]</sup> studied this problem when  $Re=100$ , and found that the critical spacing ratio is identified between 3.75 and 4.0, higher than the typical critical spacing ratio of 3.6, and that a unique reattachment point and a separation point appear at the downstream cylinder. Wu and Hu<sup>[2]</sup> investigated this problem at Reynolds numbers of 200 and 500 by employing the finite volume method and adaptive triangle meshing technology, and drew the conclusions that the lift and drag forces curves change at different space distance. Carmo and Meneghini<sup>[3]</sup> also studied this problem at the Reynolds numbers from 160 to 320. They found that, when the Reynolds number is greater than 190, three-dimensional effects occur, thus two-dimensional numerical simulations can not accurately satisfy the requirement of determining the critical spacing ratio.

Spectral element method (SEM) was first proposed by Patera<sup>[4]</sup>, who applied this approach to Navier-Stokes equations under one- and two-dimensions. Spectral element is a high-order element combining the geometrical facility of the finite element method and the high accuracy of the spectral method, by increasing the order of the polynomial  $p$ . It can achieve a high convergence ability and computing efficiency even though the mesh is coarse. Blackburn and Karniadakis<sup>[5]</sup> studied the problem of vortex-induced vibration (VIV) of a single cylinder in two- and three-dimensions, and verified the efficiency of the SEM.

The current work pays attention to the problem of flow around two circular cylinders which are particularly arrayed as a tandem configuration at Reynolds number  $Re=200$  using the SEM. But to start with, the problem of flows past one circular cylinder is simulated as a validation to examine the SEM computational code. Then using this method, the effects of gap spacing ratio of the two cylinders on the flow characteristics are investigated. In the current case, the numerical simulations are carried out associated with the center-to-center space ratios of  $L/D = 1.2, 1.5, 2.0, 3.6, 4.0, 6.0, 8.0$  and  $10.0$ . To show the flow pattern near the wake region, flow vorticity contours are clearly presented. The corresponding flow indexes such as the mean-averaged force coefficients and the frequencies of lift fluctuation are also obtained for all the cylinders.

The framework of this paper is organized as follow: In Section 2 we introduces the governing equations for incompressible viscous flow and provides the detailed processes of the SEM. The verifications of the computer code are achieved through the benchmark problem of flow over a single cylinder in Section 3. Section 4 presents and discusses the descriptions and results for the two tandem cylinders array with various spacing ratios. Main conclusions of the current work are finally summarized in Section 5.

## 2 GOVERNING EQUATIONS

In Cartesian coordinates, the non-dimensional tensorial forms of the Navier-Stokes governing equations for the incompressible viscous fluid flow in a domain  $\Omega$  and a time interval  $[0, T]$  are written as:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

The above equations are solved by employing the *Semtex* code. Released by Prof. H.M. Blackburn (Monash University, Australia), the *Semtex* is a quadrilateral spectral element DNS code that combines the standard nodal Gauss–Lobatto–Legendre (GLL) basis functions and Fourier expansions in a homogeneous direction<sup>[6]</sup>. More literatures related with *Semtex* include references<sup>[7-10]</sup>. In the current study, it should be noted that we choose 7 order GLL basis functions and incremental time step of 0.005 for all the computations.

Some definitions of the flow parameters required in the computations are given:

$$C_D = \frac{2F_D}{\rho U_\infty^2 D}, \quad C_L = \frac{2F_L}{\rho U_\infty^2 D}, \quad St = \frac{f_s D}{U_\infty}, \quad C_p = \frac{2(p - p_\infty)}{\rho U_\infty^2}, \quad (3)$$

with the corresponding explanations as:  $C_D$ , drag coefficient;  $C_L$ , lift coefficient;  $St$ , Strouhal number;  $C_p$ , pressure coefficient;  $F_D$ , force in the stream-wise direction;  $F_L$  force in the transverse direction;  $f_s$ , vortex shedding frequency;  $\rho$ , the fluid density;  $U_\infty$ , characteristic velocity;  $p_\infty$ , characteristic pressure; and  $D$ , the characteristic length scale (Here it is treated as the cylinder diameter).

### 3 VALIDATION STUDIE

The problem of unsteady flow over a single cylinder is chosen as a workbench reference to validate the numerical code for further investigations. As shown in Figure 1, a computational domain  $50D \times 40D$  is presented, where  $D$  is the diameter of the cylinder with the center position  $(0, 0)$ . The inlet boundary is positioned  $20D$  upstream from the cylinder center with the velocity boundary condition:  $u=1, v=0$ . And the boundary conditions of the outlet  $30D$  downstream from the cylinder center point is prescribed as  $\partial u/\partial x=0, \partial v/\partial x=0, p=0$ . Concerning the upper and lower boundaries, they are both  $20D$  away from the horizontal centerline, with a slip boundary condition as:  $\partial u/\partial y=0$  and  $v=0$ . On the cylinder surface, no-slip conditions are applied as  $u=0, v=0$ .

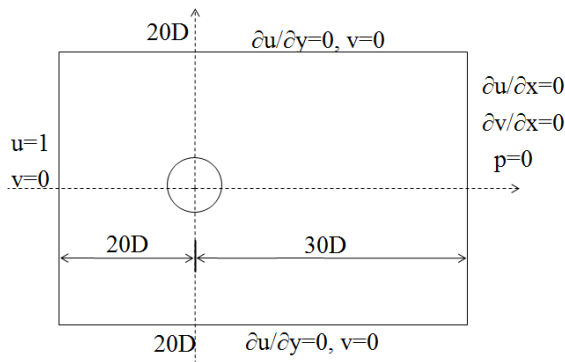


Figure 1. Schematic diagram of the computational domain and boundary conditions.

There are totally 354 spectral elements in the computational domain, as illustrated in Figure 2(a). In the region around the cylinder, the mesh grids are assigned as thin as  $0.1D$  in order to obtain more accurate results (Figure 2(b)).

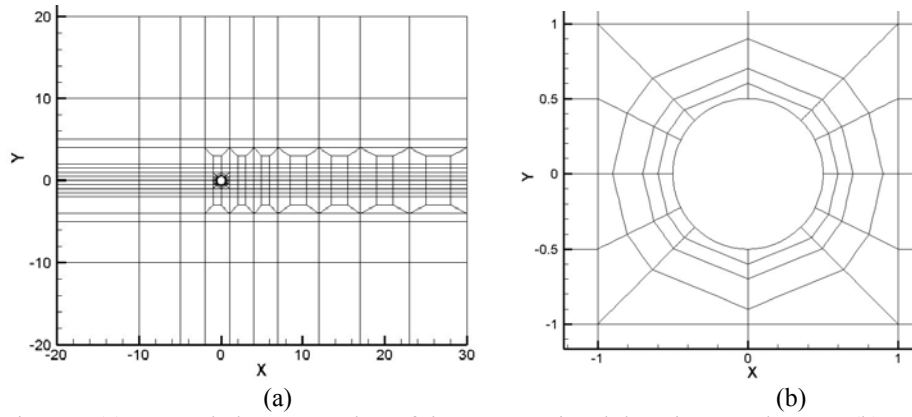


Figure 2.(a) Spectral element meshes of the computational domain, 354 elements (b) zoomed-in view of the mesh around the cylinder.

We compare the computed mean drag coefficient  $C_D$  and Strouhal number  $St$  at  $Re=200$  with the data from literatures<sup>[11-14]</sup> as shown in Table 1. The mean drag coefficient  $C_D$  is 1.346 while others range from 1.31 to 1.35. And the  $St=0.195$  varies little to the exited results by Meneghini *et al.*<sup>12</sup> and Franke *et al.*<sup>13</sup>. Moreover, the current curve of  $St$  and  $Re$ , which is also chosen as a reference, is explored and plotted in Figure 3 with the corresponding data from references<sup>[15-17]</sup>. With the increase of  $Re$  ranging from 60 to 120, the obtained  $St$  is found to become larger. Meanwhile, our computed curve is quite close to the results provided by Young and Ni<sup>17</sup>. These evidences mentioned above can be good illustrations for the effectiveness of the *Semtex* computational code for the solutions of laminar flow.

Table 1.Comparisons of the mean drag coefficients  $C_D$  and Strouhal numbers  $St$  at  $Re=200$

| Parameters                              | $C_D$ | $St$  |
|-----------------------------------------|-------|-------|
| Franke <i>et al.</i> <sup>[13]</sup>    | 1.31  | 0.194 |
| Farrant <i>et al.</i> <sup>[14]</sup>   | 1.37  | 0.196 |
| Meneghini <i>et al.</i> <sup>[12]</sup> | 1.30  | 0.196 |
| Braza <i>et al.</i> <sup>[11]</sup>     | 1.35  | 0.200 |
| Present                                 | 1.346 | 0.195 |

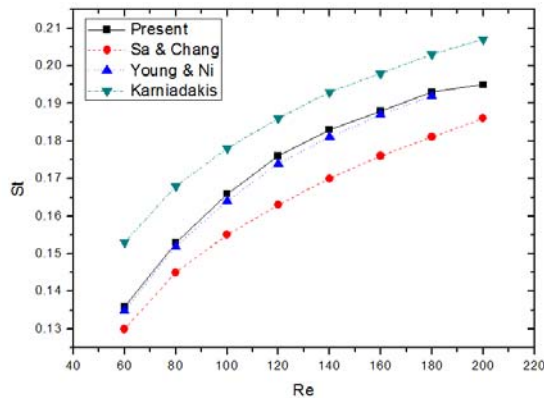


Figure 3.The relationship between the Strouhal number  $St$  and Reynolds number  $Re$  from 60 to 200 for the problem of flow past a single cylinder. Comparisons are made between the present results and collected data from references<sup>[15-17]</sup>.

## 4 FLOW AROUND TWO TANDEM CIRCULAR CYLINDERS

### 4.1 Problem descriptions

In this subsection, we present the problem of flow past two tandem cylinders ( $Re=200$ ). Details of the computational models and one sample of the mesh grids are respectively shown in Figure 4(a) and (b). The width and length of the domain and boundary conditions are of the same as those in the above single cylinder case. Two cylinders are marked as “1” and “2” with the distance  $L$  between their circular centers. The inlet boundary is  $20D$  upstream from the center of the cylinder “1” while the outlet is  $30D$  downstream. In the following, different value is relatively set for the distance  $L/D$  as 1.2, 1.5, 2.0, 3.6, 4.0, 6.0, 8.0 and 10.0. In the example of  $L/D=2.5$ , there are totally 432 mesh grids while there are around 400~500 grids in the cases with other spacing ratios. Meanwhile, high resolutions of grid refinement are adopted around the cylinders and in the wake flow regions.

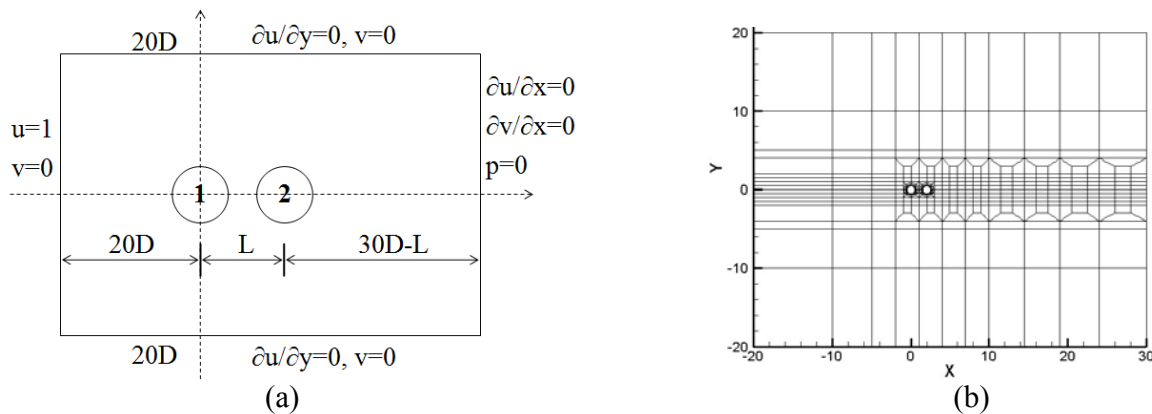


Figure 4.(a) Schematic of the computational model (b) the mesh grids of the spectral element for the case of flow past two tandem cylinders with  $L/D=2$ .

### 4.2 Flow patterns

The computational vorticity patterns at various spacing ratios are shown in Figure 5(a)-(h). It can be observed that there is a spacing ratio, smaller than which there is no vortex shedding from the upstream cylinder while larger than which periodical vortex shedding occurs. From there figures the critical spacing ratio is estimated at around  $L/D \approx 3.6$ . As  $L/D < 3.6$ , the shear layers generated from the upside and downside of the front cylinder also attach on the upper and lower sides of the rear cylinder. In the far region behind the rear cylinder, an obvious Kármán vortex street can be observed. At the critical spacing ratio  $L/D = 3.6$ , the vortex begins to shed from the upstream cylinder. When the spacing ratio is higher, for example,  $L/D = 4$ , there is observed Kármán vortex in the gap region between the two cylinders. The vortex from the cylinder “1” periodically impinges on the front surface of the rear cylinder, affecting the flow pattern in the wake region. As the spacing ratio increases, the vortex from the front cylinder seems to have a higher influence on the wake flow. At  $Re=200$  in which slight three-dimensional effect has occurred, the wake flow begins to show more complexity in the far region.

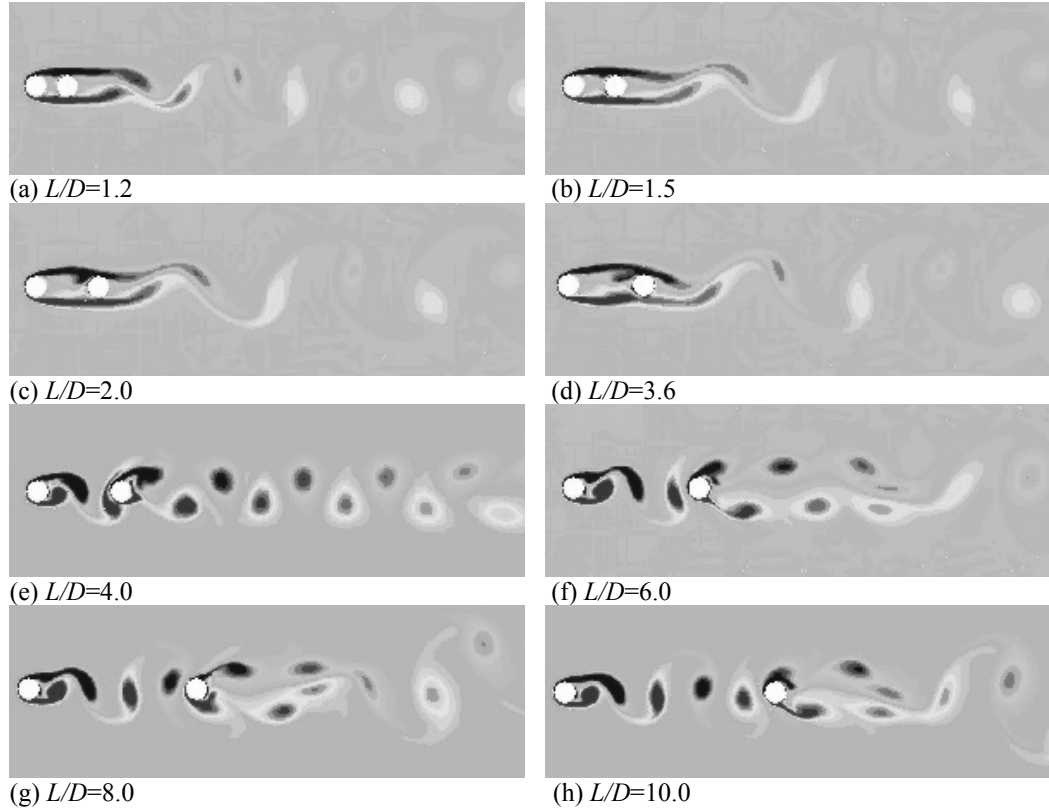


Figure 5. Vorticity contours with different spacing ratios: (a)  $L/D=1.2$ ; (b)  $L/D=1.5$ ; (c)  $L/D=2.0$ ; (d)  $L/D=3.6$ ; (e)  $L/D=4.0$ ; (f)  $L/D=6.0$ ; (g)  $L/D=8.0$  and (h)  $L/D=10.0$ .

#### 4.3 Lift and drag coefficients

Figure 6 shows the mean drag coefficients  $C_D$  for both cylinders as function of the spacing ratio between two cylinders in tandem. As can be seen, the mean drag coefficients  $C_D$  for both cylinders are smaller than that for the single one, while the one for the upstream cylinder is larger than that for the downstream one. The value of  $C_{D1}$  for the upstream cylinder decreases gradually with the increasing spacing ratio, and reaches its minimum value at  $L/D=3.6$ . At larger spacing ratio  $L/D=3.6\sim 4.0$ ,  $C_{D1}$  increases sharply, reaching the peak at  $L/D=4.0$ . Subsequently, it increases to a asymptotic value respectively, which should presumably be that of a single cylinder. For the downstream cylinder, the value of  $C_{D2}$  is much smaller than that of the upstream one. Also, at  $3.6 < L/D < 4$ , the  $C_{D2}$  sharply increases like the behavior of  $C_{D1}$ . As discussed above, this is because of the transition of the flow patterns. In this spacing ratio region, obvious vortex shedding takes place on the first cylinder and strikes on the rear cylinder. These changes are believed as the main reason of the sharp changes in the drag coefficients. At higher spacing ratio as  $L/D > 4$ , the vortex shedding modes become stable which is associated with the plat curves of  $C_{D1}$  and  $C_{D2}$ . Similar performances can also be seen from the relationships of RMS of drag and lift coefficients vs spacing ratios, as shown in Figure 7(a) and (b). There is an obvious changes in the region of  $3.6 < L/D < 4$ , in which the flow patterns transform. These results are other evidences that vortices patterns determine the flow characteristic parameters.

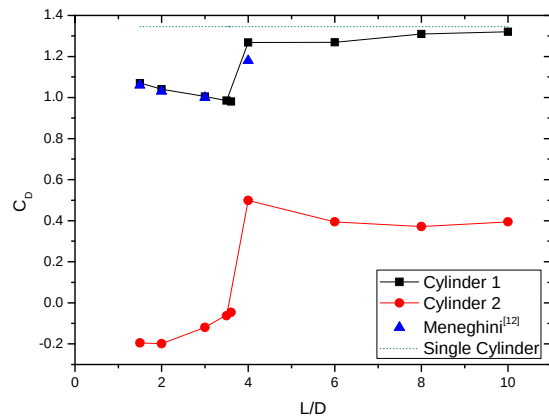
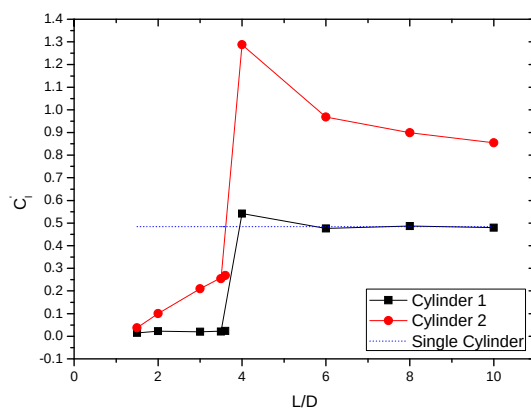
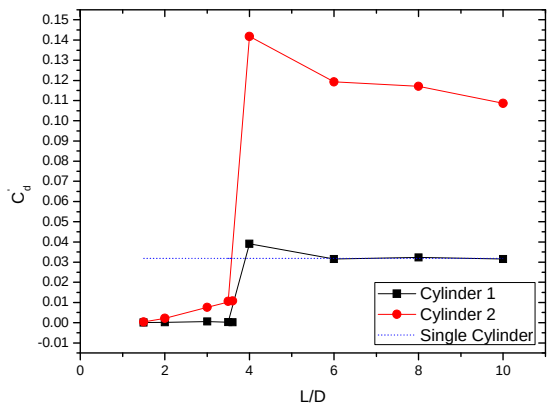


Figure 6. Variation of mean drag coefficients with different spacing ratios.



(a) Figure 7. Variation of (a) the RMS. drag coefficients (b) the RMS. lift coefficients with different spacing ratios

#### 4.4 Strouhal number

Since all the parameters have been non-dimensionalized, the Strouhal number,  $St$ , is equal to the vortex shedding frequency which is obtained through the Fourier transform on the time dependent curve of lift coefficient.

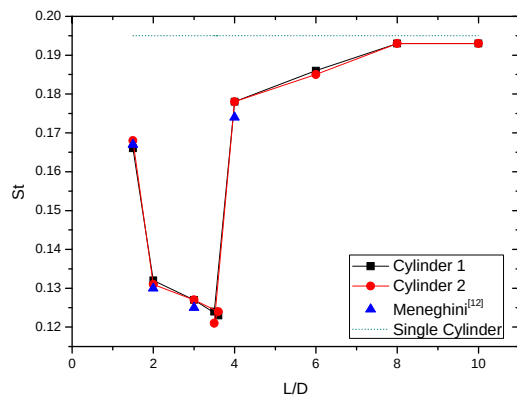


Figure 8. Variation of the Strouhal numbers,  $St$ , with different spacing ratios,  $L/D$

The curves of  $St$  and  $L/D$  for the two cylinders are demonstrated in Figure 8 and they are found to be much closed to the corresponding data from Meneghini *et al.*<sup>12</sup>. The Strouhal number shows a decreasing trend as  $L/D < 3.6$ , and it begins to rapidly increase with higher  $L/D$  value, especially at  $3.6 < L/D < 4.0$ . It is believed that this strong change is mainly caused by the transformation of flow patterns. When  $L/D > 8$ , the Strouhal number becomes an almost constant value around 0.195, approaching the one of the single cylinder case.

## 5 CONCLUSIONS

In this current study, spectral element method is employed to investigate the flow around two tandem circular cylinders at a Reynolds number of 200. The spacing ratios are found to have a great impact on the flow characteristics and flow patterns. A critical spacing ratio of around 3.6 is observed. At smaller spacing ratio as  $L/D < 3.6$ , no obvious vortex shedding is observed around the upstream cylinder and it only occurs on the downstream cylinder. As  $L/D > 4.0$ , vortices are generated around both the front and rear cylinders. Meanwhile, the vortex from the front cylinder strikes on the surface of the rear cylinder, giving rise to higher drag coefficients and larger vortex shedding frequencies. Good agreements are found between the current study and data from existed literatures. The SEM is tested as an effective and reliable approach to cope with the fluid flow problems at low Reynolds numbers.

## 6 ACKNOWLEDGEMENTS

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