

Investigation of vortex-induced vibration of twin box girders

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ABSTRACT: Dynamic wind tunnel tests of twin box girders with a space ratio of $L/D=1.70$ have been performed in this study. The results show that although the strength of the vortex is very weak under stationary conditions, it gives rise to vortex-induced vibrations with a lock-in range of $0.570 \leq U_r \leq 0.668$ in dynamic testing. To further study the flow characteristics around the twin box girder when undergoing vortex-induced vibration, a hybrid method combining experiments with numerical simulations is employed. The energy transfer between the flow and motion of the body, and evolution of flow patterns over vortex-induced vibration process are analyzed based on the computational results. The results indicate that with an increase in oscillation amplitude, strong vortices form in the gap between the two box girders. These vortices impinge on the windward wall of the downstream box girder and cause the flow to separate and re-attach periodically around the windward corners of the downstream box girder.

KEYWORDS: twin box girders; vortex-induced vibration; vortex structures;

1 INTRODUCTION

With increases in the spans of bridges, bridges become more flexible and have small damping. Therefore, the bridges more readily oscillate dramatically when subjected to wind. In order to improve the aerodynamic stability of long-span bridge, twin-separated box steel girder configuration may be a good choice. For the kind of bridge, the bridge deck is composed of two parallel longitudinal girders with an open space between them. The two parallel girders are connected by transverse cross-beams. Now some super-long span bridges have chose this section configuration, for instance, Xihoumen suspension Bridge (main span: 1650m, China), Hong Kong Stonecutters cable-stayed Bridge (main span: 1018m, China), Gwangyang suspension Bridge (main span: 1545m, Korea). Although it is proved that twin-separated box girder has higher critical flutter speed than single box girder (Ge and Xiang, 2009), the flow characteristics around the bridge deck become more complicated due to the effects of gap between two separated box girders.

It is well known that when the vortex shedding frequency is close to the natural frequency of the body, it can cause vortex-induced resonance. Although vortex-induced vibration is limited amplitude vibration and does not cause collapse for the bridge, it can result in large displacements and discomfort to the drivers. In addition, the vortex-induced oscillations commonly occur in the low wind speed region, so the occurrence probability of vortex-induced vibration is high, resulting in long-term fatigue damage. The main objective of the paper is to investigate the flow characteristics around twin box girders and deeply understand the mechanism of vortex-induced vibration of twin box girders.

2 EXPERIMENTAL INVESTIGATION OF VORTEX-INDUCED VIBRATIONS OF TWIN BOX GIRDERS

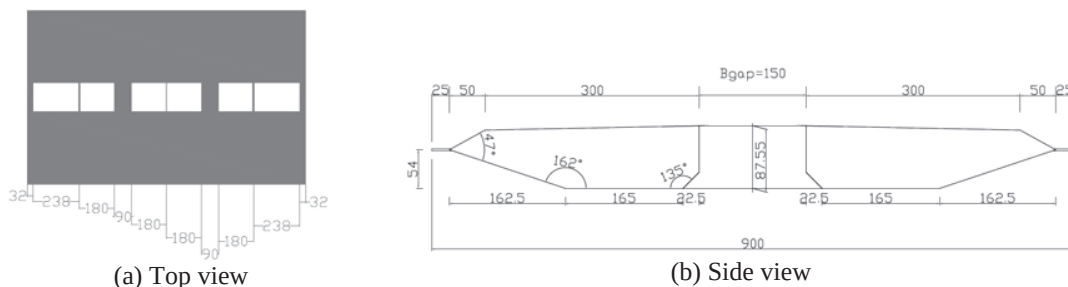
2.1 Experiment set-up

The experiments are conducted in a closed circuit wind tunnel, which has a small rectangle testing section of 4 m in width, 3 m in height and 25 m in length and a large rectangle testing section of 6 m in width, 3.5 m in height and 50 m in length (see Figure 1). In the small testing section, the maximum wind velocity can be 50 m/s, the turbulence intensity of the free stream can be no stronger than 0.46% and the non-uniformity of free-stream is less than 1%. In this study, all experiments are carried out in the small testing section.

The model is composed of two parallel box girders with an open space of $L=150$ mm. The two parallel girders are connected by transverse cross-beams every 360 mm. The length and height of the model are 1440 mm and 87.55 mm, respectively. Detailed geometry of the section model is shown in Figure 2. In the dynamic test, the model is supported by 8 springs allowing a vertical degree of freedom with a natural frequency of 4.272 Hz and damping ratio of 0.21%. The lower surface of the section model is 1.20 m from the bottom of the wind tunnel. The geometric blockage ratio in the testing section is 2.9%; thus the blockage effects are negligible.

Two one-dimensional hotwire probes (Dantec 55P11) are used in the experiment. One is placed at 1.5 m upstream from the leading edge of the section to measure the oncoming free-stream velocity, while the other probe is moved to different locations near the section model to obtain the vortex-shedding information. The sampling frequencies of both hotwire probes were set at 1 kHz.

When carrying out the vortex-induced vibration test, the inflow velocity is increased step-wise, and the increment of each step is approximately 0.05 m/s over the range of $2.19 \leq U \leq 2.65$ m/s ($1.3e5 \leq Re \leq 1.59e5$).



(a) Top view
Figure 1 Geometry information of the section model

2.2 Vortex-induced response

Figure 2 presents the time history of vertical acceleration when the twin box girder is undergoing vortex-induced vibrations. The vertical displacement of oscillation is obtained by integrating the vertical acceleration twice. Figure 3 shows the relationship between the RMS of displacement response and reduced wind velocity ($U_r = U/f_{\text{vertical}}B$, in which U_r is the reduced wind velocity and f_{vertical} is the vertical nature frequency of the model). The response amplitude increases in two steps with an increase in the reduced inflow velocity up to $U_r=0.650$. Over the range of $0.57 \leq U_r \leq 0.603$, the amplitude increases extremely slowly, whereas over the range of $0.616 \leq U_r \leq 0.650$, the amplitude increases very rapidly. At $U_r=0.668$, the amplitude undergoes a small reduction. When the reduced inflow velocity increases to 0.689, the amplitude of vortex-induced vibration abruptly drops to zero, which suggests that the vortex exciting force suddenly

disappears. In the whole process of vortex-induced vibration, the largest RMS of non-dimensional displacement, $RMS(y)/D$, is 0.029 at $U_r=0.650$, which is much smaller than that of rigid cylinders, where the maximum is 0.11 at the same Skop-Griffin parameter of $S_G=1.875$ (Sarpkaya, 2004). In the range of $0.57 \leq U_r \leq 0.603$, the vortex-shedding frequency is controlled by the oscillation frequency of the body, which is identical to the natural vertical frequency of the system. Beyond this range, the vortex-shedding frequency follows Strouhal's law. The variation of the dimensionless vortex-shedding frequency with the reduced wind velocity is shown in Figure 4.

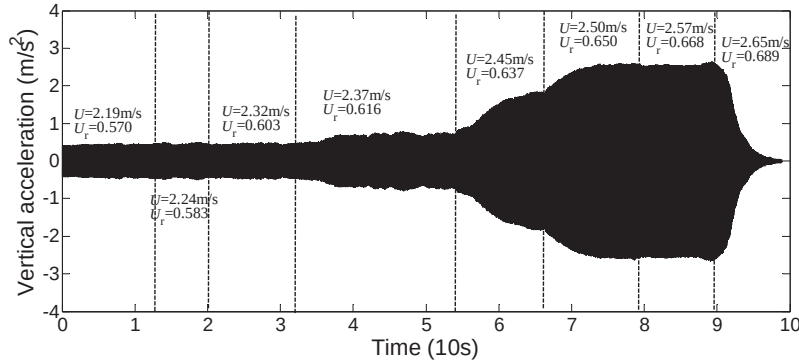


Figure 2. Time histories of vertical acceleration.

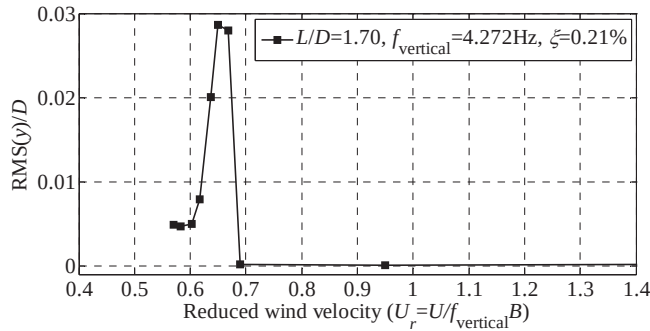


Figure 3. Vortex response of the twin separated box girders.

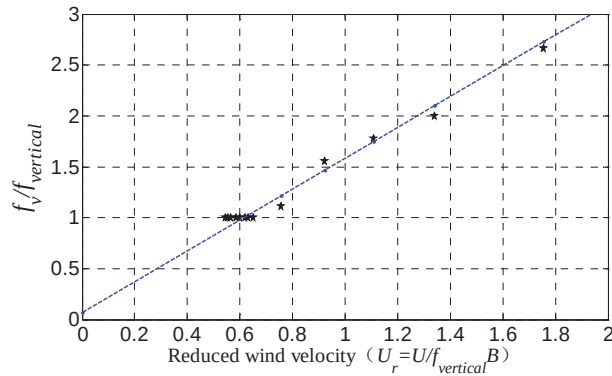


Figure 4. Variation of the dimensionless vortex-shedding frequency with the reduced wind velocity (f_v is the vortex-shedding frequency).

3 NUMERICAL INVESTIGATION OF VORTEX-INDUCED VIBRATIONS OF A TWIN BOX GIRDER

3.1 Computational method

In the wind tunnel test, the motion of a structure is easy to measure. However, it is very difficult to accurately measure the fluid structures of a flow field, which usually requires very expensive instrument, e.g., PIV. To obtain characteristics of flow structures when the body is undergoing vortex-induced vibration, a hybrid method that combined experiments with numerical simulations (Li et al., 2010) is employed in this paper. For the problem of flow-induced vibration of a bluff body, the analytical model includes two parts: one is fluid dynamics based on the incompressible Navier-Stokes equations; and the other is the dynamics based on the equation of the motion of the body. The two parts are coupled through the no-slip condition, i.e., the velocities of the fluid and the body are identical at the interface. In traditional computational process, the fluid and structure motion equations are solved simultaneously. For the hybrid method, the motion of the structure is already known, which is first obtained from the wind tunnel test or field monitoring, i.e., the flow velocities on the interface are also known. Therefore, the flow-induced vibration problem becomes a moving boundary problem of the flow. Only fluid dynamic equations need to be solved and the motion of the interface is measured from the wind tunnel test or field monitoring, so it is more accurate than the traditional computational method to obtain the flow characteristics.

The computational domain is discretized with an unstructured mesh, as shown in Figure 5. The majority of y^+ values, which are nearest to the wall, are in the range of 0.105-1.2 (only 5.67% of y^+ values higher than 1.2). The numerical simulations are performed with ANSYS CFX 11 code based on the finite volume method. The large eddy simulation (LES) with a Smagorinsky subgrid scale model is employed in the computational process. The advection term is discretized using the Central Difference Scheme in the spatial domain, and the transient term is discretized by the Second Order Backward Euler Scheme in the time domain. The time step is set at 0.001s, and the convergence criteria is such that if the RMS of normalized residual taken over the whole domain is less than 10^{-4} , the iteration process stops.

At the inlet boundary, a steady uniform flow velocity is given. At the outlet boundary, an opening pressure condition is applied, and for the upper and lower surfaces of the computational domain, symmetric conditions are employed.

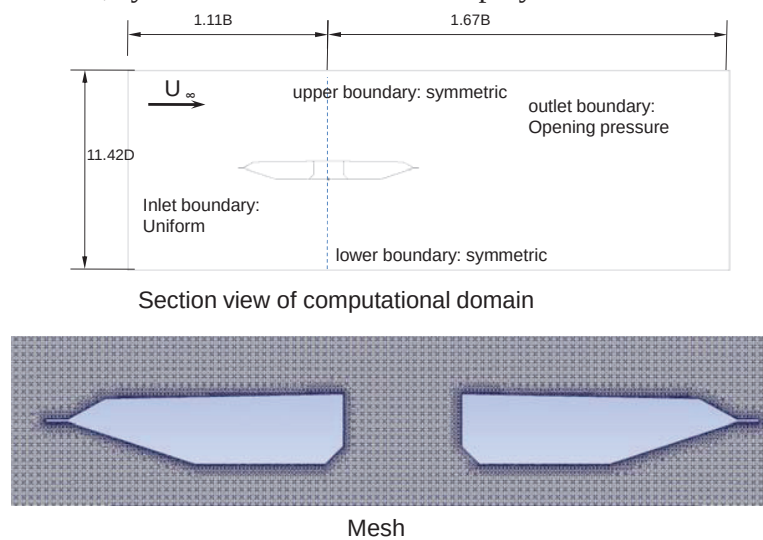


Figure 5. Section view and mesh of the computational domain.

3.2 Energy transfer between flows and the twin box girder

In the vortex-induced vibration process, if the work performed by the aerodynamic force on the body is positive, i.e., the energy of flows is transferred to body, the vibration of the body will gradually increase. The energy transfer is mainly influenced by the amplitude of the fluctuating force and the phase between the fluctuating force and body motion.

The aerodynamic force is calculated by integrating the surface pressure of the twin separated box girders. Lift force coefficient per length is defined as:

$$C_L = \frac{F_L}{\frac{1}{2} \rho_{air} U^2 B L}, \quad (1)$$

where, C_L is the lift force coefficient, F_L is the lift force, U is the inflow velocity, B is the width of model and L is the length of the model.

The phase between the lift force and vertical displacement is calculated by the following equation:

$$\theta(\omega) = \begin{cases} \arccos \frac{\text{real}(P_{ly}(\omega))}{\sqrt{(\text{real}(P_{ly}(\omega)))^2 + (\text{imag}(P_{ly}(\omega)))^2}} & \text{if } \text{imag}(P_{ly}) \geq 0 \\ -\arccos \frac{\text{real}(P_{ly}(\omega))}{\sqrt{(\text{real}(P_{ly}(\omega)))^2 + (\text{imag}(P_{ly}(\omega)))^2}} & \text{if } \text{imag}(P_{ly}) < 0 \end{cases}, \quad (2)$$

where, $\theta(\omega)$ is the phase lag between lift force and vertical displacement, P_{ly} is the cross power spectrum of lift force and vertical displacement and ω is the circular frequency of vertical vibration (ω is equal to the oscillation frequency).

The total work of lift force performed on the twin box girder is obtained as follows:

$$W = \frac{1}{N} \int_{y(KT)}^{y(KT+NT)} F_L dy, \quad (3)$$

where, W is the work of lift force on the body, T is the period of vertical vibration and K and N are the numbers of cycles.

Figure 6 shows the mean lift force coefficient, RMS of fluctuating lift force coefficient, phase angle between lift force and vertical displacement and work of lift force acting on the twin box girder at $U_r=0.570, 0.583, 0.603, 0.616$ and 0.650 . At $U_r=0.570, 0.583$ and 0.603 , the phase between lift force to vertical displacement are $-72.0^\circ, -95.0^\circ$ and -110.0° , respectively, which are nearly in phase with the vertical velocity of body vibration, indicating that the aerodynamic lift force mainly provides negative damping characteristics. However, the net work acting on the structure is very low, which is attributed to a small-fluctuation lift force, so the amplitude of vibration is small at the beginning of oscillation. At $U_r=0.650$, although the phase of the lift force to vertical displacement is about -144.0° , which deviates from the direction of the vertical velocity, the large-fluctuation lift force resulted in large work of lift force on the body, so a dramatic response of the body is observed. It is found that the phase lags between lift force and vertical displacement are very different from those for a circular cylinder, in which phase lags are less than 45.0° in initial branch of oscillation, and lead to an increase in oscillation (Feng, 1968). Unlike a circular cylinder, no phase jump over the vortex-induced vibration process appears for the box girder.

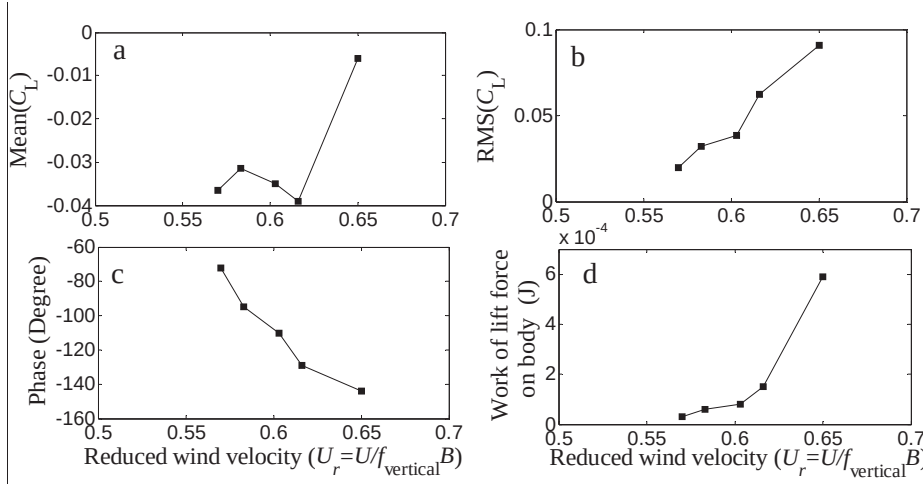


Figure 6. Lift force characteristics and work performed by the lift force at $U_r=0.570, 0.583, 0.603, 0.616$ and 0.650 . (a) mean lift force coefficients, (b) RMS of fluctuating lift force coefficient, (c) phase between lift force and displacement, and (d) work performed by the lift force

The work of lift force on the body can also be rewritten as:

$$\begin{aligned}
 W_k &= \frac{1}{N} \int_{y(KT)}^{y(KT+NT)} \int_S \left\{ -p\delta_{jk} + 2\mu \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \right\} n_j dS dy \\
 &= \int_S \frac{1}{N} \int_{y(KT)}^{y(KT+NT)} \left\{ -p\delta_{jk} + 2\mu \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \right\} n_j dy dS
 \end{aligned} \quad (4)$$

To obtain the energy transfer over the whole surface of the body, a work density function $P(x,y,z)$ on the surface of the twin box girder is defined as:

$$P(x, y, z) = \frac{1}{N} \int_{y(KT)}^{y(KT+NT)} \left\{ -p\delta_{jk} + 2\mu \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \right\} n_j dy. \quad (5)$$

The work density of flow over the surface of the twin box girder at $U_r=0.570$ and $U_r=0.650$ at $z=630$ mm is shown in Figure 7. The positive values of the work density show that the flow energies are transferred to body, while the negative values indicate that the energy of body motion is transferred to the flows. At $U_r=0.570$, the beginning of vortex-induced vibration, the energy of flows is transferred to the body in the regions of $X/B=[0.217 \ 0.412]$ for the lower surface and $X/B=[0.190 \ 0.450]$ for the upper surface due to the influence of shedding vortices in the near wake. However, for the case of $U_r=0.650$ (the amplitude of response is largest), besides the work density becoming much larger, the region where the energy of the flows transfers to the body is located mainly near the gap. This implies that the vortices in the gap play a critical role to vortex-induced vibrations of twin box girders.

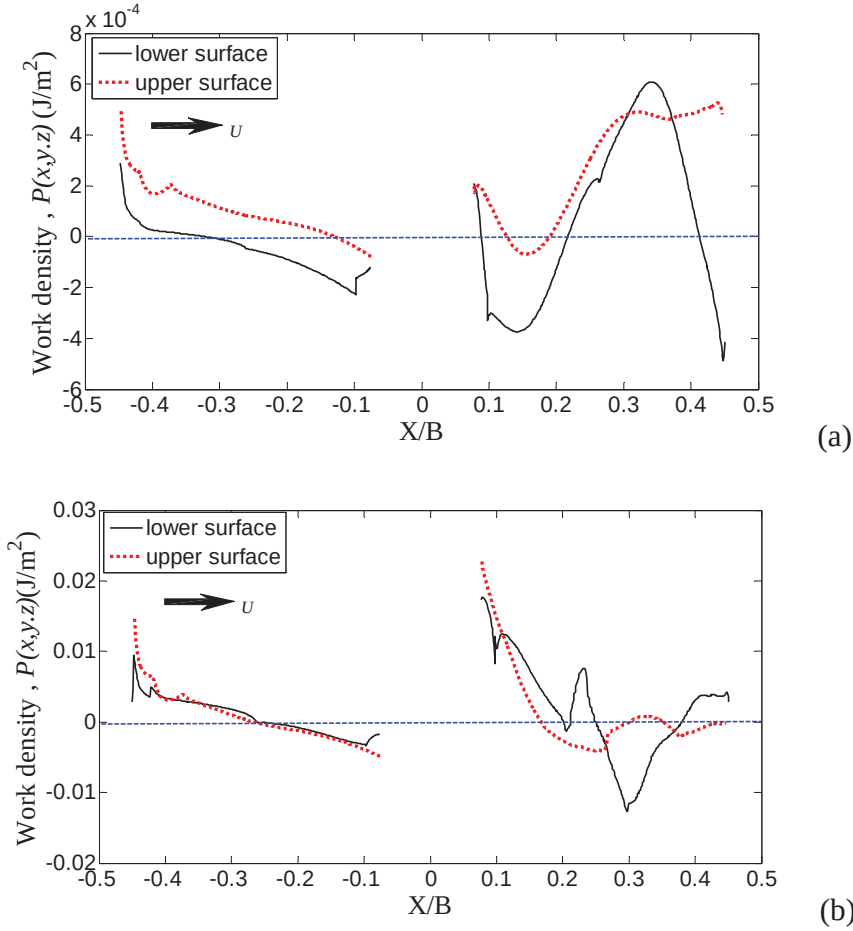


Figure 7. Work density over the surface of the twin box girder at $z=630$ mm (a) $U_r=0.570$, and (b) $U_r=0.650$.

3.3 Flow pattern evolution when the twin box girder undergoes vortex-induced vibration

In the vortex-induced vibration process, the structure motion and vortex interact with each other, forming a feedback loop system. Therefore, the vortex structure and vortex-shedding modes have significant effects on the structural oscillation. Although the vortex structures can be recognized according to many criteria, for example, Q-criterion (Hunt, 1988), Δ -criterion (Dallmann, 1983, and Chong, 1990), λ_2 -criterion (Jeong and Hussain, 1995), etc., there still lacks a common agreement, and these methods have their respective advantages and limitations. In this study, the λ_2 -criterion is selected to obtain the vortex structure. Jeong and Hussain (1995) argued that, if discarding unsteady straining and viscous effects, the existence of a minimum pressure value and existence of a vortex core are consistent. By taking the gradient of the Navier-Stokes equation and decomposing it into symmetric and asymmetric parts, the following equation can be deduced:

$$\frac{DS_{ij}}{Dt} - \nu S_{ij, kk} + \Omega_{ik} \Omega_{kj} + S_{ik} S_{kj} = -\frac{1}{\rho} p_{,ij} \quad (6)$$

where, S_{ij} is the strain tensor, $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, Ω_{ij} is the rotation tensor,

$$\Omega_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \text{ and } p_{,ij} \text{ is the Hessian pressure.}$$

The occurrence of a local pressure minimum in a plane requires two positive eigenvalues of the tensor $p_{,ij}$. Discarding the first term of Eq.(6), which represents an unsteady irrotational straining effect and the second term, which represents viscous effects, yields only $S^2 + \Omega^2$ to determine the existence of a local pressure minimum. Therefore, the vortex core definition criterion is that the vortex is the location with two negative eigenvalues of $S^2 + \Omega^2$. Assuming λ_1 , λ_2 and λ_3 are the eigenvalues of $S^2 + \Omega^2$ and $\lambda_1 \geq \lambda_2 \geq \lambda_3$, the vortex identification criterion is equivalent to the condition that $\lambda_2 < 0$.

Figure 8 presents the vortex core structures at $U_r=0.570$ and $U_r=0.650$. At the initial stage of vortex-induced vibration, i.e., $U_r=0.570$, the body motion leads to the boundary layers of the upstream box girder becoming unstable and generates vortices on the upper and lower sides of the gap. It is worth noting that these vortices are generated due to the flow by the body motion disturbance rather than the direct interaction of the upper and lower shear layers. The shedding vortices propagate downstream along the upper and lower surfaces of the downstream box girder and then merge with the vortices generated in the inclined regions of the downstream box girder. Finally, the merging vortices are shed alternately in the near wake of the downstream box girder with the 2S shedding mode, i.e., when the displacement is largest, the single vortex sheds from the opposite side. Due to the asymmetrical geometry of the wind nose (inclined regions of box girder), asymmetrical vortex structures are generated in the upper and lower sides of the near wake. For the lower side of the nose, the width of the inclined plane is very long ($L_{\text{inclined}}/B=0.18$), so the nascent vortex on the lower side of the wake is long and narrow, and the energy distribution of the vortex is not uniform. This special kind of vortex structure results in instability of the primary vortex on the upside of wake and rapid translation into 3-D structures. However, for the upper side of the nose, the dimensionless width of the inclined plane L_{inclined}/B is only 0.056, so the primary vortex is more regular and stays stable when propagating downstream.

When the amplitude of the vortex-induced vibration is largest, the vortex-shedding phenomena and vortex structures change significantly. The interaction between the upper and lower boundary shear layers of the upstream box girder is enhanced through the gap, and thus, strong vortices generate around the gap. These vortices propagate downstream along the surfaces of the downstream box girder and finally break into small vortices in the nose region. In the near wake of the downstream box girder, because of body motion disturbance, the primary vortices, which appear at the initial stage of vortex-induced vibrations, disappear, and the flow transforms into turbulent flow. As shown in Figure 9, the vortex-shedding phenomena around the gap are very complicated. As the body reaches its maximum displacement, the nascent vortex of the side impinges on the windward wall of downstream box girder and cuts off the vorticity supply of the opposite vortex, so a free vortex forms. In addition, the flow from the opposite side separates around the windward corner of the downstream box girder and a new vortex begins to form at that moment. After approximately 1/4 of a cycle, i.e., the displacement of body is zero, the separated flow begins to re-attach, and the flow re-attaches on the surface of the downstream box girder completely as the body reaches the maximum displacement on the same side.

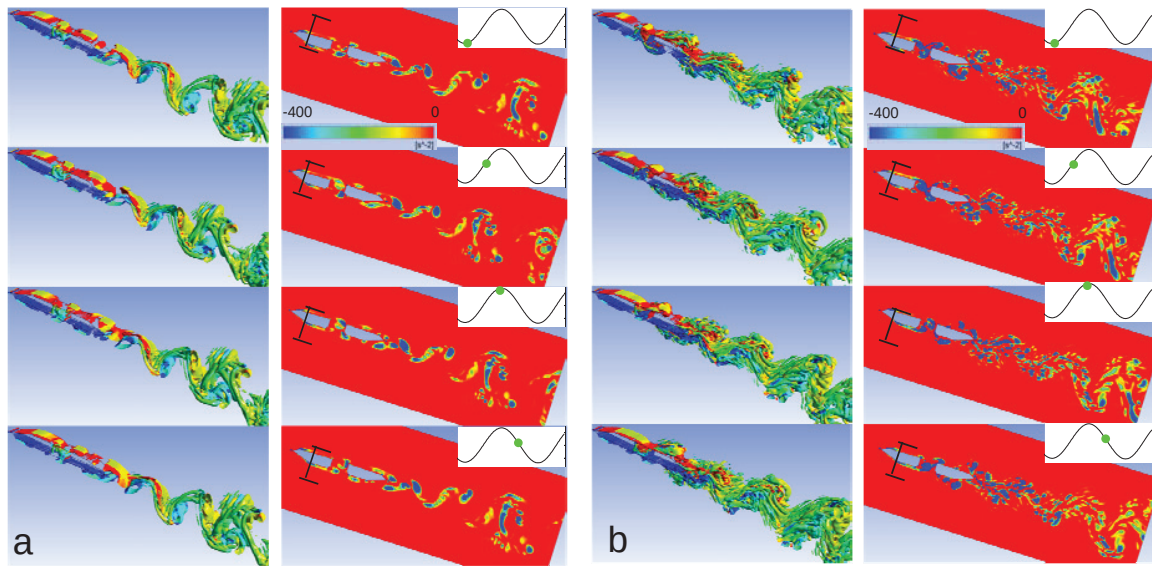


Figure 8. Vortex core structures, Left: Iso-surfaces of $\lambda_2 = -10 [\text{s}^{-2}]$; Right: the contour plots of λ_2 at section of $Z = 630 \text{ mm}$ (a) $Ur = 0.570$ and (b) $Ur = 0.650$.

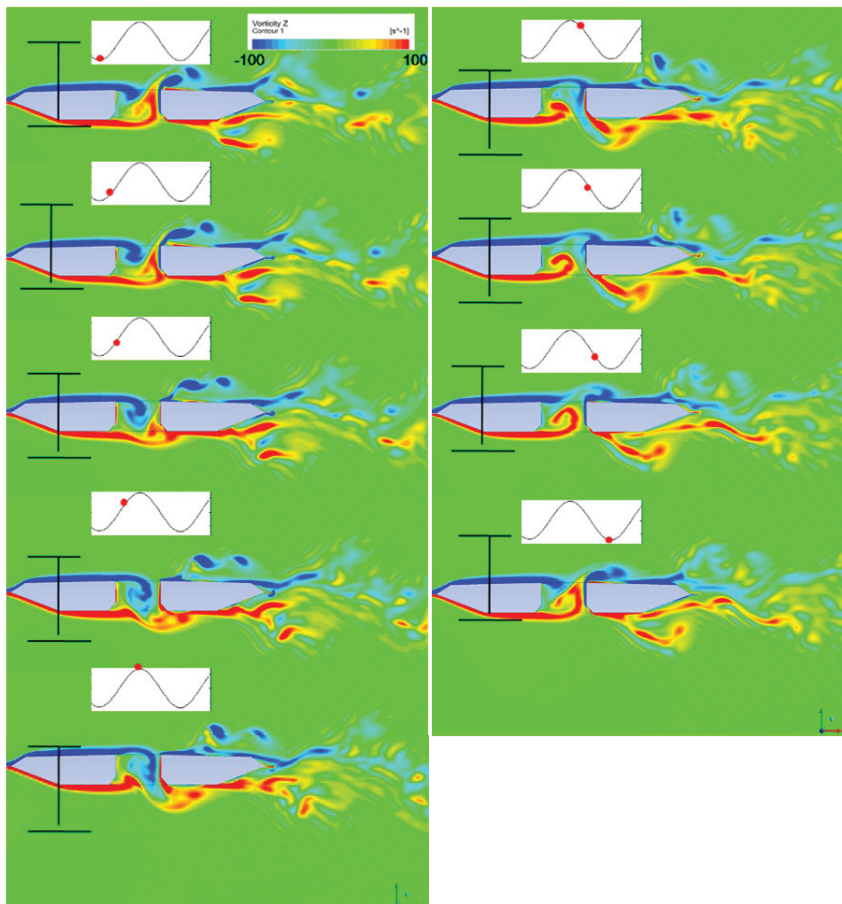


Figure 9. Instantaneous vorticity contours in a periodic vibration at $z = 630 \text{ mm}$.

4 CONCLUSION

The experimental investigations of vortex-induced vibrations of a twin box girder model with a gap ratio $L/D=1.70$ are carried out in this study. The following conclusions can be obtained:

Under stationary conditions, there is no occurrence of regular Karman vortices in the gap, but regular vortex-shedding phenomena with $St=0.15$ appear in the near wake of the downstream box girder. Although the strength of the primary vortex is very weak in the stationary test, vortex-induced vibration of the twin box girder occurs in the dynamic test. The synchronization range is $0.570 \leq U_r \leq 0.668$. Furthermore, the largest RMS of non-dimensional displacement is only 0.029 at $U_r=0.650$, which is much smaller than that of circular cylinders.

A hybrid method is employed to investigate the flow characteristics around the twin box girders when undergoing vortex-induced vibrations. The results show that the regular vortices are generated only in the near wake of the downstream box girder with the 2S shedding mode at the initial stage of vortex-induced vibration. However, as the amplitude of oscillation becomes large, strong vortices are generated in the gap and the flow of near wake transforms into turbulent flow. The vortices impinge on the windward wall of the downstream box girder when propagating downstream and cause the flow to separate and re-attach periodically around the windward corners of the downstream box girder. Due to the effects of vortices in the gap, the pressure shows high fluctuating characteristics near the windward corner of the downstream box girder. The energy of body oscillation is generated mainly by the pressures of these regions.

5 ACKNOWLEDGEMENTS

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