

# Numerical estimation of added mass for circular membrane vibrating in still air

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**ABSTRACT:** The added mass has significant effect on the vibration of the membrane structures, which depends on the mode shapes and can not be ignored during their vibration analysis. In this paper, the added mass of the circular membrane is investigated by the proposed numerical analysis method. Using the Realizable  $k-\varepsilon$  and RNG  $k-\varepsilon$  model of the FLUENT commercial computational Codes associated with the Dynamic-mesh technique to simulated the air loading on the circular membrane and using FEM to obtain the dynamic matrix equation for the circular membrane, the natural frequencies in consequence of considering the effect of the surrounding air is observed. An existing test result is employed to verify the proposed method. The problem presented herein may be considered as the air-structure coupled problem.

**KEYWORDS:** added mass, circular membrane, numerical estimation

## 1 INTRODUCTION

When structures vibrate in a certain kind of fluid, part of the surrounding fluid will be invoked and will vibrate together with structures; this effect is called added mass. Generally, the added mass of building structures vibrating in air, such as steel structures or concrete structures, can be ignored, because their material densities are much greater than that of air. However, the added mass has a significant influence on the vibration of membrane structures due to the light weight of their material.

Actually, the added mass of typical objects, such as cylinders and spheres, moving in fluid with acceleration has been widely investigated. Currently, research on the added mass of membranes is still limited. A study by Minami [1] indicated that the added mass has no relationship to the vibration frequency and amplitude but only to the air density and the dimension of the membrane. The added mass is equivalent to the air uniformly distributed on the membrane with an estimated height equal to 68% of the length of the membrane. Irwin and Wardlaw [2] presented an empirical equation for estimating the added mass of the Montreal Stadium. Li, et al [3] proposed a simplified added mass model which can take into account complicated vibration mode shape of flat membranes, i.e., the added mass above each vibration region is equal to the uniformly distributed air with height of  $0.65l$ , in which  $l$  is the diameter of the inscribed circle of the region. The vibration regions are separated by the boundary lines which are made up of the points at which the displacement in the mode shape is zero.

A numerical method for added masses of the circular membrane vibrating in still air is developed by using the FLUENT commercial computational Codes in this paper with the Realizable  $k-\varepsilon$  and RNG  $k-\varepsilon$  model associated with the Dynamic-mesh technique. The vibration of the circular membrane can be simulated and the air loading on the circular membrane can be obtained. Based on the concept of kinetic energy, the added mass of the circular membrane can be derived out. The dynamic matrix equation for the structure is obtained by means of FEM and the natural frequencies in consequence of considering the

effect of the surrounding air is observed. Finally, an existing test result is employed to validate the efficiency of the proposed method.

## 2 DETERMINATION OF THE ADDED MASS

In the case of an incompressible inviscid stationary fluid, the change rate in time of kinetic energy of any part of the fluid is equal to the work done by the pressures on its surface.

$$\frac{dE}{dt} = - \int_A V_N \Delta p dA \quad (1)$$

where  $E$  is the kinetic energy,  $A$  is the surface bounding the fluid region and  $V_N$  denotes the velocity of the fluid particle in the direction of the normal  $N$ . The last integral thus expresses the rate at which the pressure  $\Delta p$ ; exerted from outside an element of the boundary, is doing work. Assuming the fluid-solid interface of the membrane can be expressed as

$$\frac{dE}{dt} = - \int_A \frac{\partial h}{\partial t} \Delta p dA \quad (2)$$

Where  $\partial h / \partial t$  is the velocity of the membrane in the normal direction.  $h$  is the surface displacement of the circular membrane.

On the other hand, the change rate of kinetic energy in time of the oscillating circular membrane having a mass per unit area equivalent to the added mass,  $M_a$ , can be given

$$\frac{dE}{dt} = - M_a \int_A \frac{\partial h}{\partial t} \frac{\partial^2 h}{\partial t^2} dA \quad (3)$$

So an expression for the added mass per unit area can be given in the following form:

$$M_a = \frac{\int_A \frac{\partial h}{\partial t} \Delta p dA}{\int_A \frac{\partial h}{\partial t} \frac{\partial^2 h}{\partial t^2} dA} \quad (4)$$

Eq.(4) indicates the essential dependence of the added mass of the oscillating circular membrane upon the fluid loading,  $\Delta p$ . The understanding of the properties of added mass will allow the fluid loading to be obtained in the most suitable form.

## 3 FORMULATIONS OF FLUID STRUCTURE INTERACTION (FSI)

In theoretical considerations, to take into account the fluid effect on the surrounded vibrating structure, the system has to be treated as a fluid structure interaction problem. In that case, the structural dynamics equation has to be coupled with the fluid equations.

It is well known that the discretized structural dynamics equation can be formulated as follows:

$$[K]\{u\} + [C]\{\dot{u}\} + [M]\{\ddot{u}\} = \{P\} \quad (5)$$

In which  $[K]$  is the structural stiffness matrix;  $[C]$ , the structural damping matrix;  $[M]$ , the structural mass matrix, and  $\{u\}$ ,  $\{P\}$  are the displacement and force vector, respectively.

The equation of motion of an undamped dynamic system in matrix notation is

$$[K]\{h\} + [\bar{M}]\left\{\frac{\partial^2 h}{\partial t^2}\right\} = \{0\} \quad (6)$$

where  $[\bar{M}] = [M] + [M_a]$ .

The membrane mass matrix is taken as diagonal with the masses corresponding to the collocation points. It can be transformed to the standard eigenvalue problem

$$([K] - \omega^2 [\bar{M}])\{h\} = \{0\} \quad (7)$$

Where  $\omega$  is the eigenvalue, and the square root of the eigenvalue is the natural frequency considering the effect of the surrounding air.

## 4 DETERMINATION OF THE FLUID LOADING

### 4.1 Mathematical model

The finite volume method is employed to resolve the fluid governing equations, transforming a continuum domain into a discretized space of small cells where an algebraic system of equations is resolved after linearization and discretization of the original mathematical equations for the mass and momentum conservation. Considering the present vibration of the circular membrane, the following basic characteristics have been implemented in the model:

- Full-scale simulation of the circular membrane over a 3D domain is used segregated solver which solves the equation set sequentially.
- The governing equation of the flow is described by incompressible viscous Navier-Stokes equation.
- Unsteady resolution of the circular membrane using the smoothing techniques and remeshing techniques for dynamic meshes.
- Implementation of a parametric UDF to simulate the vibration of circular membrane can update the position of each node based on the deflection due to fluid-structure interaction.

### 4.2 Turbulent model

Up to know, significant progress in the field of CFD (Computational Fluid Dynamics) has been achieved, and it has been used as an efficient tool for prediction of fluid loads. Generally, the governing equation of the flow is described by incompressible viscous Navier-Stokes equation. Because Reynolds-averaged Navier-Stokes (RANS) modeling can reasonably simulate mean wind characteristics and require less computing time than other CFD methods, it becomes the most widely used method in many industrial applications. For numerical simulation, the governing equations of the incompressible turbulent wind flow around circular membrane are presented by the RANS equations as follows:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad (8)$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} (-\rho \overline{u_i' u_j'}) \quad (9)$$

The turbulent kinetic energy, the  $k$  equation in Realizable  $k$ - $\varepsilon$  model is the same as that in the standard  $k$ - $\varepsilon$  model and the RNG  $k$ - $\varepsilon$  model. However, the form of the  $\varepsilon$  equation in Realizable  $k$ - $\varepsilon$  model is quite different from those in the standard and RNG-based  $k$ - $\varepsilon$  models. One of the noteworthy features is that the production term in the equation does not involve the production of  $k$ ; i.e. It is believed that the present form better represents the spectral energy transfer. It is a mathematical theory which can be used to derive a turbulence model which is similar to the  $k$ - $\varepsilon$  model. Following improvements compared to standard  $k$ - $\varepsilon$  model: (1). Additional term in  $\varepsilon$  equation improves the accuracy when modelling rapidly strained flows. (2). The effect of swirl turbulence is included in the RNG model. (3). The RNG model enables to use Prandtl numbers (turbulent viscosity/turbulent heat conductivity) as variables. (4). Enables to use lower Reynolds numbers than in standard model. The analytical derivation results in a model with constants different from those in the standard  $k$ - $\varepsilon$  model, and additional terms and functions in the transport equations for  $k$  and  $\varepsilon$ .

### 4.3 SIMPLIC algorithm

The pressure-velocity coupling was resolved using the SIMPLIC algorithm and the under relaxation factors were kept within typical ranges (0.3 for the pressure correction equation and 0.7 for the momentum equation), which helped to speed up the convergence process.

#### 4.4 Dynamic meshes

Dynamic and deformable meshes allow the displacement of the domain boundaries in CFD simulations, adjusting the associated mesh to the new locations accordingly. With the dynamic mesh model in FLUENT, it allows an arbitrary motion of fluid boundaries, and the response of the fluid to the prescribed boundary motion can be captured. Meanwhile, the motion control of the circular membrane can be realized by using DEFINE GRID MOTION UDF in FLUENT, and a grid motion UDF can update the position of each node based on the deflection due to fluid-structure interaction.

Different schemes for the mesh reconstruction can be found in the literature, according to the basic characteristics of the moving mesh. In the present study, two fundamental methods have been employed for efficient treatments of the vibration of the circular membrane. One is smoothing techniques: where the initial connectivity of the mesh nodes is remained unchanged. The nodes are repositioned as if they were connected by small springs. Though there is no restriction about the original mesh type, it can be used only in the case of small deformations. The other is remeshing techniques: which is an alternative to the smoothing algorithm when there are excessively large deformations in the model. The mesh has to be locally rebuilt after the changes in the boundaries are completed.

#### 4.5 Boundary conditions

It is necessary to impose the boundary condition artificially because the computational domain can not be infinite. The entrance boundary condition is defined by pressure-inlet where the specification of a static (gauge) pressure at the inlet boundary is set to zero. The exit boundary condition is defined by pressure-outlet where the specification of a static (gauge) pressure at the outlet boundary is set to zero. The boundary condition at the top and on both sides of flow field is symmetric boundary which is equivalent to the free-slip wall. The no-slip boundary condition is enforced at wall of the circular membrane surface and the ground.

### 5 RESULTS AND COMPARISON

Li, et al [3] investigated the vibration of circular membrane vibrating in air and in vacuum. A vacuum chamber was designed to test the vibration of a circular flat membrane in still air with various air pressures. The membrane was clipped by a top circle and a bottom circle. The pressures was imposed uniformly by lifting an inner circle. As Table 1 shown, the vibration of the membrane was measured in 1atm, and 0.6atm. Two membrane materials were used in the tests, including a latex and a rubber membrane, and five prestress levels of the membrane were tested. The natural frequencies of the membrane in 1atm, and 0.6atm were given in Table 2. Using the Realizable  $k-\varepsilon$  model and the RNG  $k-\varepsilon$  model of FLUENT commercial computational Codes to derive the aerodynamic pressure, and associating with FEM to derive the natural frequencies in consequence of considering the effect of the surrounding air, the results are shown in Table 3. The fundamental frequency in vacuum was estimated with Finite Element Method (FEM), as listed in Table 4.

Table 1 Test cases

| Air pressure | Latex membrane Prestress (MPa) |                  |                  | Rubber membrane Prestress (MPa) |                  |
|--------------|--------------------------------|------------------|------------------|---------------------------------|------------------|
|              | $\sigma_1=0.092$               | $\sigma_2=0.178$ | $\sigma_3=0.297$ | $\sigma_4=0.471$                | $\sigma_5=0.828$ |
| 1 atm        | A1                             | B1               | C1               | D1                              | E1               |
| 0.6 atm      | A2                             | B2               | C2               | D2                              | E2               |

Table 2 Natural frequency of the circular membrane vibrating in 1atm, and 0.6atm

| mode                                     | case  |       |       |       |       |       |       |       |       |       |
|------------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                                          | A1    | A2    | B1    | B2    | C1    | C2    | D1    | D2    | E1    | E2    |
| 1 <sup>st</sup> mode                     | 11.05 | 12.25 | 15.46 | 18.10 | 21.22 | 24.34 | 35.58 | 39.95 | 51.20 | 53.99 |
| 2 <sup>nd</sup> and 3 <sup>rd</sup> mode | 23.11 | 24.51 | 32.60 | 35.86 | 44.63 | 49.31 | 67.42 | 73.04 | 97.38 | 102.4 |
| 4 <sup>th</sup> and 5 <sup>th</sup> mode | -     | -     | -     | -     | -     | -     | 97.38 | -     | 151.1 | -     |
| 6 <sup>th</sup> mode                     | 33.55 | 39.74 | 47.63 | 57.70 | 71.47 | 78.91 | -     | -     | -     | -     |

Table 3 Natural frequency of the circular membrane vibrating in 1atm, and 0.6atm using Realizable  $k-\varepsilon$  model and RNG  $k-\varepsilon$  model

| Case | The result of using Realizable $k-\varepsilon$ model |                                          |                                          |                      | The result of using RNG $k-\varepsilon$ model |                                          |                                          |                      |
|------|------------------------------------------------------|------------------------------------------|------------------------------------------|----------------------|-----------------------------------------------|------------------------------------------|------------------------------------------|----------------------|
|      | 1 <sup>st</sup> mode                                 | 2 <sup>nd</sup> and 3 <sup>rd</sup> mode | 4 <sup>th</sup> and 5 <sup>th</sup> mode | 6 <sup>th</sup> mode | 1 <sup>st</sup> mode                          | 2 <sup>nd</sup> and 3 <sup>rd</sup> mode | 4 <sup>th</sup> and 5 <sup>th</sup> mode | 6 <sup>th</sup> mode |
|      | $f_1$ (Hz)                                           | $f_2$ (Hz)                               | $f_3$ (Hz)                               | $f_4$ (Hz)           | $f_1$ (Hz)                                    | $f_2$ (Hz)                               | $f_3$ (Hz)                               | $f_4$ (Hz)           |
| A1   | 12.80                                                | 23.28                                    | 33.94                                    | 37.56                | 12.85                                         | 23.39                                    | 34.31                                    | 37.64                |
| A2   | 15.14                                                | 26.79                                    | 38.94                                    | 42.55                | 15.19                                         | 27.08                                    | 39.07                                    | 42.67                |
| B1   | 17.81                                                | 32.38                                    | 47.21                                    | 52.25                | 17.88                                         | 32.54                                    | 47.72                                    | 52.35                |
| B2   | 21.05                                                | 37.26                                    | 54.17                                    | 59.19                | 21.13                                         | 37.66                                    | 54.35                                    | 59.36                |
| C1   | 23.00                                                | 41.83                                    | 60.98                                    | 67.49                | 23.09                                         | 42.03                                    | 61.64                                    | 67.62                |
| C2   | 27.20                                                | 48.13                                    | 69.97                                    | 76.45                | 27.29                                         | 48.65                                    | 70.20                                    | 76.67                |
| D1   | 37.47                                                | 65.70                                    | 93.26                                    | 102.12               | 37.58                                         | 65.62                                    | 93.97                                    | 102.25               |
| D2   | 42.23                                                | 72.11                                    | 101.67                                   | 110.31               | 42.32                                         | 72.60                                    | 101.86                                   | 110.50               |
| E1   | 49.68                                                | 87.01                                    | 123.65                                   | 135.40               | 49.83                                         | 87.01                                    | 123.44                                   | 135.58               |
| E2   | 55.96                                                | 95.61                                    | 134.80                                   | 146.26               | 56.11                                         | 96.26                                    | 135.06                                   | 146.51               |

Table 4 Natural frequencies of the circular membrane vibrating in vacuum

| Prestress (MPa)  | Finite Element Analysis |                                          |                                          |                      | Fitting result of test |                                          |                                          |                      |
|------------------|-------------------------|------------------------------------------|------------------------------------------|----------------------|------------------------|------------------------------------------|------------------------------------------|----------------------|
|                  | 1 <sup>st</sup> mode    | 2 <sup>nd</sup> and 3 <sup>rd</sup> mode | 4 <sup>th</sup> and 5 <sup>th</sup> mode | 6 <sup>th</sup> mode | 1 <sup>st</sup> mode   | 2 <sup>nd</sup> and 3 <sup>rd</sup> mode | 4 <sup>th</sup> and 5 <sup>th</sup> mode | 6 <sup>th</sup> mode |
| $\sigma_1=0.092$ | 24.45                   | 39.08                                    | 52.48                                    | 56.43                | 24.45                  | 33.38                                    | -                                        | 61.96                |
| $\sigma_2=0.178$ | 34.00                   | 54.36                                    | 73.00                                    | 78.50                | 34.00                  | 50.97                                    | -                                        | 92.92                |
| $\sigma_3=0.297$ | 43.98                   | 70.02                                    | 82.50                                    | 101.39               | 43.98                  | 72.18                                    | -                                        | 110.18               |
| $\sigma_4=0.471$ | 55.43                   | 88.42                                    | 118.75                                   | 127.69               | 55.43                  | 87.13                                    | 114.24                                   | -                    |
| $\sigma_5=0.828$ | 73.49                   | 117.24                                   | 157.45                                   | 169.30               | 73.51                  | 121.13                                   | -                                        | -                    |

Comparing the result of the natural frequencies in Table 2 and Table 3, the FLUENT results have a good agreement with the experimental results and the result of using the Realizable  $k-\varepsilon$  model and the RNG  $k-\varepsilon$  model is almost the same. The proposed numerical method is effective and can be used for the analysis of free vibrations of flat membrane. Comparing the result of the natural frequencies vibrating in 1atm and 0.6atm and the result of the natural frequencies vibrating in vacuum, it can find the effect of the air surrounding the membrane structure on the natural frequencies of the structure is very significant.

## 6 CONCLUSIONS

The numerical method for obtaining added masses of the circular membrane vibrating in still air is proposed in this paper. The aerodynamic pressure associated with the structure deformations is derived by the FLUENT commercial computational Codes and the natural

frequencies in consequence of considering the effect of the surrounding air is observed by FEM. The result based on this method is verified by comparing with the existing test results. Through the numerical example calculated with the use of the air mass, it can find that the effect of the air surrounding the membrane structure on the natural frequencies of the structure is very significant.

## 7 ACKNOWLEDGEMENTS

The authors are grateful to the financial support by the national Natural Science Foundation of China (No.50978192), the Research Fund of the State Key Laboratory of Disaster Reduction in Civil Engineering (No.SLDRCE09-B-01), and the Kwang-Hua Fund of College of Civil Engineering, Tongji University, and the Global COE Program of Tokyo Polytechnic University awarded by the Ministry of Education, Culture, Sports, Science and Technology, Japan.

## 8 REFERENCES

- [1] H. Minami. Added mass of a membrane vibrating at finite amplitude, *Journal of Fluids and Structures*, 1998, Vol.12, pp. 919-932
- [2] H.P.A.H. Irwin, R.L. Wardlaw. A wind tunnel investigation of a retractable fabric roof for the Montreal Olympic stadium. *Proceedings of the 5th International Conference*, Colorado, USA: Cermak J. E., 1979, pp. 925-938.
- [3] Y.Q. Li, L. Wang, Z.Y. Shen, Y. Tamura. Added mass estimation of flat membranes vibrating in still air, *Journal of Wind Engineering and Industrial Aerodynamics*, 2011, Vol. 99, pp.815-824.