

Unsteady buffeting prediction of bridges using proper orthogonal decomposition

Thai-Hoa Le^{a)}, Yukio Tamura^{b)}

^{a)} *Wind Engineering Research Center, Tokyo Polytechnic University
1583 Iiyama, Atsugi-shi, Kanagawa 243-0297, Japan*

^{b)} *Department of Architectural Engineering, Tokyo Polytechnic University
1583 Iiyama, Atsugi-shi, Kanagawa 243-0297, Japan*

ABSTRACT: This paper presents the unsteady buffeting response prediction of bridges in the time domain using the proper orthogonal decomposition. Digital simulation of the turbulence wind fields is carried out at the bridge nodes using the spectral representation techniques via Cholesky decomposition and modal factorization. Covariance matrix-based proper orthogonal decomposition is used to decompose the unsteady buffeting forces via the impulse response functions, then formulate the buffeting response prediction of bridges in the time domain. The impulse response functions of unsteady buffeting forces are identified via aerostatic force coefficients and aerodynamic admittance functions. The Newton-beta integration method is also applied to obtain the buffeting response solution in the time domain. Numerical example of full-scale bridge is taken into account for illustration and discussions.

KEYWORDS: Buffeting analysis; Unsteady buffeting forces; Time domain; Proper Orthogonal Decomposition; Covariance proper transformation; Roger approximation

1 INTRODUCTION

Buffeting response prediction of long-span bridges subjected to the turbulent-induced forces (the buffeting forces) requires essential concern among wind-induced vibrations and their aerodynamic responses. General formulation and framework of the buffeting response prediction of bridges has been proposed by A.G. Davenport in the frequency domain (Davenport 1962). Recently, the state-of-the-art buffeting response analysis of bridges in coupling with the aeroelastic forces in the time domain has been carried out by some authors (ex., Boonyapinyo et al. 1999, Chen et al. 2000). Computational difficulties are to decompose spatially-correlated full-scale buffeting forces into the structurally generalized coordinates which are decomposed thanks to the structural modal transformation in order to estimate the generalized responses in the generalized coordinates and then the global responses of structures. In conventional methods in both the frequency-domain and the time-domain, the so-called joint acceptance function (JAF) has been used to decompose the full-scale buffeting forces into the structurally generalized coordinates.

Proper orthogonal decomposition (POD) (Lumley 1970), has been applied in many engineering fields (Liang et al. 2002). In the wind engineering topics, the proper orthogonal decomposition has been applied for analysis and order-reduced synthesis of the random pressure fields (Tamura et al. 1999), representation and simulation of the random fields (Tubino and Solari 2005), stochastic dynamic response (Carassale et al. 1999, Solari and Carassale 2000, Chen and Kareem 2005). Basing on the basic matrix of either the zero-time-lag covariance matrix or cross spectral density matrix of the random field, the proper orthogonal decomposition has been branched by either the covariance proper transformation or the spectral proper transformation. New approach of the gust response analysis of

structures has been proposed recently by Carassale et al. 1999 in which the structural modes decomposed by the structural modal transformation are associated with either the covariance turbulent loading modes or the spectral turbulent loading modes decomposed from proper transformations in order to determine the generalized responses and the global responses of structures. Combination manner between the structural modes and the turbulent loading modes for predicting the stochastic response of structures is named by Carassale et al. 1999 and Solari and Carassale 2000 as the double modal transformations (DMT). So far, almost literatures on the proper orthogonal decomposition application to the stochastic response of structures are formulated based on the spectral proper transformation in the frequency domain (Carassale et al. 1999, Solari and Carassale 2000, Chen and Kareem 2005). The frequency-domain buffeting response of long-span bridge using the spectral proper transformation has been formulated previously somewhere (ex., Tubino and Solari, 2007). The stochastic response of structures formulated in the time domain using the covariance proper transformation is very promising because of its direct and capable solutions for nonlinear problems and unsteady buffeting aerodynamics. However, digital simulation of the time series of the turbulence wind fields and the direct integration methods of generalized dynamic equations are usually required that are associated with time-consuming and difficult computation. Time-domain buffeting analysis is always promising for advanced treatment of nonlinear problems, unsteady buffeting response and buffeting-induced vibration countermeasures. Moreover, the unsteady buffeting forces in the time domain via the impulse response functions are better approach for the unsteady buffeting response problem due to its advantage for modeling so-called “memory effect” in the unsteady fluid that could capture the past history of response.

In the paper, the unsteady buffeting response prediction of bridges in the time domain using the proper orthogonal decomposition has been formulated. The spectral branched proper orthogonal decomposition has been used to digitally simulated the turbulence wind fields on the bridge, while the covariance branched proper orthogonal decomposition is used to decompose the unsteady buffeting forces via the impulse response functions, then the buffeting response prediction has been formulated in the time domain. The impulse response functions used for the unsteady buffeting forces have been estimated indirectly using the aerostatic force coefficients, the aerodynamic admittance functions and the Roger rational approximation. Buffeting responses of the bridges have been solved directly in the time domain using the Newton-beta integration technique. Numerical example of medium-span cable-stayed bridge has been taken into account for results and discussions.

2 UNSTEADY BUFFETING FORCES

The quasi-steady buffeting forces per unit deck length (lift, drag and moment) are determined due to the quasi-steady theory that is corrected by frequency-dependant aerodynamic admittance functions as follows (see Figure 1):

$$L_b(t) = \frac{1}{2} \rho U^2 B [C_L \chi_{Lu}(n) \frac{2u(t)}{U} + (C'_L + C_D) \chi_{Lw}(n) \frac{w(t)}{U}] \quad (1a)$$

$$D_b(t) = \frac{1}{2} \rho U^2 B [C_D \chi_{Du}(n) \frac{2u(t)}{U} + (C'_D - C_L) \chi_{Dw}(n) \frac{w(t)}{U}] \quad (1b)$$

$$M_b(t) = \frac{1}{2} \rho U^2 B^2 [C_M \chi_{Mu}(n) \frac{2u(t)}{U} + C'_M \chi_{Mw}(n) \frac{w(t)}{U}] \quad (1c)$$

where C_L, C_D, C_M : aerodynamic static coefficients (at balanced angle of attack); C'_L, C'_D, C'_M : first-order derivatives; $\chi_{Fu}(n)$ ($F = L, D, M; u = u, w$): aerodynamic transfer functions between turbulent components and turbulent-induced forces (their absolute magnitudes refer as aerodynamic admittance functions).

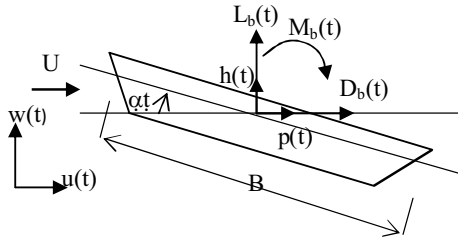


Figure 1. Buffeting forces on bridge deck

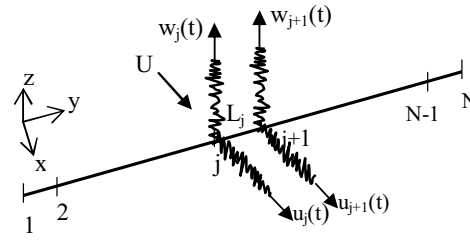


Figure 2. Turbulence wind fields

Quasi-steady buffeting forces, however, do not exactly reflect unsteady characteristics as nature of these random forces, which depend on not only geometric configuration of bridge decks, oncoming wind turbulence, but frequency-dependant parameters due to wind-structure interaction and bluff body flow. Additionally, it is indicated that unsteady fluid dynamics at low, medium flow velocity ranges must account for both past and present motion histories, thus the so-called “memory effect” of the unsteady fluid flow should be included in modeling of unsteady buffeting forces. So far, the unsteady buffeting forces can be determined via either the indicial response functions (Scanlan 1993) or the impulse response functions (Lin 1984, Chen and Kareem 2000) thanks to the convolution integration operation. Recently, the impulse response functions are usually used for modeling the unsteady buffeting forces in the time domain as follows:

$$L_b(t) = \frac{1}{2} \rho U^2 \left\{ \int_{-\infty}^t I_{Lu}(t-\tau) \frac{u(\tau)}{U} d\tau + \int_{-\infty}^t I_{Lw}(t-\tau) \frac{w(\tau)}{U} d\tau \right\} \quad (2a)$$

$$D_b(t) = \frac{1}{2} \rho U^2 \left\{ \int_{-\infty}^t I_{Du}(t-\tau) \frac{u(\tau)}{U} d\tau + \int_{-\infty}^t I_{Dw}(t-\tau) \frac{w(\tau)}{U} d\tau \right\} \quad (2b)$$

$$M_b(t) = \frac{1}{2} \rho U^2 \left\{ \int_{-\infty}^t I_{Mu}(t-\tau) \frac{u(\tau)}{U} d\tau + \int_{-\infty}^t I_{Mw}(t-\tau) \frac{w(\tau)}{U} d\tau \right\} \quad (2c)$$

where $I_{Lu}, I_{Lw}, I_{Du}, I_{Dw}, I_{Mu}, I_{Mw}$: impulse response functions which are usually obtained thanks to available admittance functions such following formulae as (Chen and Kareem 2002):

$$\bar{I}_{Lu} = 2BC_L \chi_{Lu}(n), \quad \bar{I}_{Lw} = B(C'_L + C_D) \chi_{Lw}(n) \quad (3a)$$

$$\bar{I}_{Du} = 2BC_D \chi_{Du}(n), \quad \bar{I}_{Dw} = B(C'_D - C_L) \chi_{Dw}(n) \quad (3b)$$

$$\bar{I}_{Mu} = 2B^2 C_M \chi_{Mu}(n), \quad \bar{I}_{Mw} = B^2 C'_M \chi_{Mw}(n) \quad (3c)$$

where $\bar{I}_{Fv}$: Fourier transform of the impulse response functions ($F = L, D, M$ $v = u, w$):

$$\bar{I}_{Fv} = \int_0^{\infty} I_{Fv}(t) e^{-i\omega t} dt = \int_0^{\infty} I_{Fv}(s) e^{-iKs} ds$$

However, the aerodynamic admittance functions determined by either empirical formula or experimental measurement are expressed at discrete values of reduced frequency. Therefore, it is essential to approximate discrete frequency-dependant function to continuous one. Rational function approximation, known as the Roger's approximation is the mostly utilized for this purpose. Aerodynamic transfer functions can be expanded using rational function approximation:

$$\chi_{Fv}(n) = A_{Fv,1} + (iK) \sum_{j=1}^{m_{Fv}} \frac{A_{Fv,j+1}}{iK + d_{Fv,j}} \quad (4)$$

where $A_{Fv,1}, A_{Fv,j+1}, d_{Fv,j}$ frequency-dependant coefficients ($j = 1, 2, \dots, m_{Fv}$) determined by nonlinear curve-fitting technique.

Then the impulse response functions and the unsteady buffeting forces (here only $I_{Lw}(s)$,

$L_{Lw}(t)$ are expended as example) can be determined as follows (Chen and Kareem, 2002):

$$I_{Lw}(s) = B(C_D + C_L') \left[\left(A_{Lw,1} + \sum_{j=1}^{m_{Lw}} A_{Lw,j+1} \right) \delta(s) - \sum_{j=1}^{m_{Lw}} A_{Lw,j+1} d_{Lw,j} e^{-d_{Lw,j}s} \right] \quad (5a)$$

$$I_{Lw}(s) = \frac{1}{2} \rho U^2 B(C_D + C_L') \left[\left(A_{Lw,1} + \sum_{j=1}^{m_{Lw}} A_{Lw,j+1} \right) \frac{w(t)}{B} - \sum_{j=1}^{m_{Lw}} \frac{d_{Lw,j} U}{B} \phi_{Lw,j}(t) \right] \quad (5b)$$

$$\dot{\phi}_{Lw,j}(t) = -\frac{d_{Lw,j} U}{B} \phi_{Lw,j}(t) + A_{Lw,j+1} \frac{w(t)}{U} \quad (5c)$$

where $\phi_{Lw,j}(t)$: augmented aerodynamic state coefficient.

3 PROPER ORTHOGONAL DECOMPOSITION

The main idea of the covariance matrix-based proper orthogonal decomposition and its covariance proper transformation is to find out a set of orthonormal basic vectors which can expand a multi-variate spatially-correlated random turbulent field into a sum of products of these basic orthogonal vectors (turbulent loading modes) and single-variate uncorrelated random processes (principal coordinates) based on the covariance matrix of this random field. The covariance matrix-based orthogonal vectors are found as the eigenvector solutions of the eigen problem of the zero-time lag covariance matrix $R_v(0)$ of N-variate random turbulent process $v(t)$:

$$R_v \Theta_v = \Gamma_v \Theta_v \quad (6)$$

where $R_v(0)$: zero-time-lag covariance matrix of $v(t)$; Γ_v, Θ_v : covariance eigenvalue and eigenvector matrices $\Theta_v = [\theta_{v_1}, \theta_{v_2}, \dots, \theta_{v_N}]$, $\Gamma_v = \text{diag}(\gamma_{v_1}, \gamma_{v_2}, \dots, \gamma_{v_N})$. Because the zero-time lag covariance matrix is positive-definite symmetrical squared one, thus its covariance eigenvalues are real and positive, and covariance eigenvectors are also real, satisfy the orthonormal conditions:

$$\Theta_v \Theta_v^T = I; \quad \Theta_v C_v \Theta_v^T = \Gamma_v \quad (7)$$

Then, the random turbulence field and its covariance matrix can be expressed such optimum approximation as follows:

$$v(t) = \Theta_v x_v(t) \approx \sum_{j=1}^{\tilde{N}} \theta_{vj} x_{vj}(t) \quad (8a)$$

$$R_v(0) = \Theta_v \Gamma_v \Theta_v^T(n) = \sum_{j=1}^{\tilde{N}} \theta_{vj} \gamma_{vj} \theta_{vj}^T \quad (8b)$$

where $\hat{x}_v(t) = \{\hat{x}_{v_1}(t), \hat{x}_{v_2}(t), \dots, \hat{x}_{v_N}(t)\}^T$: covariance matrix-based principal coordinates (shortly, the covariance principal coordinates) as the N-variate uncorrelated Gaussian random subprocesses that represents as image of the random turbulent field in the covariance-based space; $\tilde{N}$: number of truncated covariance eigenvectors ($\tilde{N} \ll N$). The covariance principal coordinates can be determined from observed data under following expression:

$$x_v(t) = \Theta_v^{-1} v(t) = v(t) \Theta_v = \sum_{i=1}^N v_i(t) \theta_{vi} \quad (9)$$

These approximations are known as the covariance proper transformation in the time domain. Two N-variate turbulent processes $u(t)$, $w(t)$ acting on the N structural nodes:

$u(t) = \{u_1(t), u_2(t), \dots, u_N(t)\}^T$ and $w(t) = \{w_1(t), w_2(t), \dots, w_N(t)\}^T$ are represented under the zero-time lag covariance matrices $R_u(0), R_w(0)$.

Two N-variate Gaussian random turbulent processes $u(t)$, $w(t)$ can be transformed and approximated into the covariance principal coordinates due to the covariance proper transformation:

$$u(t) = \Theta_u x_u(t) \approx \sum_{j=1}^{\tilde{N}} \theta_{u_j} x_{u_j}(t) \quad (10a)$$

$$w(t) = \Theta_w x_w(t) \approx \sum_{j=1}^{\tilde{N}} \theta_{w_j} x_{w_j}(t) \quad (10b)$$

where $x_u(t), x_w(t)$: the covariance principal coordinates of u-,w-turbulences, respectively.

As the same way, the covariance matrices can be approximated as follows:

$$R_u(0) = \Theta_u \Gamma_u \Theta_u^{*T} \approx \sum_{j=1}^{\tilde{N}} \theta_{u_j} \gamma_{u_j} \theta_{u_j}^{*T} \quad (11a)$$

$$R_w(0) = \Theta_w \Gamma_w \Theta_w^{*T} \approx \sum_{j=1}^{\tilde{N}} \theta_{w_j} \gamma_{w_j} \theta_{w_j}^{*T} \quad (11b)$$

where $\Gamma_u, \Gamma_w, \Theta_u, \Theta_w$: covariance eigenvalues and corresponding covariance eigenvectors (or covariance turbulent modes): $\Gamma_u = \text{diag}(\gamma_{u_1}, \gamma_{u_2}, \dots, \gamma_{u_N})$, $\Gamma_w = \text{diag}(\gamma_{w_1}, \gamma_{w_2}, \dots, \gamma_{w_N})$, $\Theta_u = \{\theta_{u_1}, \theta_{u_2}, \dots, \theta_{u_N}\}^T$, $\Theta_w = \{\theta_{w_1}, \theta_{w_2}, \dots, \theta_{w_N}\}^T$ determined from the covariance matrix-based proper orthogonal decomposition of the random turbulent processes $u(t)$, $w(t)$.

4 TURBULENCE WIND SIMULATION

In the unsteady buffeting response prediction of bridges, the turbulence wind fields $u(t)$, $w(t)$ require to be digitally simulated at the bridge nodes. Digital simulation techniques via the spectral representation can be used basing on Cholesky decomposition and modal factorization. The turbulence wind fields can be expressed in the frequency domain using the Cholesky decomposition as follows (Le 2007), see Figure 2:

$$v_j(t) = \sqrt{2(\Delta f)} \sum_{k=1}^j \sum_{l=1}^{\tilde{N}} \sqrt{S_{v_j}(f)} G_{jk}(f_{kl}) \cos(2\pi f_{kl} t + \phi_{kl}) \quad (12)$$

where $v_j(t)$: simulated turbulence process at node j ($v(t) = u(t), w(t)$); j : index of structural node;

k : index of moving node; l : index of moving point in frequency range; $\tilde{N}$: number of frequency intervals; Δf : frequency interval $\Delta f = f_{up}/\tilde{N}$; f_{up} : upper cutoff frequency; f_{kl} :

frequency point on frequency range $f_{kl} = (l-1)\Delta f + k\Delta f/N$; $S_{v_j}(f)$: auto spectral density function of turbulence processes $v(u(t), w(t))$ at bridge node j ; $G_{jk}(f_{kl})$: coefficient matrix

containing coherence values and dimension of bridge decks; N : number of deck nodes; ϕ_{kl} : random phase angles, uniformly distributed over $[0, 2\pi]$ generated randomly by the Monte Carlo technique.

One can use the modal factorization or the spectral matrix-based proper orthogonal decomposition to simulate the turbulence wind fields as following expression (Le 2007), see Figure 2:

$$v_i(t) = 2\sqrt{\Delta f} \sum_{j=1}^{\tilde{N}} \sum_{l=1}^{\tilde{N}} \psi_{v_j}(f_l) \sqrt{\lambda_{v_j}(f_l)} \cos(2\pi f_l t + \phi_l) \quad (13)$$

where Δf_l : frequency interval at point l ; Δf : constantly frequency interval $\Delta f = f_{up}/\tilde{N}$ and

$f_l = (l-1)\Delta f$, f_{up} : upper cut-off frequency; $\psi_{v_j}(f_l)$: spectral vector of the turbulence process at frequency f_l ; $\lambda_{v_j}(f_l)$: spectral eigenvalue of the turbulence process at frequency f_l ; ϕ_l : phase angle considered as random variable uniformly distributed over $[0, 2\pi]$. The random phase angles can also be digitally generated by the Monte Carlo technique.

Accuracy of simulated turbulence processes must be verified by comparing between power spectral densities of simulated processes and targeted power spectral densities, and between coherence of simulated processes and targeted coherence as well.

5 BUFFETING RESPONSE PREDICTION

Multi-degree-of-freedom motion (MDOF) equations of structures subjected to the full-scale buffeting forces can be expressed thanks to finite element method (FEM):

$$M\ddot{U} + C\dot{U} + KU = F_b(t) \quad (14)$$

where $U = \{h(t)^T, p(t)^T, a(t)^T\}^T$: displacement vector containing three vertical, lateral and rotational displacements; $\dot{U}, \ddot{U}$: velocity and acceleration vectors; $F_b(t)$: full-scale buffeting forces.

Structurally global responses can be expressed into the mass matrix-normalized generalized coordinates as follows:

$$U(t) = \Phi \xi(t) = \sum_{i=1}^{\bar{M}} \phi_i \xi_i \quad (15)$$

where $\xi(t) = \{\xi_1(t), \xi_2(t), \dots, \xi_M(t)\}$: structural generalized coordinates; $\Phi = [\phi_1, \phi_2, \dots, \phi_M]^T$: modal matrix; $\bar{M}$: number of truncated structural modes ($\bar{M} \ll M$);

Therefore, single-degree-of-freedom (SDOF) motion equation of the i-th generalized coordinate is determined:

$$\ddot{\xi}_i(t) + 2\zeta_i \omega_i \dot{\xi}_i(t) + \omega_i^2 \xi_i(t) = \frac{1}{2} \phi_i^T \rho U B [C_u u(t) + C_w w(t)] \quad (16)$$

where ω_i, ζ_i : circular frequency, damping ratio of the i-th generalized coordinate.

Using the covariance proper transformation, the turbulence wind fields $u(t)$, $w(t)$ can be approximated by uncorrelated subprocesses and covariance eigenvectors (or covariance turbulent modes). Thus, the SDOF motion equation is expressed such following forms as:

$$\ddot{\xi}_i(t) + 2\zeta_i \omega_i \dot{\xi}_i(t) + \omega_i^2 \xi_i(t) = \frac{1}{2} \rho U B \left[\phi_i^T C_u \sum_{j=1}^{\tilde{M}} \theta_{uj} y_{uj}(t) + \phi_i^T C_w \sum_{j=1}^{\tilde{M}} \theta_{wj} y_{wj}(t) \right] \quad (17a)$$

$$\ddot{\xi}_i(t) + 2\zeta_i \omega_i \dot{\xi}_i(t) + \omega_i^2 \xi_i(t) = \frac{1}{2} \rho U B \left[\sum_{j=1}^{\tilde{M}} A_{uij} y_{uj}(t) + \sum_{j=1}^{\tilde{M}} A_{wij} y_{wj}(t) \right] \quad (17b)$$

where $A_{uij} = \phi_i C_u \theta_{uj}$; $A_{wij} = \phi_i C_w \theta_{wj}$: cross modal participation coefficients accounting for interaction between structural modes and covariance turbulent modes; C_u, C_w : full-scale force coefficients relating to wind velocity, aerodynamic force coefficients and deck length;

$y_u(t) = \{y_{u_1}(t), y_{u_2}(t), \dots, y_{u_N}(t)\}^T$, $y_w(t) = \{y_{w_1}(t), y_{w_2}(t), \dots, y_{w_N}(t)\}^T$: covariance principal coordinates, which are determined from original turbulent fields as follows

$$y_u(t) = u(t) \Theta_u; \quad y_{u_j}(t) = u(t) \theta_{u_j} \quad (18a)$$

$$y_w(t) = w(t) \Theta_w; \quad y_{w_j}(t) = w(t) \theta_{w_j} \quad (18b)$$

Therefore, the SDOF motion equations associated with vertical, horizontal and rotation structural principal coordinates can be expressed as follows:

$$\ddot{\xi}_{hi}(t) + 2\zeta_{hi} \omega_{hi} \dot{\xi}_{hi}(t) + \omega_{hi}^2 \xi_{hi}(t) = \frac{1}{2} \rho U B \left[\sum_{j=1}^{\tilde{M}} A_{hij}^L y_{uj}(t) + \sum_{j=1}^{\tilde{M}} A_{hij}^L y_{wj}(t) \right] \quad (19a)$$

$$\ddot{\xi}_{pi}(t) + 2\zeta_{pi} \omega_{pi} \dot{\xi}_{pi}(t) + \omega_{pi}^2 \xi_{pi}(t) = \frac{1}{2} \rho U B \left[\sum_{j=1}^{\tilde{M}} A_{pij}^D y_{uj}(t) + \sum_{j=1}^{\tilde{M}} A_{pij}^D y_{wj}(t) \right] \quad (19b)$$

$$\ddot{\xi}_{ai}(t) + 2\zeta_{ai} \omega_{ai} \dot{\xi}_{ai}(t) + \omega_{ai}^2 \xi_{ai}(t) = \frac{1}{2} \rho U B \left[\sum_{j=1}^{\tilde{M}} A_{aij}^M y_{uj}(t) + \sum_{j=1}^{\tilde{M}} A_{aij}^M y_{wj}(t) \right] \quad (19c)$$

where $A_{uij}^L, A_{wij}^L, A_{hij}^D, A_{pij}^D, A_{aij}^M, A_{wij}^M$: cross modal participation coefficients between structural modes and covariance turbulent modes which are correspondent to u-, w-turbulences and turbulent-induced lift, drag, moment $A_{uij}^L = \phi_{hi} C_u^L \theta_{uj}$; $A_{wij}^L = \phi_{hi} C_w^L \theta_{wj}$; $A_{hij}^D = \phi_{pi} C_u^D \theta_{uj}$; $A_{pij}^D = \phi_{hi} C_w^D \theta_{wj}$;

$$A_{aij}^M = \phi_{ai} C_u^M \theta_{uj}; \quad A_{wij}^M = \phi_{ai} C_w^M \theta_{wj}$$

The SDOF motion equations system is formulated in the time domain. Finding out solution of this system can be obtained based on the direct integration methods such as the Newmark-beta method, the fourth-order Runge-Kutta method and so on. Finally, the structural global responses (vertical, lateral and rotational displacements) of bridges can be determined based on the superposition principle as follows:

$$U_h = \sum_{i=1}^{\bar{M}_h} \phi_{hi} \xi_{hi}(t) ; U_p = \sum_{i=1}^{\bar{M}_p} \phi_{pi} \xi_{pi}(t) ; U_a = \sum_{i=1}^{\bar{M}_a} \phi_{ai} \xi_{ai}(t) \quad (20)$$

where U_h, U_p, U_a : global vertical, lateral and rotational displacements, respectively; ϕ_h, ϕ_p, ϕ_a : structural modes corresponding to three responses; $\bar{M}_h, \bar{M}_p, \bar{M}_a$: numbers of component modes in combination of responses.

6 RESULTS AND DISCUSIONS

A medium-span cable-stayed bridge has been taken for numerical example using the computational procedures. Span arrangement is 40.5+97+40.5=178m, while there are 30 total nodes in the bridge deck (nodes 8, 23 at pylons). The global responses of bridge are required to predict at the mean velocity range between 5m/s and 40m/s. 3-D frame model has built thanks to the finite element method (FEM), see Figure 3. First ten structural modes have been analyzed. Natural frequencies of the first ten structural modes vary between 0.61Hz and 1.86Hz. Structural damping ratios are assumed to be 0.005 for all modes. Aerodynamic static coefficients of cross section at balanced attack angle and their first derivatives have been experimentally determined: $C_L = 0.158$, $C_D = 0.041$, $C_M = 0.174$, $C'_L = 3.73$, $C'_D \approx 0$, $C'_M = 2.06$. One-sided auto spectral density functions of u-, w-turbulent components are obtained due to the Kaimail's and the Panofsky's spectral models, respectively (Simiu and Scanlan 1976). Coherence function is applied the Davenport's empirical exponential function with decay factors with decay factors $c_u = 10$, $c_w = 6.5$ (Davenport 1962). Aerodynamic admittance function is used the Liepmann's empirical function as approximation of the Sears' function (Liepmann 1958), while the mechanical admittance function is determined corresponding to structural generalized coordinates.

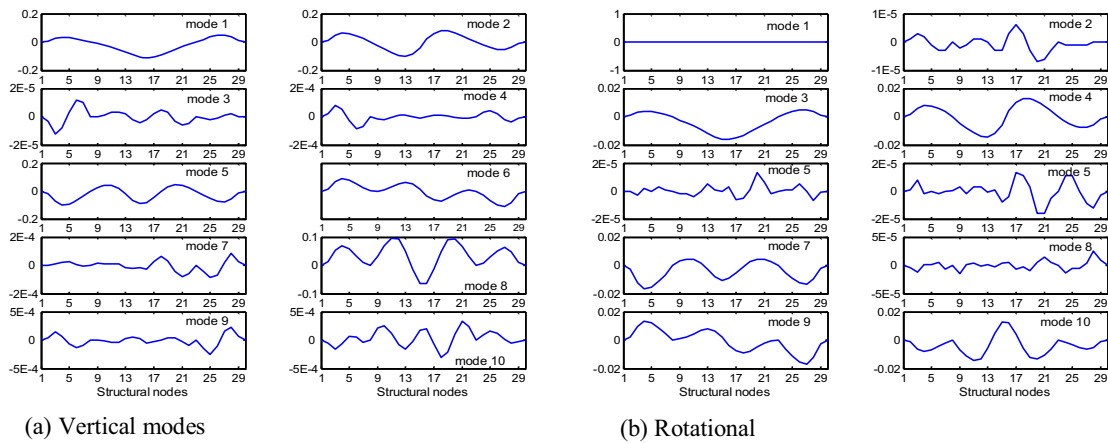


Figure 3. Normalized bridge mode shapes : a. vertical modal displacement, b. rotational modal displacement

At the beginning, the turbulence wind fields at the 30 deck nodes $u(t) = \{u_1(t), u_2(t), \dots, u_{30}(t)\}^T$ and $w(t) = \{w_1(t), w_2(t), \dots, w_{30}(t)\}^T$ have been simulated using the above-mentioned digital simulation procedures. Figure 4 shows simulated turbulence processes at some deck nodes during 100-second interval, at the mean velocity $U=20\text{m/s}$ (other simulated turbulence processes at other deck nodes and other wind velocities omitted for a sake of brevity).

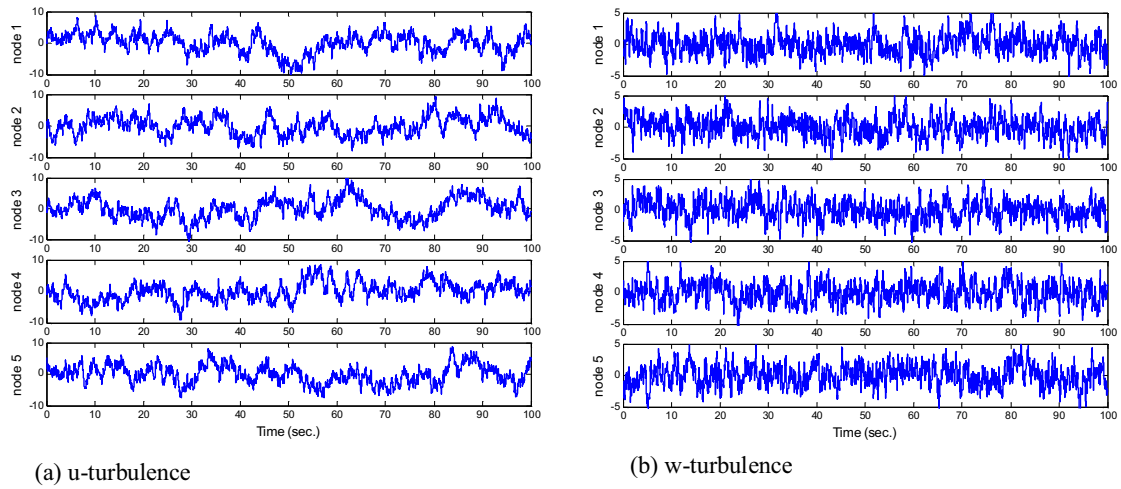


Figure 4. Simulated turbulence processed at some deck nodes at mean velocity $U=20\text{m/s}$

Simulated turbulence fields $u(t)$, $w(t)$ have been used to formulate the zero-time-lag covariance matrix. The covariance eigenvectors (or covariance turbulence modes) and covariance eigenvalues have been found out. From the covariance eigenvalues, eigenvectors and the simulated turbulence wind fields, the covariance principal coordinates then can be determined. Figure 5 shows the first ten covariance turbulence modes of u -, w -turbulence wind fields.

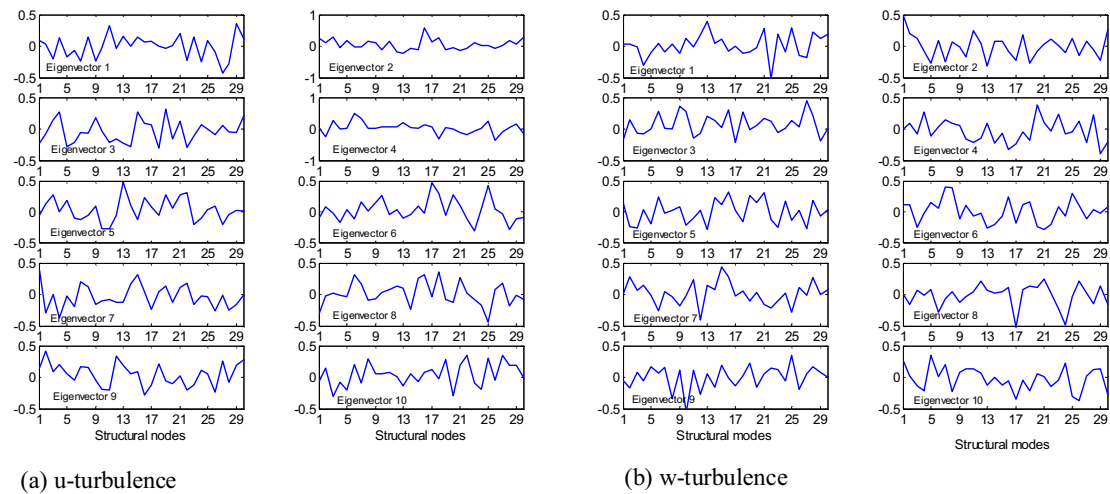


Figure 5. Covariance turbulence modes of u -, w -turbulence wind fields

Time histories of the global buffeting responses of bridge (the vertical, rotational and lateral displacements) can be solved via the direct integration method. Figure 6 indicates the time histories of the vertical and rotational displacements at the main span's middle node No.15 at the wind velocity $U=20\text{m/s}$ during the 100-second period. The maximum response values at all bridge deck nodes at all investigated mean wind velocities can be determined from these time histories of the global buffeting responses.

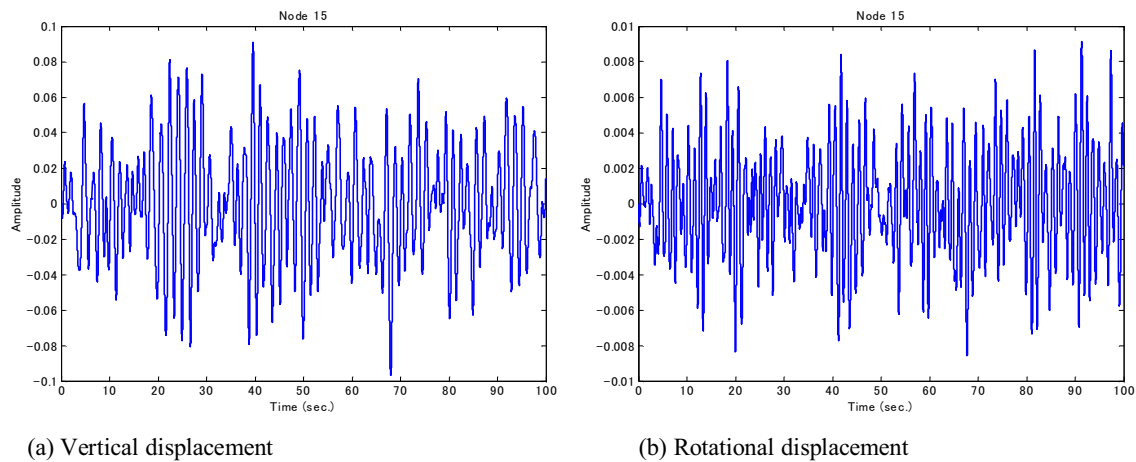


Figure 6. Time histories of global buffeting responses at mid-span node 15 at mean velocity $U=20\text{m/s}$

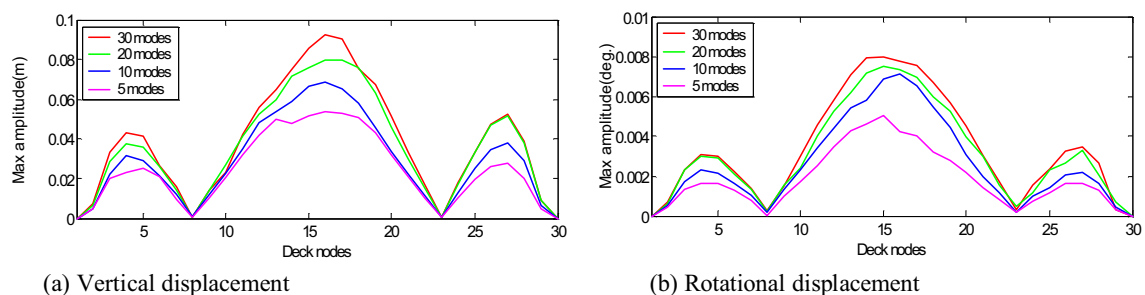


Figure 7. Effect of number of turbulence modes on global responses at all deck nodes at mean velocity $U=20\text{m/s}$

Effect of number of the covariance turbulence modes on the buffeting responses of bridge has been investigated. Figure 7 indicates the maximum structural responses on the bridge deck at $U=20\text{m/s}$ corresponding to number of the turbulence modes ($\tilde{M} = 30; 20; 10 \text{ and } 5$, in which 30 modes imply for targeted response). Apparently, maximum buffeting responses reduce with respect to decrease of truncated number of the turbulence modes. Number of turbulence modes in some extent does influence considerably on the buffeting responses of bridge. Concretely, the maximum vertical amplitudes at midspan node 15 are 9.3cm (100%), 8.0cm (86%), 6.9cm (74%) and 5.3cm (57%) and the maximum torsional amplitudes at node 15 are 0.0078^0 (100%), 0.0075^0 (96%), 0.0071^0 (71%) and 0.0049^0 (63%) corresponding to 30, 20, 10 and 5 turbulence modes taken into account. Therefore, higher number of low-order covariance turbulence modes should be used for estimating the buffeting responses of bridge in the time domain.

7 CONCLUSION

New approach for the quasi-steady and unsteady buffeting response prediction of bridges has been formulated in the time domain using the covariance matrix-based proper orthogonal decomposition. Time histories of the turbulence wind fields on the bridge decks have been digitally simulated via the spectral matrix-based proper orthogonal decomposition. Effect of number of the covariance turbulence modes on the simulated turbulence wind fields and

global buffeting responses of bridges also is investigated accordingly. The computational procedure is promising for the both quasi-steady and unsteady buffeting response analysis of bridges in the time domain with its essential advantages to treat with aerodynamic and geometrical nonlinearities, and unsteady buffeting forces as well. Moreover, it seems that many number of the covariance turbulence modes should be required for more accurate estimation of the buffeting response of bridges. In the other words, the first covariance turbulence mode does not play significant role in its contribution for the buffeting response of bridges in the time domain.

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REFERENCES

- V. Boonyapinyo, T. Miyata, H. Yamada, H., Advanced aerodynamic analysis of suspension bridges by state-space approach, *J. Struct. Engrg.*, ASCE 125, 12(1999) 1357-1366.
- L. Carassale, G. Piccardo, G. Solari, Double modal transformation and wind engineering applications, *J. Engrg. Mech.*, ASCE 127, 5 (1999) 432-439.
- X. Chen, M. Matsumoto, A. Kareem, Time domain flutter and buffeting response analysis of bridges, *J. Engrg. Mech.*, ASCE 126, 1 (2000) 7-16.
- X. Chen, A. Kareem, Advances in modeling of aerodynamic forces on bridges, *J. Engrg. Mech.*, ASCE 128, 11(2002) 1193-1203.
- X. Chen, A. Kareem, Proper orthogonal decomposition-based modeling, analysis, and simulation of dynamic wind load effects on structures, *J. Engrg. Mech.*, ASCE 131, 4(2005) 325-339.
- A.G. Davenport, Buffeting of a suspension bridge by storm winds, *J. Struct. Div.*, ASCE 88, 3(1962) 233-268.
- T.H. Le, Unsteady buffering forces and gust response prediction of bridges with proper orthogonal decomposition applications, Doctoral dissertation, Kyoto University 2007.
- Y.C. Liang, P. Lee, W.Z. Lim, K. Lee, C.G. Wu, Proper orthogonal decomposition and its applications – part I: Theory, *J. Sound Vib.* 252, 3(2002) 527-544.
- J.L. Lumley, *Stochastic Tools in Turbulence*, Academic Press 1970.
- M. Matsumoto, H. Shirato, T.H. Le, Time and frequency domain gust response prediction using proper orthogonal decomposition, The 12th Int'l Conference on Wind Engineering (ICWE12), Cairne Australia, 2007, July 1-6.
- G. Solari, L. Carassale, Modal transformation tools in structural dynamics and wind engineering, *Wind & Structures*, 3, 4 (2000) 221-241.
- Y. Tamura, S. Suganuma, H. Kikuchi, K. Hibi, Proper orthogonal decomposition of random wind pressure field, *J. Fluid Struct.*, 13 (1999) 1069-1095.
- F. Tubino, G. Solari, Gust buffeting of long-span bridges: Double Modal Transformation and effective turbulence, *J. Engrg. Struct.* 29 (2007) 1698-1707.