

Unsteady aerodynamic forces on long-span curved roof

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ABSTRACT: The present paper discusses the characteristics of unsteady aerodynamic forces on long-span vaulted roofs, based on a wind tunnel experiment and a CFD simulation. A forced vibration test is carried out in the wind tunnel to investigate the effects of wind speed, vibration amplitude, reduced frequency and rise/span ratio of the roof on the unsteady aerodynamic forces. Furthermore, the influence of the unsteady aerodynamic forces on the dynamic response of practical long-span vaulted roofs is investigated using the results obtained from the wind tunnel experiments. The CFD simulation is used effectively to evaluate the unsteady aerodynamic forces on the vibrating roof in a wider reduced frequency range.

KEYWORDS: Unsteady aerodynamic force, Long-span curved roof, Wind tunnel experiment, Dynamic response, Computational Fluid Dynamics (CFD)

1 INTRODUCTION

Many long-span structures with curved roofs, such as stadiums, auditoriums, airports and railway stations, have been constructed in recent years, because they can provide a large space with no intermediate columns. Being light and flexible, such roofs are vulnerable to dynamic wind actions. Furthermore, the wind-structure interaction may affect the response significantly. The interaction is represented by the unsteady aerodynamic forces, or the motion-induced forces, which may increase or decrease the response of the structure. Many researches have been made of the unsteady aerodynamic forces on long-span bridges and tall buildings (e.g. Taniike et al. ^[1]). The results indicate that unstable vibration may be induced by the negative aerodynamic damping in some cases. By comparison, the number of researches on long-span roofs is quite limited. Daw and Davenport ^[2] carried out a forced vibration test on a semi-circular roof to investigate the dependence of unsteady aerodynamic forces on the turbulence intensity, wind speed, vibration amplitude and geometric details of the roof. Ohkuma et al. ^[3] investigated the mechanism of aeroelastic instability for long-span flat roofs using a forced vibration test in a wind tunnel. They discussed the effects of unsteady aerodynamic forces on the wind-induced responses, based on the wind tunnel experiment as well as on the dynamic response analysis. At present, however, the characteristics of unsteady aerodynamic forces on long-span curved roofs are not understood well. So, it is necessary to investigate this problem further for developing more reasonable methods of response analysis for these roofs.

The objective of the present study is to describe the characteristics of unsteady aerodynamic forces acting on a vaulted roof vibrating in the first anti-symmetric mode. A forced-vibration test is carried out in a turbulent boundary layer. The effects of wind speed, vibration amplitude and frequency on the characteristics of unsteady aerodynamic forces are investigated. However, the range of these parameters involved in the wind tunnel experiments is limited. Therefore, a CFD simulation is carried out to investigate the problem in more detail. In the simulation, the parameters are varied over a wider range.

2 UNSTEADY AERODYNAMIC FORCE

The displacement of structure in the j -th mode may be represented by the following equation,

$$z_j(s, t) = \phi_j(s) x_j(t) \quad (1)$$

where ϕ_j and x_j are the mode shape and generalized displacement of the j -th mode, respectively; and s represents the circumferential coordinate taken along the roof.

Applying a modal analysis to the equation of motion for the roof, we obtain the following equation of motion for the j -th generalized displacement:

$$\ddot{x}_j(t) + 2\zeta_j \omega_j \dot{x}_j(t) + \omega_j^2 x_j(t) = F_j(t) / M_j \quad (2)$$

$$F_j(t) = F_{wj}(t) + F_{Aj}(x_j, \dot{x}_j, \ddot{x}_j, \dots) \quad (3)$$

where M_j = generalized mass; ω_j = natural circular frequency; ζ_j = critical damping ratio; and F_j = generalized force. F_{wj} represents the fluctuating wind force due to the oncoming flow and wake instability, while F_{Aj} the unsteady aerodynamic force due to the wind-structure interaction.

In the case of the forced-vibration test, a steady vibration in the first anti-symmetric mode represented by a sine curve is applied to the roof. The unsteady aerodynamic force F_{Aj} (here $j = 1$) may be obtained from Eq. (4) by using the Fourier series at the frequency f_m of the forced vibration:

$$F_{Aj}(t) = F_{Rj} \cos 2\pi f_m t - F_{Ij} \sin 2\pi f_m t \quad (4)$$

$$F_{Rj} = \frac{1}{T} \int_{-T}^T F_j(t) \cos 2\pi f_m t dt \quad (5)$$

$$F_{Ij} = \frac{1}{T} \int_{-T}^T F_j(t) \sin 2\pi f_m t dt \quad (6)$$

where F_{Rj} and F_{Ij} are the in-phase and out-of-phase components of the unsteady aerodynamic force, respectively.

The effect of unsteady aerodynamic force may be represented by the aerodynamic stiffness and damping coefficients a_{kj} and a_{cj} , which are given by the following equations^[4]:

$$a_{kj} = \frac{1}{q_H A_s (x_0 / L)} \frac{2}{T} \int_0^T F_j(t) \cos 2\pi f_m t dt \quad (7)$$

$$a_{cj} = \frac{1}{q_H A_s (x_0 / L)} \frac{2}{T} \int_0^T F_j(t) \sin 2\pi f_m t dt \quad (8)$$

where q_H = velocity pressure at the mean roof height H ; A_s = roof area; x_0 = vibration amplitude; L = span of the roof; f^* = reduced frequency defined by $f_m H / U_H$, with U_H being the mean wind speed at the mean roof height H .

The generalized force F_j may be described in terms of the external and internal pressures p_e and p_i , as shown in Eq. (9):

$$F_j(t) = \int_0^{R_s} [p_{e_j}(s, t) - p_{i_j}(s, t)] \phi_j(s) ds \quad (9)$$

where R_s = total length of the vaulted roof. Internal pressure p_i is ignored in the present study, because the first anti-symmetric mode under consideration causes no change of internal volume.

3 WIND TUNNEL EXPERIMENT

3.1 Experimental apparatus and procedures

The experiments were carried out in an Eiffel-type wind tunnel with a working section 6.5m in length and 1.0m×1.4m in cross-section. A turbulent boundary layer with a power-law exponent of $\alpha=0.23$ was generated on the wind tunnel floor by using a set of turbulence-generating spires installed at the entrance of the working section, and a number of roughness blocks distributed on the floor. The profiles of the mean wind speed and turbulence intensity are shown in Fig. 1. The reference wind speed was measured at a height of $Z_G = 500\text{mm}$. The longitudinal velocity spectrum, not shown here to save space, was found to be generally consistent with the so-called Karman type spectrum.

The wind tunnel model was a vaulted roof made of 0.8mm thick polyester film, as shown in Figs. 2 and 3. A pair of end plates was used to make the flow two-dimensional. Two models with rise/span ratios of 0.15 and 0.20 were tested. Each model had 12 pressure taps of 1mm diameter distributed along the roof's centerline. The pressure taps were connected to pressure transducers in parallel via 80cm lengths of flexible vinyl tubing. The tubing effects were numerically compensated by using the gain and phase-shift characteristics of the pressure measuring system used in the experiment. The signals from the pressure transducers were sampled simultaneously at a rate of 500Hz for a period of approximately 60s. Table 1 summarizes the range of experimental parameters involved in the experiment.

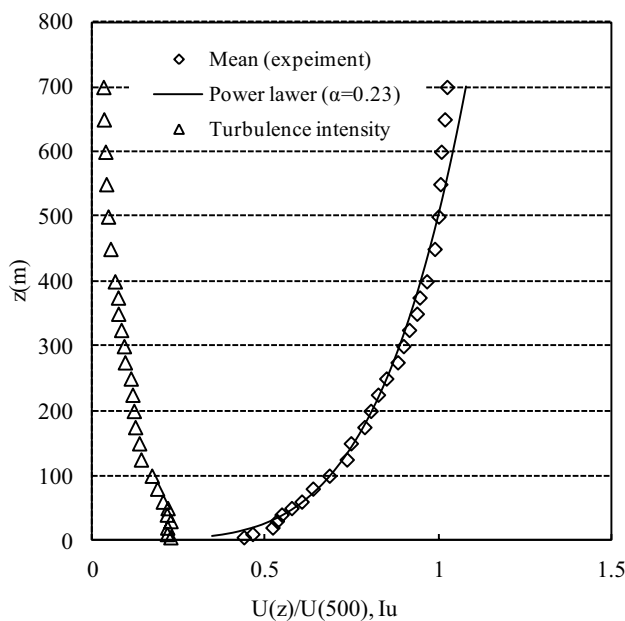


Fig.1 Profiles of the mean wind speed and turbulent intensity

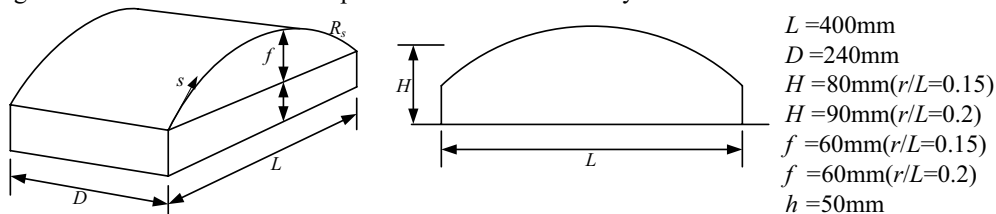


Fig.2 Geometry of the experimental model

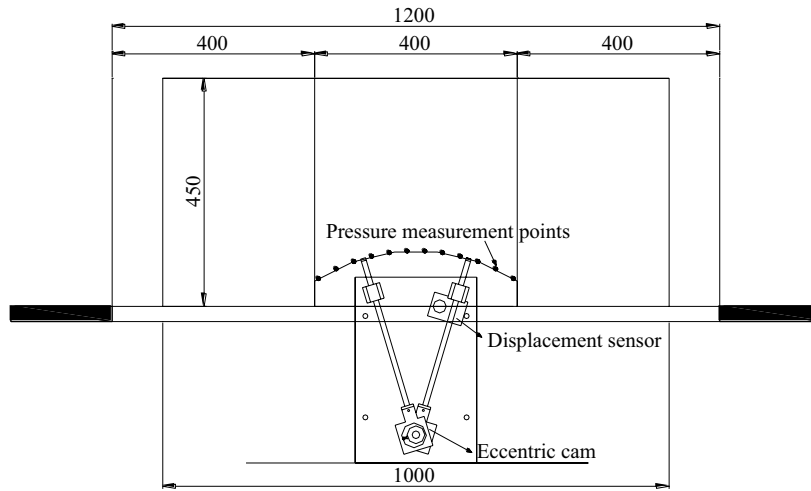


Fig.3 Experimental setup of the forced vibration test

Table 1 Parameters of experiment

Rise/span ratio r/L	0.15, 0.20
Wind speed U_H (m/s)	5.0, 7.0, 10.0
Amplitude of the forced vibration x_0 (mm)	1.0, 2.5, 4.0
Forced vibration frequency f_m (Hz)	5 to 25 at an increment of 1 Hz

3.2 Results and Discussion

Fig.4 shows the variation of the aerodynamic stiffness coefficient a_k with the reduced frequency f^* for various wind speeds (Fig. 4(a)) and vibration amplitudes (Fig. 4(b)); the rise/span ratio is 0.15. The value of a_k generally increases with an increase in f^* . As the reduced frequency decreases, the value of a_k approaches the quasi-steady value (dashed line in the figure). Similar results were observed for $r/L=0.20$. Within the limits of the present experiment, the value of a_k is generally positive, which may reduce the total stiffness of the structural system resulting in a lower natural frequency.

Plotted on Fig.5 are the variations of aerodynamic damping coefficient a_c with the reduced frequency f^* for various wind speeds and vibration amplitudes; the rise/span ratio is again 0.15. The values of a_c are generally negative except for small f^* values, which may result in an increase in the total damping of structural system. The magnitude of a_c increases as the f^* value increases. Similar results were observed for $r/L=0.20$. It can be seen that the effects of wind speed and vibration amplitude on the aerodynamic stiffness and damping coefficients are not so significant and the values of a_k and a_c are mainly dependent on f^* .

4 PREDICTION OF DYNAMIC RESPONSE

This section outlines a method for predicting the dynamic response of a full-scale vaulted roof to turbulent winds, in which the effects of the unsteady aerodynamic forces are taken into account. As an example, we consider a long-span membrane structure with the same shape as that used in the wind tunnel experiment. In general, the natural frequency of membrane structure with a span of approximately 100m is in a range from 0.5Hz to 1.5Hz and the mass from 2 kg/m² to 15 kg/m². The dynamic responses of such structures in a turbulent wind are evaluated by using the averaged

curves of aerodynamic stiffness and damping coefficients obtained from the wind tunnel experiments.

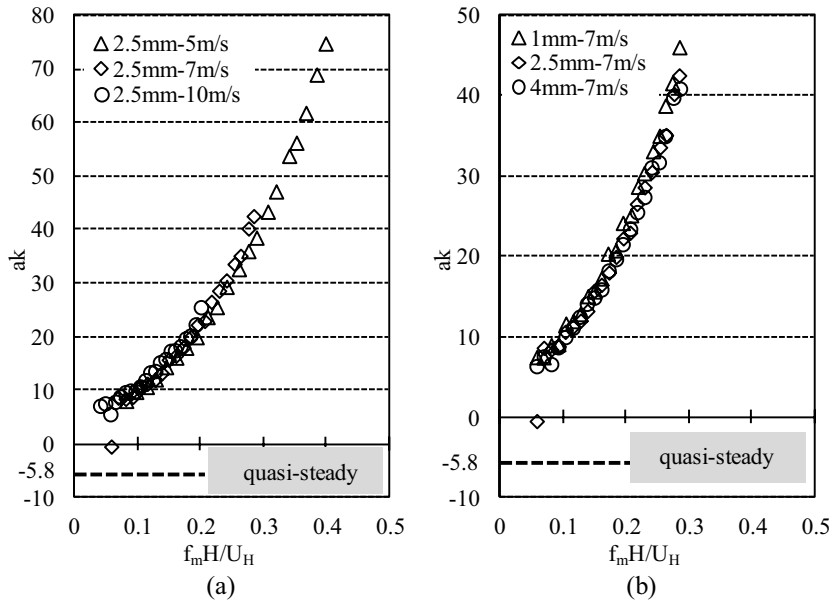


Fig.4 Aerodynamic stiffness coefficient versus f^* ($r/L=0.15$)

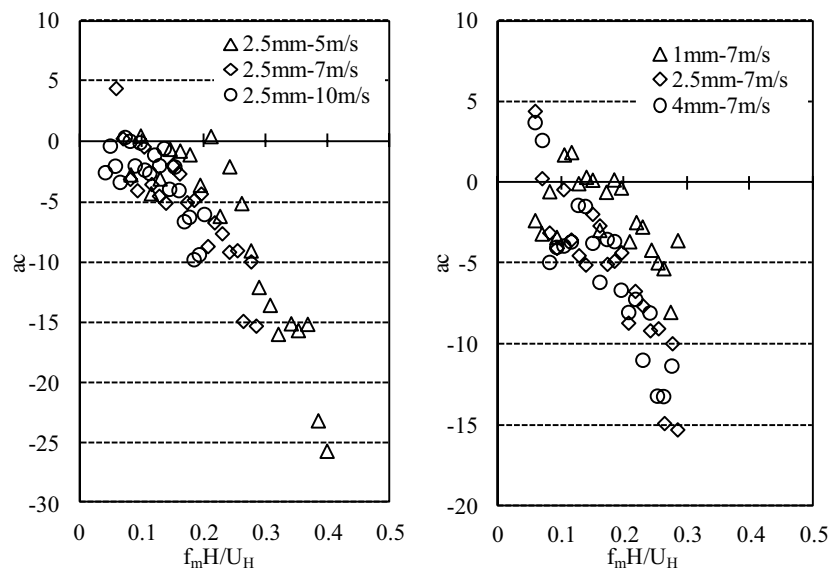


Fig.5 Aerodynamic damping coefficient versus f^* ($r/L=0.15$)

Aerodynamic stiffness (K_{aj}) and aerodynamic damping ratio (ζ_{aj}) are given by Eqs. (10) and (11). The mechanical admittance including the aerodynamic stiffness and damping is defined by Eq. (12), in which we assume that the structural damping ratio is $\zeta_s = 1\%$ and the natural frequency of the first anti-symmetric mode is $f_s = 0.5\text{Hz}$. Only the first vibration mode ($j=1$) is considered, because the roof response to turbulent wind forces is thought to be dominated by this mode^[6].

$$\zeta_{a_j}(f) = -\frac{1}{16\pi^2} \frac{\rho_a}{\rho_{s_j}} \left(\frac{U_H}{f_{s_j} H} \right)^2 \frac{H}{L} a_{c_j}(f) \quad (10)$$

$$\frac{K_{a_j}(f)}{K_{s_j}} = -\frac{1}{8\pi^2} \frac{\rho_a}{\rho_{s_j}} \left(\frac{U_H}{f_{s_j} H} \right)^2 \frac{H}{L} a_{k_j}(f) \quad (11)$$

$$|\chi_j(f)|^2 = \frac{1}{\left[1 - \left(\frac{f}{f_{s_j}} \right)^2 + \frac{K_{a_j}(f)}{K_{s_j}} \right]^2 + 4 \left(\xi_{s_j} + \xi_{a_j}(f) \right)^2 \left(\frac{f}{f_{s_j}} \right)^2} \quad (12)$$

Fig.6 illustrates the mechanical admittance functions plotted against frequency f for various wind speeds, where the structural mass is assumed $M_s=4\text{kg/m}^2$. As the wind speed increases, the resonant frequency decreases and the peak value of the mechanical admittance function at the resonant frequency increases. The decrease in the resonant frequency is due to the positive aerodynamic stiffness coefficient.

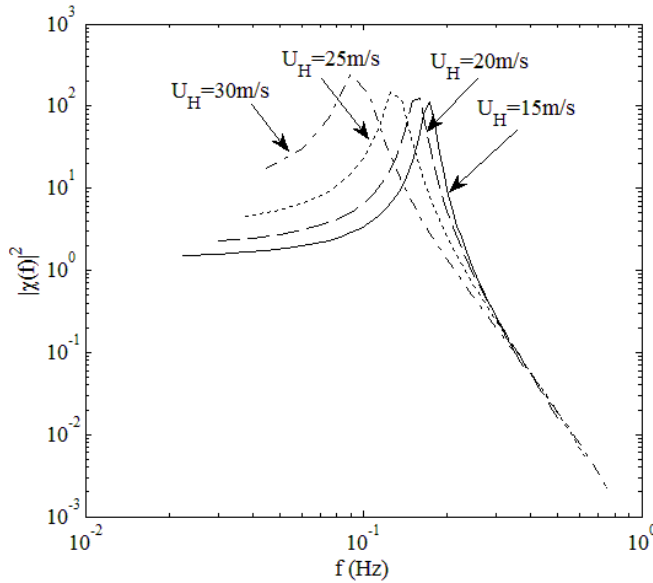


Fig.6 Variation of mechanical admittance function with wind speed

The variation of mechanical admittance function with the roof's mass is illustrated in Fig.7, in which we assume that $U_H=20\text{m/s}$. The resonant frequency increases and the resonant peak value of the mechanical admittance function decreases as roof's mass increases. This feature implies that the increase in mass is quite effective for reducing the aerodynamic excitation of the roof. In other words, the effect of unsteady aerodynamic forces on the response of the roof becomes less significant for heavier roofs.

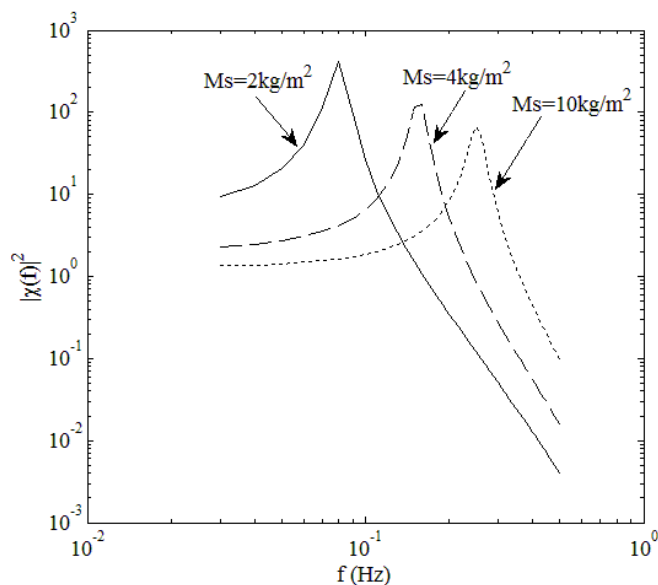


Fig.7 Variation of mechanical admittance function with roof's mass

5 CFD SIMULATION

The unsteady aerodynamic force acting on a vibrating roof in the first anti-symmetric mode is numerically simulated by using a commercial software 'STAR-CD'. As the first step of study, the wind tunnel experiment using a model with $r/L=0.15$ is reproduced. The amplitude x_0 of vibration is fixed to 4.0mm in this study.

5.1 Computational conditions

The large eddy simulation (LES) with the Smagorinsky sub-grid model ($C_s=0.12$) is used to simulate the flow field around the model. The computational conditions are summarized in Table 2. The computational domain and mesh arrangement are shown in Fig.8. It is important to provide proper inflow condition for predicting the wind loads on structures precisely, particularly for LES. In this study, the inflow turbulence is generated in a preliminary computational domain where roughness blocks are arranged to generate a turbulent boundary layer similar to the wind tunnel flow [7, 8]. The profile of the mean wind speed and turbulent intensity at the inlet of the computational domain are compared with those of the wind tunnel flow in Fig. 9. Although there exists a difference between CFD simulation and wind tunnel experiment for the profile of mean wind speed, the agreement is quite good for the turbulence intensity.

5.2 Results and discussion

Fig.10 shows the distributions of the mean wind pressure coefficients along the centerline of the vibrating roof obtained from the CFD simulation and the wind tunnel experiment, in which the results for the frequencies of 10Hz and 20Hz are plotted. A generally good agreement between these two results can be seen. The difference is somewhat larger near the top of the roof; the CFD values are approximately 10% larger in magnitude than the experimental ones. This difference

may be due to a difference in surface roughness of the roof between CFD simulation and wind tunnel experiment.

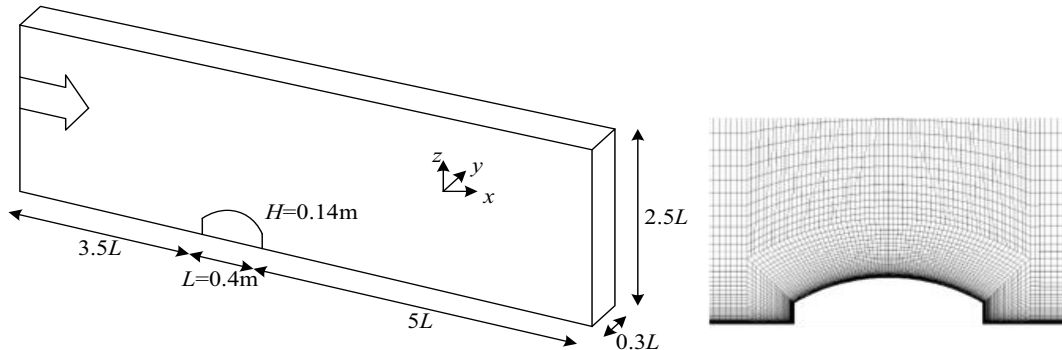


Fig.8 Computational domain and mesh arrangement

Table 2 Computational conditions

Computational domain	$9.5L(x) \times 0.3L(y) \times 2.5L(z)$
Inlet boundary	Inflow turbulence is generated in preliminary computational domain
Upper boundary	Zero normal velocity and zero normal gradients of other variables
Side boundary	Cyclic boundary conditions
Outlet boundary	Zero normal gradients of all variables
Floor and model surfaces	no-slip condition
Grid discretization	$260 (x) \times 12 (y) \times 64 (z) = (199680)$
Convection schemes	Second-order centered difference scheme
Time differential schemes	MARS method
Numerical algorithm	PISO algorithm
Time step	$\Delta t = 2.0E-04$ second (Courant Number : $9.1E-02$)

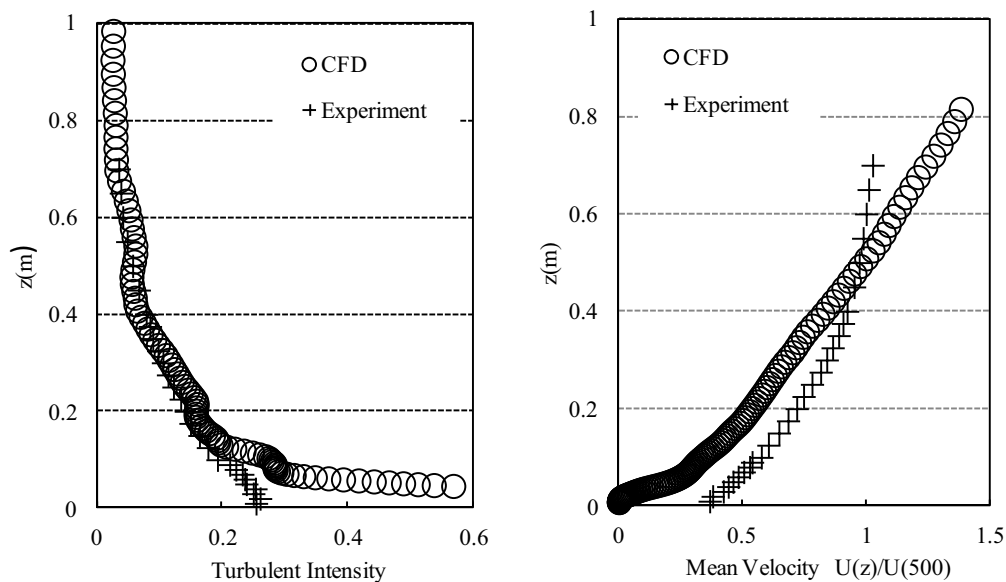


Fig.9 Comparisons of mean wind speed and turbulent intensities of CFD with wind tunnel test

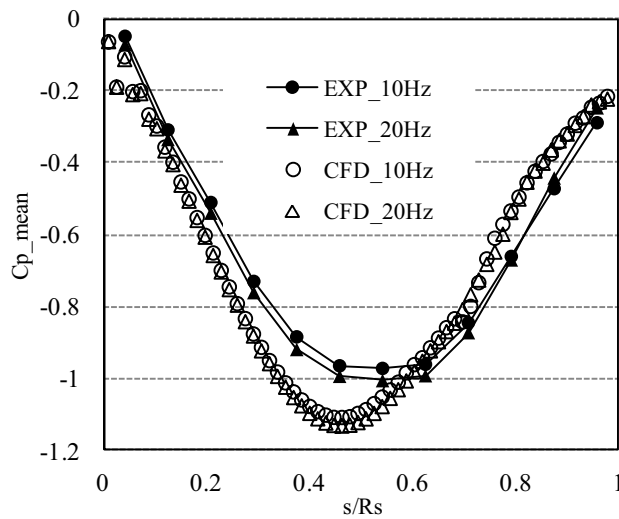


Fig.10 Comparisons between CFD simulation and wind tunnel experiment for the mean wind pressure coefficient

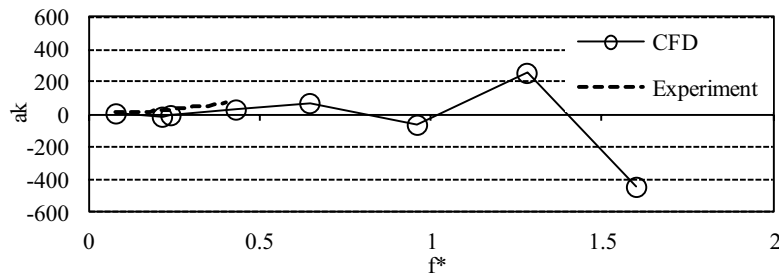


Fig.11 Comparison of aerodynamic stiffness coefficient computed by CFD with wind tunnel test

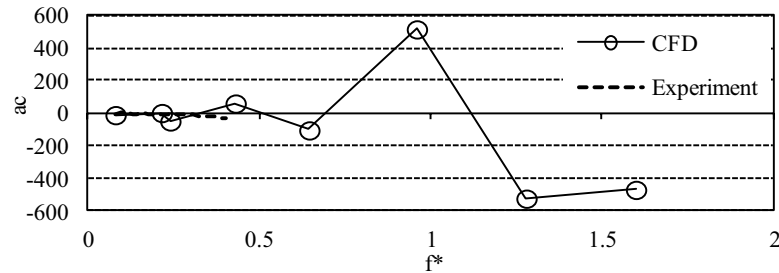


Fig.12 Comparison of aerodynamic damping coefficient computed by CFD with wind tunnel test

Based on the results of the CFD simulation, the aerodynamic stiffness and damping coefficients are calculated by using Eqs. (7) and (8). The aerodynamic stiffness coefficients a_k obtained from the LES and wind tunnel experiment are plotted in Fig. 11 for several reduced frequencies. The both results for the reduced frequency range of 0 to 0.4 are generally consistent with each other. A comparison for the aerodynamic damping coefficients a_c between the LES and wind tunnel experiment is shown in Fig. 12. Although the simulation value of a_c at $f^*=0.4$ is positive, the general trend of a_c with f^* in the lower f^* range is consistent with the experimental result. In a higher reduced frequency range, such as $f^*>0.5$ for example, the magnitudes of a_k and a_c obtained from the CFD simulation fluctuate significantly with increasing f^* . It is due to the flow-structure interaction. Such an interaction may be related to the convection velocity of pressure field on the

roof as well as to the roof's vibration speed relative to the wind speed ^[9]. Future studies will be necessary to discuss this problem in detail.

6 CONCLUSIONS

The unsteady aerodynamic force on a long-span vaulted roof has been investigated based on a wind tunnel experiment as well as on a CFD simulation. The main results obtained in the present study may be summarized as follows:

- 1) The aerodynamic stiffness and damping coefficients vary with the reduced frequency. The coefficients are influenced by the wind speed, rise/span ratio and vibration amplitude only slightly.
- 2) Within the limits of the present experiment, the aerodynamic stiffness is generally negative, which decreases the total stiffness of system. However, the aerodynamic damping is positive, which may result in an increase of the total damping of system.
- 3) As the mass of the roof increases, the effect of the unsteady aerodynamic forces on the roof's response to turbulent winds becomes less significant. And the unsteady aerodynamic forces reduced the resonant frequency and change the resonant peak.
- 4) The CFD simulation is a useful tool to evaluate the unsteady aerodynamic forces on the vibrating roof in a turbulent flow. The general trends of the aerodynamic stiffness and damping coefficients with the reduced frequency obtained from the CFD simulation are consistent with that from the wind tunnel experiment.

In our future study, we will construct a model of the unsteady aerodynamic force and incorporate it into the wind-resistant design of long-span curved roofs.

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