

A new state-space model for self-excited forces and straight forward analysis of bridge deck flutter

Zeng-wei Guo^a, Yao-jun Ge^a

^a *State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China*

ABSTRACT: Self-excited forces of bridge can be expressed in state space domain according to the state space description of dynamic system in modern control theory, and its formulation is depended on rational functions for approximating aerodynamic transfer function. In order to discuss time and frequency-domain response characteristics of state space formulation of self-excited force, flutter analysis of a simple supported beam with idealized thin plane section is conducted in frequency and time domain simultaneous, using the Roger's and Karpel's rational function approximations for aerodynamic transfer function. It is observed that frequency-response characteristics of the two formulations are both favorable, although flutter results based on Karpel's rational function approximations are more sensitive to residual of flutter derivatives. Besides, time-response characteristics of the two formulations are similar. As a result of unknown initial conditions of the aerodynamic state, structural response may be erroneous in first time period, which can be solved by restricting the lower limit of the aerodynamic parameters associated with aerodynamic states.

KEYWORDS: bridge, flutter, state-space, self-excited force, time and frequency characteristic.

1 INTRODUCTION

Flutter instability is one of the critical concerns for design and construction of long span bridges since it may lead bridge deck to oscillate divergently at some critical wind speed, and eventually result in a total collapse of structure. Since the collapse of Tacoma Narrow Bridge in 1940, great effort has been made to clarify the generation mechanism of this catastrophic aerodynamic behavior of bridge deck. Scanlan primarily proposed a formulation of self-excited force in terms of structural motions and introduced flutter derivatives to describe its unsteady fluid memory effect (Scanlan and Tomko, 1971). Subsequently, massive flutter analysis procedures through multimode or full mode technique have been developed based on this formulation (Ding et al, 2002b; Dung et al, 1998; Ge and Tanaka, 2000; Jain et al, 1996). These procedures must be conducted in frequency domain, since flutter derivatives obtained from sectional model wind tunnel tests are frequency dependent. However, Structural nonlinearities and turbulence effects, which may be prominent for long span bridges, cannot be considered in the frequency-domain analysis. It is necessary to cast the self-excited forces and governing equations of motion in the time domain to take account for structural nonlinearities and turbulence effects in flutter analysis. Corresponding to flutter derivatives, time-domain self-excited forces were expressed in terms of convolution integrals of impulse functions or indicial functions to take into account the effect that self-excited force variation lags behind structural motions. Most researchers (Caracoglia and Jones, 2003; Chen and Kareem, 2002; Costa and Borri, 2006; Wilde and Fujino, 1998) have dealt with convolution integrals of impulse or indicial functions using the following two methods: (1) adopting recursive algorithm similar to the famous Newmark- β method, (2) introducing additional aerodynamic state to represent this convolution integral and proposing state-space formulations.

The assessment of self-excited force in the time domain requires identification of aerodynamic impulse or indicial response functions. However, there are no theoretical solutions of the-

se functions for bluff bridge decks and it is also difficult to determinate these functions directly in wind tunnel tests. In aerospace engineering, Roger approximated the frequency-dependent Theodorsen function in terms of rational functions, and transformed the frequency-dependent aeroelastic forces of airplane into time domain based on the inverse Laplace transform. Analogously, these aerodynamic impulse or indicial response functions for bluff bridge sections can be determined by means of the inverse Laplace transform of Scanlan's formulation, if flutter derivatives can be approximated with continuous functions. Lin introduced rational functions for approximating flutter derivatives in bridge engineering (Lin and Yang, 1983), and there were mainly two formulations for these rational functions, which were noted as Roger's formulation and Karpel's formulation. However, time and frequency response characteristics of self-excited force state space model, which may influence the flutter analysis result both in frequency and time domain, have never been discussed in literatures.

The present paper derives the governing equations of flutter in state space domain based Roger's and Karpel's rational function approximations, and discusses their differences. Taking a simple supported beam with idealized thin plane section as example, we clarify the time and frequency response characteristics of these two state space model of self-excited force.

2 STATE-SPACE FORMULATIONS OF SELF-EXCITED FORCE

For complex sinusoidal motions with circular frequency ω , the self-excited forces per unit length are commonly expressed in terms of flutter derivatives as (Scanlan, 2001):

$$L = \frac{1}{2} \rho U^2 B \left(KH_1^* \frac{\dot{h}}{U} + KH_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} + KH_5^* \frac{\dot{p}}{U} + K^2 H_6^* \frac{p}{B} \right) \quad (1a)$$

$$D = \frac{1}{2} \rho U^2 B \left(KP_1^* \frac{\dot{p}}{U} + KP_2^* \frac{B\dot{\alpha}}{U} + K^2 P_3^* \alpha + K^2 P_4^* \frac{p}{B} + KP_5^* \frac{\dot{h}}{U} + K^2 P_6^* \frac{h}{B} \right) \quad (1b)$$

$$M = \frac{1}{2} \rho U^2 B^2 \left(KA_1^* \frac{\dot{h}}{U} + KA_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B} + KA_5^* \frac{\dot{p}}{U} + K^2 A_6^* \frac{p}{B} \right) \quad (1c)$$

where ρ =air mass density; U =wind velocity; B = bridge deck width; $K=\omega B/U$ =reduced frequency; $H^* i, A^* i, P^* i (i=1,2,\dots,6)$ =frequency dependent flutter derivative; h, p, α =vertical, lateral and pitching displacement respectively, and the head dot denotes partial differentiation with respect to time t .

By taking the Laplace transform of Eq. (1), the self-excited force formulated utilizing flutter derivatives can be given in a matrix form by

$$\tilde{\mathbf{F}}_{se} = \frac{1}{2} \rho U^2 \mathbf{b}^T \mathbf{Q} \mathbf{b} \tilde{\mathbf{\Lambda}} \quad (2)$$

$$\text{where } \tilde{\mathbf{F}}_{se} = \begin{Bmatrix} \tilde{L} \\ \tilde{D} \\ \tilde{M} \end{Bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & B \end{bmatrix}; \quad \tilde{\mathbf{\Lambda}} = \begin{Bmatrix} \tilde{h} \\ \tilde{p} \\ \tilde{\alpha} \end{Bmatrix}; \quad \mathbf{Q} = \begin{bmatrix} K^2 H_4^* + pKH_1^* & K^2 H_6^* + pKH_5^* & K^2 H_3^* + pKH_2^* \\ K^2 P_6^* + pKP_5^* & K^2 P_4^* + pKP_1^* & K^2 P_3^* + pKP_2^* \\ K^2 A_4^* + pKA_1^* & K^2 A_6^* + pKA_5^* & K^2 A_3^* + pKA_2^* \end{bmatrix}$$

=aerodynamic transfer function; $p=sB/U$, $s=(\zeta+i)\omega$, ζ =damping ratio; the superscript indicates the Laplace transformation superscripts.

Approximation of unsteady aerodynamics forces as continuous rational functions allows self-excited forces to be cast in state space realization. The most common forms of these approximating functions used currently in bridge engineering for aerodynamic transfer matrix $\mathbf{Q}$ are (Chen and Kareem, 2002; Wilde and Fujino, 1998):

$$\hat{\mathbf{Q}} = \mathbf{A}_1 + \mathbf{A}_2 p + \mathbf{A}_3 p^2 + \sum_{l=1}^m \frac{\mathbf{A}_{l+3} p}{p + \lambda_l} \quad (\text{Roger's formulation}) \quad (3a)$$

$$\hat{\mathbf{Q}} = \mathbf{A}_1 + \mathbf{A}_2 p + \mathbf{A}_3 p^2 + \mathbf{D}(p\mathbf{I} + \mathbf{\Lambda})^{-1} \mathbf{E} p \quad (\text{Karpel's formulation}) \quad (3b)$$

Where $\mathbf{Q}$ =rational function approximation of aerodynamic transfer function $\mathbf{Q}$; $\mathbf{A}_1, \dots, \mathbf{A}_{l+3}$, $\mathbf{D}$, $\mathbf{E}$, $\mathbf{\Lambda}$, $\lambda_1, \dots, \lambda_m$ =frequency-independent ration function coefficients; m indicates the number of the partial fractions.

By defining the partial fractions as follows:

$$\tilde{\Phi}_{se,l} = \frac{\mathbf{b}^T \mathbf{A}_{l+3} \mathbf{b}}{p + \lambda_l} (p\tilde{\mathbf{\Lambda}}) \quad (\text{Roger's formulation}) \quad (4a)$$

$$\tilde{\Phi}_{se} = (p\mathbf{I} + \mathbf{\Lambda})^{-1} \mathbf{E} \mathbf{b} (p\tilde{\mathbf{\Lambda}}) \quad (\text{Karpel's formulation}) \quad (4b)$$

and substituting Eq. (3) and (4) into Eq. (2), subsequently inverse Laplace transform of Eq. (4), the following state space formulation of self-excited forces can be obtained:

$$\begin{Bmatrix} \mathbf{F}_{se} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} = \frac{1}{2} \rho U^2 \left(\begin{bmatrix} \mathbf{b}^T \mathbf{A}_1 \mathbf{b} & \frac{B}{U} \mathbf{b}^T \mathbf{A}_2 \mathbf{b} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\ & & \frac{\lambda_1 U}{B} \mathbf{I} & & \\ & & & \ddots & \\ & & & & \frac{\lambda_m U}{B} \mathbf{I} \end{bmatrix} \mathbf{X}_R + \begin{bmatrix} \mathbf{0} & (\frac{B}{U})^2 \mathbf{b}^T \mathbf{A}_3 \mathbf{b} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{b}^T \mathbf{A}_4 \mathbf{b} & & \mathbf{I} & & \\ \vdots & & & \ddots & \\ -\mathbf{b}^T \mathbf{A}_{l+3} \mathbf{b} & & & & \mathbf{I} \end{bmatrix} \dot{\mathbf{X}}_R \right) \quad (\text{Roger's formulation}) \quad (5a)$$

$$\begin{Bmatrix} \mathbf{F}_{se} \\ \mathbf{0} \end{Bmatrix} = \frac{1}{2} \rho U^2 \left(\begin{bmatrix} \mathbf{b}^T \mathbf{A}_1 \mathbf{b} & \frac{B}{U} \mathbf{b}^T \mathbf{A}_2 \mathbf{b} & \mathbf{b}^T \mathbf{D} \\ \mathbf{0} & \mathbf{0} & \frac{U}{B} \mathbf{\Lambda} \end{bmatrix} \mathbf{X}_K + \begin{bmatrix} \mathbf{0} & (\frac{B}{U})^2 \mathbf{b}^T \mathbf{A}_3 \mathbf{b} & \mathbf{0} \\ -\mathbf{E} \mathbf{b} & \mathbf{0} & \mathbf{I} \end{bmatrix} \dot{\mathbf{X}}_K \right) \quad (\text{Karpel's formulation}) \quad (5b)$$

Additional aerodynamic states are introduced to express the aerodynamic phase lag behind structural motions. Obviously, addition of each partial fraction for approximating flutter derivatives may result in an additional new aerodynamic state in the state space equations of bridge deck flutter. Since there are three nonzero components of each nodal self-excited force vector, the number of additional aerodynamic state may be $3m$ and m for Roger's formulation and Karpel's formulation respectively. For Roger's formulation, variance of each aerodynamic state influences only one self-excited force component, and each self-excited force component is determined by only m independent aerodynamic states. However, for Karpel's formulation, variance of each aerodynamic state may influence all self-excited force components, and each self-excited force component may be influenced by all aerodynamic states.

3 FLUTTER ANALYSIS BY STATE SPACE APPROACH

The governing equations of flutter of a bridge deck in the smooth flow are given in matrix form as:

$$\mathbf{M}\ddot{\mathbf{Y}} + \mathbf{C}\dot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = \mathbf{F}_{se} \quad (6)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ =mass, damping, stiffness matrices, respectively; $\mathbf{Y}$ =nodal displacement vector; $\mathbf{F}_{se}$ = nodal self-excited force vector.

Substituting Eq. (5) into Eq. (6) yields the following state space equations in flutter analysis

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} \quad (7)$$

where $\mathbf{A}$ =system matrix, for Roger's formulation, $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} & \frac{1}{2}\rho U^2 \bar{\mathbf{M}}^{-1} & \dots & \frac{1}{2}\rho U^2 \bar{\mathbf{M}}^{-1} \\ \mathbf{0} & \mathbf{b}^T \mathbf{A}_4 \mathbf{b} & \frac{\lambda_1 U}{B} \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{b}^T \mathbf{A}_{m+3} \mathbf{b} & \mathbf{0} & \dots & \frac{\lambda_m U}{B} \mathbf{I} \end{bmatrix},$

and for Karpel's formulation of approximating flutter derivatives
$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} & \frac{1}{2}\rho U^2 \bar{\mathbf{M}}^{-1} \mathbf{b}^T \mathbf{D} \\ \mathbf{0} & \mathbf{E} \mathbf{b} & -\frac{U}{B} \Lambda \end{bmatrix};$$

$$\mathbf{X} = \{\mathbf{Y} \quad \dot{\mathbf{Y}} \quad \mathbf{X}_{se}\}^T; \quad \bar{\mathbf{K}} = \mathbf{K} - \frac{1}{2}\rho U^2 \mathbf{b}^T \mathbf{A}_1 \mathbf{b}; \quad \bar{\mathbf{C}} = \mathbf{C} - \frac{1}{2}\rho U B \mathbf{b}^T \mathbf{A}_2 \mathbf{b}; \quad \bar{\mathbf{M}} = \mathbf{M} - \frac{1}{2}\rho B^2 \mathbf{b}^T \mathbf{A}_3 \mathbf{b}.$$

Flutter analysis can be conducted in both frequency and time domain using this preceding frequency independent state space equation.

3.1 Flutter analysis in frequency domain

Flutter analysis in frequency domain can be converted to be an algebraic eigenvalue problem, since real part of each complex eigenvalues of system matrix $\mathbf{A}$ indicates the logarithm decay rates of corresponding modal of structure, which represents its dynamic stability. As the system matrix $\mathbf{A}$ is frequency independent and parameterized only by wind velocity, complex analysis can be carried out straightforward without frequency iterative to determine the critical flutter wind speed.

The diagonal elements of system matrix $\mathbf{A}$ determine the behavior of structural states and aerodynamic states, and the non-diagonal elements represent coupling effects between structural states and aerodynamic states. If wind speed is not so high, the coupling effects between structure and fluid may be not significant, the non-diagonal terms of system matrix $\mathbf{A}$ are close to zero and $\mathbf{A}$ can be appropriately considered to a diagonal matrix. Therefore, eigenvalues of $\mathbf{A}$ generally consist of $2n$ complex-conjugate pairs of eigenvalues corresponding to structure itself and m real values corresponding to aerodynamic states, since structural governing equations are second order differential equations, while governing equations of aerodynamic states are of first order. However, if wind speed is close to critical flutter wind velocity, aeroelastic effects may be significant and the non-diagonal elements are far from zero, so some eigenvalues corresponding to aerodynamic states may become complex, which may be mixed up with structural eigenvalues. Therefore, the inherent eigenvalues corresponding to structure should be sorted out in flutter analysis in frequency domain (Xie and Xiang, 1985).

We identify structural inherent eigenvalues based on the fact that natural mode shape of structures may not change a lot at different wind velocities, especially at two adjacent wind speeds. The procedures of identification of structural inherent eigenvalues at a certain wind speed can be summarized as follows:

(1) Conduct structural damped eigenvalue analysis at zero wind speed, and sort the eigenvalues and their corresponding eigenvectors according to the image part of eigenvalues in descending order, and then store the first n complex eigenvalues Λ_0 and their corresponding eigenvectors Φ_0 as structural inherent eigenvalues and eigenvectors.

(2) Assume a wind speed increment ΔU , and set current wind speed $U_i = i \cdot \Delta U$ ($i=1, \dots, n$).

(3) Carry out damped eigenvalue analysis at current wind speed U_i , and hunt for the first n natural eigenvectors of structure in sequence according to the sorted structural eigenvectors Φ_{i-1} using an evaluating indicator of mode similarity MSI , which is defined as

$$MSI = \frac{(\Theta_{j,i}^T \Phi_{j,i-1})^2}{(\Theta_{j,i}^T \Theta_{j,i})(\Phi_{j,i-1}^T \Phi_{j,i-1})} \quad (8)$$

Where $\Phi_{j,i-1}$ =the j th column of sorted eigenvectors Φ_{i-1} at wind speed U_{i-1} , $\Theta_{j,i}$ =the j th column of unsorted eigenvectors Φ_{i-1} at wind speed U_i , and the superscript T indicates vectors or matrix transpose.

(4) Sort the identified structural eigenvalues and their corresponding vectors according to Φ_{i-1} at wind speed U_{i-1} , and store the structural eigenvalues Λ_i and eigenvectors Φ_i at current wind speed.

If real parts of structural eigenvalues are all negative bridge will be dynamically stable, and if real part of any structural eigenvalue is positive bridge will be dynamically unstable. So the critical flutter wind speed can be defined as the wind velocity at which bridge system has only one eigenvalue λ with zero real part, and the imaginary part of this complex eigenvalue λ is flutter circular frequency.

3.2 Flutter analysis in time domain

Flutter analysis can also be conducted in time domain, in which structural and aerodynamic nonlinearities may be incorporated. In the present paper, buffeting force caused by the turbulence existing inherently in natural wind in conjunction with the structural-induced signature turbulence is not considered, and it is taken for granted that governing equation of bridge deck flutter (Eq. (6)) is homogeneous. Free vibration response to a certain initial condition of structure can be readily obtained by any time integration scheme, and subsequently damping ratio can be determined. Flutter instability of bridge deck will be happened when free vibration amplitude is constant or even increasing with time. The initial condition of structure should be given according to its flutter mode shape in order to seek out the critical flutter wind speed more precisely, with the reason that free vibration amplitude might be still decay with time at some wind speed a little larger than critical flutter wind speed with improper initial condition.

4 CASE STUDY

In this section, a simply supported line-like bridge with thin-airfoil cross section is taken as a case study. Since theoretical solution of flutter frequency and flutter wind velocity for this structure is available, this example can be used to verify the state space approach of flutter analysis and investigate the time and frequency domain response characteristics of two state-space formulations of self-excited force. The structural parameters are as follows: span $l=300\text{m}$; width of the bridge deck $B=40\text{m}$; vertically bending rigidity $EI_z=2.1 \times 10^6 \text{MPam}^4$; laterally bending rigidity $EI_y=1.8 \times 10^7 \text{MPam}^4$; torsional rigidity $GI_t=4.1 \times 10^5 \text{MPam}^4$; mass $m=20,000\text{kg/m}$; mass moment of inertia $I_m=4.5 \times 10^6 \text{kgm}^2/\text{m}$ and air mass density $\rho=1.248\text{kg/m}^3$. The structural damping is not considered in this example to make a theoretical comparison.

Theodorsen functions have been adopted for expressing self-excited forces acting on this simply supported bridge with thin-airfoil cross section, and it should be approximated by rational function firstly before flutter analysis with state space approach. The aerodynamic coefficients in rational function approximation are obtained by minimizing the optimization target J , which is defined as:

$$J(A_i, \lambda_i) = \sum_{i=1}^N w_i \left\| \mathbf{Q}_{\text{Re}}(\nu_i) - \hat{\mathbf{Q}}_{\text{Re}}(\nu_i) \right\|_F + \sum_{i=1}^N w_i \left\| \mathbf{Q}_{\text{Im}}(\nu_i) - \hat{\mathbf{Q}}_{\text{Im}}(\nu_i) \right\|_F \quad (9)$$

where $\mathbf{Q}_{\text{Re}} = \begin{bmatrix} H_4^* & H_3^* \\ A_4^* & A_3^* \end{bmatrix}$, $\mathbf{Q}_{\text{Im}} = \begin{bmatrix} H_1^* & H_2^* \\ A_1^* & A_2^* \end{bmatrix}$ = real and image part of aerodynamic transfer function,

respectively ; $\hat{\mathbf{Q}}_{\text{Re}} = \left(\frac{\nu}{2\pi} \right)^2 \mathbf{A}_1 - \mathbf{A}_3 + \sum_{l=1}^m \mathbf{A}_{l+3} \frac{\nu^2}{\lambda_l^2 \nu^2 + 4\pi^2}$, $\hat{\mathbf{Q}}_{\text{Im}} = \frac{\nu}{2\pi} \mathbf{A}_2 + \sum_{l=1}^m \mathbf{A}_{l+3} \frac{\lambda_l \nu^3}{2\pi \lambda_l^2 \nu^2 + 8\pi^3}$ = real and image part of Roger's rational function approximation of aerodynamic transfer function, respectively; $\hat{\mathbf{Q}}_{\text{Re}} = \left(\frac{\nu}{2\pi} \right)^2 \mathbf{A}_1 - \mathbf{A}_3 + \nu^2 \mathbf{D} (4\pi^2 \mathbf{I} + \Lambda^2 \nu^2)^{-1} \mathbf{E}$, $\hat{\mathbf{Q}}_{\text{Im}} = \frac{\nu}{2\pi} \mathbf{A}_2 + \nu^3 \mathbf{D} (8\pi^3 \mathbf{I} + 2\pi \nu^2 \Lambda^2)^{-1} \Lambda \mathbf{E}$ = real and

$$\text{ly}; \hat{\mathbf{Q}}_{\text{Re}} = \left(\frac{\nu}{2\pi} \right)^2 \mathbf{A}_1 - \mathbf{A}_3 + \nu^2 \mathbf{D} (4\pi^2 \mathbf{I} + \Lambda^2 \nu^2)^{-1} \mathbf{E}, \hat{\mathbf{Q}}_{\text{Im}} = \frac{\nu}{2\pi} \mathbf{A}_2 + \nu^3 \mathbf{D} (8\pi^3 \mathbf{I} + 2\pi \nu^2 \Lambda^2)^{-1} \Lambda \mathbf{E} = \text{real and image}$$

part of Karpel's rational function approximation of aerodynamic transfer function, respectively; M = the number of elements in aerodynamic matrix $\mathbf{Q}$; n = the number of reduced wind velocity; w_i = weight factor; $\|\cdot\|_F = F$ norm of matrix.

Simulated annealing algorithm is applied to solve this optimization problem, and the rational function coefficients of two formulations extracted by approximating Theodorsen functions with reduced wind velocity ranging from 0 to 20 are listed in Table 1.

Table 1 Rational function coefficients extracted by approximating Theodorsen functions

	Roger's formulation				Karpel's formulation			
	Group I		Group II		Group I		Group II	
Residual J	1.51×10^{-4}		6.1×10^{-3}		9.44×10^{-4}		7.9×10^{-3}	
$\mathbf{A}_1$	-3.54×10^{-9}	-6.2832	0.0228	-8.6669	-0.0033	-6.3451	0.0175	-8.6613
	7.12×10^{-10}	1.5708	-0.0057	2.1667	-1.67×10^{-5}	1.5756	-0.0045	2.1539
$\mathbf{A}_2$	-3.1416	-2.3562	-3.1783	-2.3563	-3.1516	-2.3498	-3.1797	-2.3545
	0.7854	-0.1963	0.7946	-0.1963	0.7789	-0.1915	0.7930	-0.1953
$\mathbf{A}_3$	-1.5708	-6.04×10^{-9}	-1.5625	-0.0085	-1.5682	-0.0030	-1.5621	-0.0090
	-1.23×10^{-9}	-0.0491	-0.0021	-0.0470	0.0031	-0.0530	-0.0014	-0.0480
$\mathbf{A}_4$ (Roger)	-1.2629	1.7891	-1.2390	1.8730	-5.0253	2.5375	-4.9185	2.5645
$\mathbf{D}$ (Karpel)	0.3157	-0.4473	0.3098	-0.4683	1.2911	-0.6000	1.2368	-0.6316
$\mathbf{A}_5$ (Roger)	-0.0943	1.0131	-0.0863	3.2749	0.2499	-0.3571	0.2517	-0.3805
$\mathbf{E}$ (Karpel)	0.0236	-0.2533	0.0216	-0.8187	-0.0333	0.4162	-0.0314	1.2746
Λ	0.6	0.091	0.5638	0.0200	0.5920	0.0830	0.5630	0.0200

Figure 1 gives the comparison of flutter derivatives based on Theodorsen function and its rational function approximation results. It is shown that four groups of aerodynamic coefficients in table 1 can all accord well with flutter derivatives.

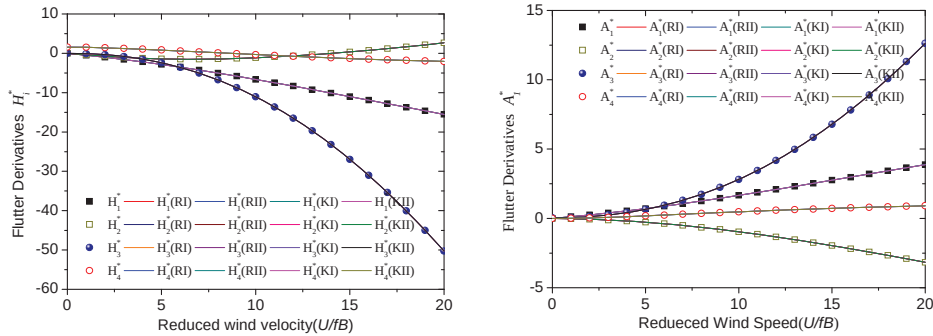


Figure 1 Rational function approximation of flutter derivatives

Applying the aerodynamic coefficients listed in Table 1 and following the identification procedures of structural inherent eigenvalues described in section 3.1, damped complex eigenvalue analysis is conducted under wind velocities from 0 to 150m/s. The critical flutter wind speed and flutter frequencies of this simple supported beam with four groups of rational function coefficients are given in Table 2.

Table 2 Flutter analysis results of the simply supported beam with four groups of rational function coefficients

	Theoretical solution	Flutter derivatives	Roger I	Roger II	Karpel I	Karpel II
Critical flutter speed(m/s)	139.90	139.79	139.79	139.72	139.86	139.75
Flutter frequency (Hz)	0.3801	0.3800	0.3800	0.3802	0.3799	0.3802

Apparently, critical flutter wind speed and flutter frequency using state space approach with four groups of coefficients are almost the same as the theoretical solution and full order solution (Ding et al, 2002a) directly using flutter derivatives. Choosing group I coefficients of Roger's formulation as an example, the first 10 conjugate pairs of structural complex eigenvalues and complex eigenvectors are also obtained, and the variation of these complex eigenvalues versus wind velocity is plotted in Fig. 1. It is observed that (i) the vibration frequencies of modes except for the first torsional and vertical modes all alert little or even remain unchanged with the increase in wind velocity; (ii) for the first vertical and torsional mode, the frequency increases and decreases in the wind velocity range, respectively; (iii) the vibration damping of first torsional mode firstly increases and then decreases with increasing wind velocities, while damping of the other modes with increasing wind velocities. As shown in Fig. 1(b), the damping of first torsional mode becomes zero at a wind velocity of 139.79 m/s, and the corresponding torsional frequency becomes 0.3940 Hz, namely flutter frequency.

4.1 Frequency-domain response characteristics of two formulations

As it is still difficult to identify aerodynamic coefficients in rational function approximation of aerodynamic transfer function of self-excited forces directly in wind tunnel test, these aerodynamic coefficients are usually obtained by fitting flutter derivatives. However, this rational function approximation of aerodynamic transfer function cannot fit flutter derivatives without residual. Precision of flutter analysis results may be determined by frequency response characteristics of two formulations and fitting residual. In order to investigate frequency response characteristics of two formulations, we chose 22 groups of aerodynamic coefficients for two formulations with fitting residual ranging from 1.0×10^{-3} to 10.0 to conduct flutter analysis in frequency domain by state-space approach. For each group of aerodynamic coefficients, the initial iterative values and stopping criteria are identical and ultimate residual is in the same level for both two state-space formulations to confirm that the difference between flutter analyses results based on two formulations is not induced by self-difference of aerodynamic coefficients. Fig. 3 gives the variation of flutter analysis results to approximation residual of flutter derivatives, in which the relative errors of critical flutter wind speed and flutter frequencies are compared with that of full order solution directly using flutter derivatives, not with theoretical solution. It is observed that (i) relative errors of critical flutter wind speed and flutter frequency for both two formulations are no larger than 10% even for approximation residual larger than 10.0; (ii) the flutter analysis results based on Karpel's formulation are more sensitive to approximation errors than that of Roger's formulation although the number of additional aerodynamic states introduced in Karpel's formulation is half of Roger's formulation.

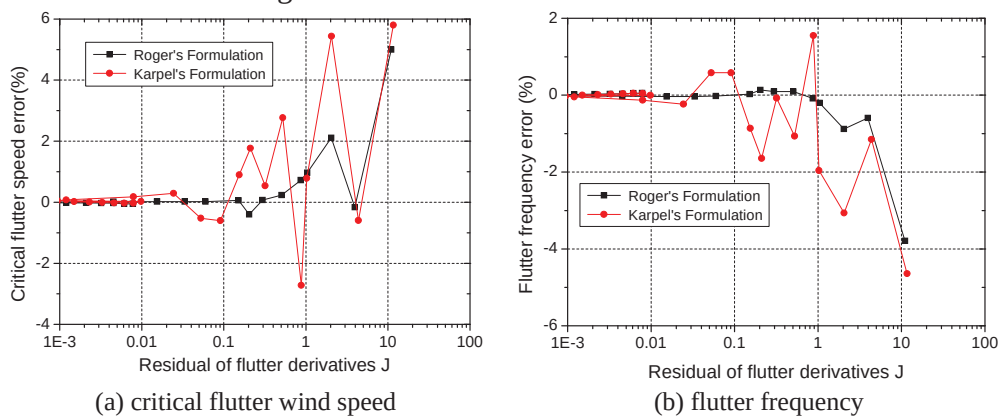


Figure 3 Variation of flutter analysis results to approximation residual of flutter derivatives

4.2 Time-domain response characteristics of two formulations

It is necessary to assume an initial condition of flutter governing equation in smooth wind flow before conducting flutter analysis in time domain. However, it is difficult to determine initial condition of aerodynamic states corresponding to the given initial condition of structural states. With these factitious initial conditions, structural responses may be erroneous in the first time period, although this incorrect response will gradually converge to physical solution with the prolonging of calculating time. Hence, it is essential to investigate the time response characteristic of two state-space formulations of self-excited forces.

In order to study time response characteristics of two self-excited force formulations, time-domain analyses of this simple supported beam have been carried out using the four groups of aerodynamic coefficients listed in table 1, respectively. For a given velocity, the time-domain structural response subjected to an initial excitation can be readily computed using a high precision direct integration time integration scheme. For convenience of comparison, time step size is assumed as 0.1s, while torsional and vertical displacement with value of 1deg and 0.01m respectively are employed as the initial conditions and applied to every node on the beam, and initial conditions of aerodynamic states are set as zero.

Under respective critical wind velocity of four groups of aerodynamic coefficients, the response time histories of mid-point of this simple supported beam are shown in Fig. 4. It is observed that time-domain response characteristics of two state-space formulations of self-excited force are almost identical and structural responses are incorrect in a long period time corresponding to the second group of coefficients for both state-space formulations. It can be noticed that the two groups of aerodynamic coefficients are basically the same except for those related to aerodynamic states no matter for Roger's formulation and Karple's formulation. Therefore, those coefficients related to aerodynamic states may determine the time-domain response characteristics of state-space formulations of self-excited forces.

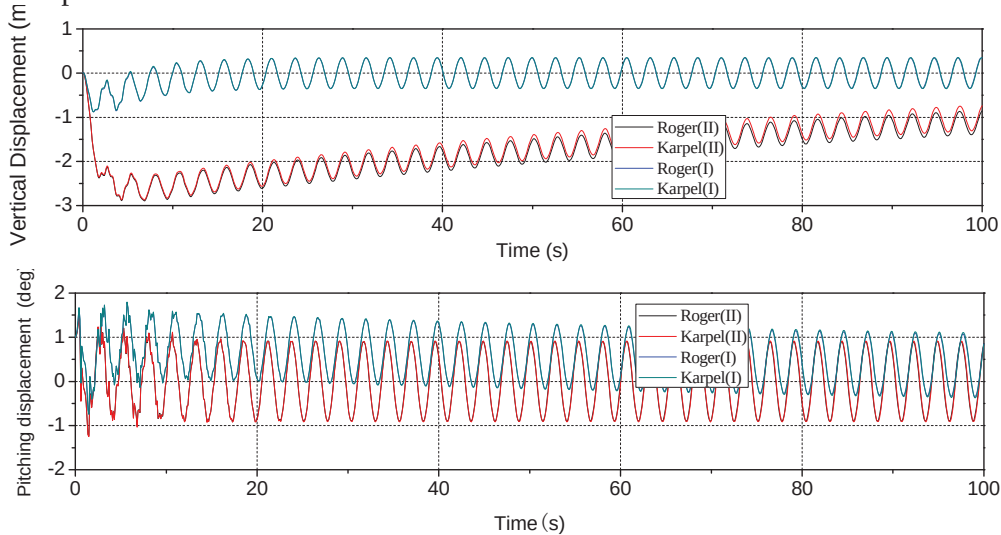


Figure 4 Time history of displacements at center span of simple supported beam

This incorrect response in first time period is mainly caused by improper initial conditions of aerodynamic states. Since these initial conditions of aerodynamic states cannot be determined in advance, it is alternative to make this incorrect response converge to physical solution as soon as possible. As the time-domain response characteristics of these two state-space formulations of self-excited force are almost identical and mainly determined by the characteristic of aerodynamic states, we choose aerodynamic states in Roger's formulation to cast around for solutions to make this initial incorrect response attenuate as soon as possible.

In the critical state of flutter, the motion of bridge deck can be viewed as a simple harmonic motion. Based on this assumption and by taking inverse Laplace transform of Eq. (4a), the aerodynamic state related to self-excited force component F caused by bridge deck motion Δ can be expressed as:

$$X_{sel}^{F\Delta} = A_{t+3}^{F\Delta} \int_{-\infty}^t e^{-\frac{\lambda_l U}{B}(t-\tau)} \omega \cos(\omega \tau) d\tau = \frac{A_{t+3}^{F\Delta} \omega e^{-\frac{\lambda_l U}{B}t}}{\left(\frac{\lambda_l U}{B}\right)^2 + \omega^2} \left[e^{\frac{\lambda_l U}{B}\tau} \left(\omega \sin \omega \tau + \frac{\lambda_l U}{B} \cos \omega \tau \right) \right]_{-\infty}^t = \frac{A_{t+3}^{F\Delta}}{\sqrt{(\lambda_l/K)^2 + 1}} \sin(\omega t + \theta) \quad (10)$$

where $\theta = \arctan(\lambda_l/K)$. However, the lower limit of this convolution integral must be larger than zero in calculation, namely convolution integral in Eq. (10) is forced to be expressed as:

$$X_{sel,0}^{F\Delta} = A_{t+3}^{F\Delta} \frac{\omega e^{-\frac{\lambda_l U}{B}t}}{\left(\frac{\lambda_l U}{B}\right)^2 + \omega^2} \left[e^{\frac{\lambda_l U}{B}\tau} \left(\omega \sin \omega \tau + \frac{\lambda_l U}{B} \cos \omega \tau \right) \right]_0^t \quad (11)$$

$$= A_{t+3}^{F\Delta} \frac{1}{\sqrt{(\lambda_l/K)^2 + 1}} \sin(\omega t + \theta) - A_{t+3}^{F\Delta} \frac{\lambda_l/K}{(\lambda_l/K)^2 + 1} e^{-\frac{\lambda_l}{K} \frac{2\pi t}{T}} = X_{sel}^{F\Delta} + X_{sel,a}^{F\Delta}$$

where $T=1/2\pi\omega$. Obviously, an additional term is introduced if this convolution integral is calculated from zero, and it can be viewed as the initial conditions of aerodynamic states. In flutter analysis, this additional term of aerodynamic state is usually unknown in advance, and it will cause the incorrect structural response. Fortunately, if λ_l is larger than zero this additional term will decay exponentially with time at a rate depending on λ_l/K and ω . The incorrect response may attenuate more quickly by increasing λ_l/K and ω . When this addition term is smaller than five percent of aerodynamic state response, the effect can be neglected. Therefore, the influence of additional term on aerodynamic state response will be smaller than 5% if Eq. (13) is satisfied,

$$\frac{X_{sel,a}^{F\Delta}}{X_{sel}^{F\Delta}} = \frac{\frac{\lambda_l/K}{(\lambda_l/K)^2 + 1} e^{-\frac{\lambda_l}{K} \frac{2\pi t}{T}}}{\frac{1}{\sqrt{(\lambda_l/K)^2 + 1}}} < \frac{\lambda_l/K}{(\lambda_l/K)^2 + 1} e^{-\frac{\lambda_l}{K} \frac{2\pi t}{T}} < \frac{1}{2} e^{-\frac{\lambda_l}{K} \frac{2\pi t}{T}} < 5\% \quad (12)$$

which demands that λ_l is larger than $0.366K/n$, and n indicates the number cycles.

5 CONCLUSION

The self-excited force of bridge deck can be described in state space domain by using rational function approximations of unsteady aerodynamic forces. This state space equation of bridge deck motion makes flutter analysis straightforward and simplifies the design of efficient vibration suppression systems. The present paper mainly focuses on the time and frequency-domain response characteristics of Roger's and Karpel's state-space formulations of self-excited force and discusses detailed operation techniques of flutter analysis by state-space approach in frequency and time domain. Flutter analysis of a simple supported beam bridge with idealized thin plane section has been conducted as a case study. The concluding remarks can be summarized as follows:

(1) The additional aerodynamic states introduced in Roger's state-space formulation are twice than that of Karpel's formulation.

(2) Frequency-domain response characteristics of two state-space formulations are both favorable, although flutter results based on Karpel's formulation are more sensitive to residual of flutter derivatives than Roger's formulation.

(3) Time-domain response characteristics of two state-space formulations are almost identical; with restricting the lower limit of the coefficients related to aerodynamic states, the time period of incorrect transient response of structure can be shorten.

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