

# Accurate equivalent static wind loads of long-span bridges based on the displacement responses

Dong Rui Ge Yao-jun Yang Yong-xin

*State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University  
Room302, Bridge Building, Tongji University, 1239 Siping Road, Shanghai, P.R. China*

**ABSTRACT:** Equivalent Static Wind load (ESWL) connects structure engineers and wind engineering researchers, and plays an important role in the field of civil engineering. The long-span bridge is a typical flexible line-like structure in horizontal direction. The displacements are usually the controlling responses in the structure design. Traditional ESWL calculating methods which are based on the most disadvantageous response of one node, such as the displacement in the mid-span or quarter-span, are single target methods. When putting the ESWLs calculated by the traditional single target methods on the bridge, except the most disadvantage node, the other nodes' responses are different from the real values. This paper firstly gets the modes and influence matrix using finite element method, then calculating the PSD matrix of the buffeting forces by coupled flutter and buffeting vibration method. According to random vibration theory and structure dynamics, the ESWLs of long-span bridges in vertical, horizontal and torsional can be expressed by the mode, influence matrix, aerodynamic mechanical admittance matrix, and PSD matrix of the buffeting forces. On the basis of the load-response relationship, the ESWLs calculated through the above method are the only accurate solution which based on the displacements of the bridge.

**KEYWORDS:** Equivalent static wind load, Long-span bridges, Displacement responses, Mode matrix, Influence matrix, Coupled flutter and buffeting vibration.

## 0. INTRODUCTION

Wind is the motion of air. Complex aerodynamic or/and aeroelastic forces can be produced when the air blows structures. When the structure stiffness is big, the wind effect is mainly static performance. On the contrary, the wind can induce significant dynamic responses. The long-span bridge is a typical line-like structure with small stiffness in horizontal, and buffeting effects can not be ignored in most cases. Estimating the buffeting responses of a long-span bridge is a complicated and difficult task, and can not be mastered by most structure engineers. So the wind engineers and researchers proposed the concept of ESWL (Equivalent Static Wind Load) to solve this problem. ESWL transfers the complex and difficult dynamic calculation into simple static analysis, and be accepted by most countries'/regions' wind load codes/specifications. ESWL can be defined as that the responses produced by the static ESWL are equal/approximate to the responses produced by the real wind load. Then the structure engineers can solve complex dynamic problems by using simple static analysis. ESWL has become the bridge which connected wind engineers and structure engineers, and plays an important role in practical projects.

The main traditional ESWL methods have GLF<sup>[1]</sup> (Gust Load Factor) method, LRC<sup>[2][3][4]</sup> (Load-Response Correlation) method, IWL (Inertia Wind Load) method, and LRC+IWL method<sup>[5][6][7]</sup>. All these methods are single-target methods. When putting the ESWLs which calculated by the above methods on the structures, only the response on the equivalent node is the same as the real one. Later Japanese researchers proposed Universal ESWL method<sup>[8]</sup>. This method firstly needs to take several different kinds of single-target ESWLs as the basic vectors of the multi-targets ESWLs. Then calculate the optimal coefficients fitting for the ultimate multi-targets ESWLs by the least square method. In fact, the Universal ESWL method is a numerical processing method and the physical meaning is not clear.

The displacement is usually the key response in the design of a long-span bridge. So, using the accurate ESWLs based on the displacement responses to design the long-span bridge can guarantee the safety of the

bridge design. Based on the long-span bridge's characteristics above, an accurate ESWL method based on the displacement responses is proposed in this paper. And a 300 meters simple bridge with ideal thin plate section is taken as an example to prove the validity and effectiveness of the method.

## 1. ESWLs BASED ON THE DISPLACEMENT RESPONSES

For linear MDOF (multi-degree-of-freedom) system, the governing equation of motion is

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = \{F(t)\} \quad (1)$$

Where,  $[M], [C], [K]$  are mass, damping, and stiffness matrices respectively;  $\{q(t)\}, \{\dot{q}(t)\}, \{\ddot{q}(t)\}, \{F(t)\}$  are dynamic displacement, speed, acceleration and force vectors of the nodes respectively.

According to mode superposition method,  $\{q(t)\}$  can be expressed by mode matrix  $[\Phi]$  and modal coordinate vector  $\{\xi(t)\}$

$$\{q(t)\} = [\Phi]\{\xi(t)\} \quad (2)$$

Submit equation (2) into equation (1), the linear MDOF system can be represented as

$$[I]\{\ddot{\xi}(t)\} + [\bar{M}]^{-1}[\bar{C}]\{\dot{\xi}(t)\} + [\bar{M}]^{-1}[\bar{K}]\{\xi(t)\} = [\bar{M}]^{-1}\{\bar{F}\} \quad (3)$$

Where,  $[I], [\Phi_q]^T[M][\Phi_q] = [\bar{M}], [\Phi_q]^T[C][\Phi_q] = [\bar{C}], [\Phi_q]^T[K][\Phi_q] = [\bar{K}]$  are identity matrix, general mass, damping, and stiffness matrices respectively;  $[\Phi_q]^T\{F(t)\} = \{\bar{F}\}$  is the general node force vector.

Using normalized general mass matrix, then equation (3) can be represented as

$$[I]\{\ddot{\xi}(t)\} + [\Lambda_c]\{\dot{\xi}(t)\} + [\Lambda_\omega]\{\xi(t)\} = \{\bar{F}\} \quad (4)$$

Where,  $[\bar{M}]^{-1}[\bar{C}] = [\Lambda_c] = \text{diag}(2\zeta_1\omega_1 \cdots 2\zeta_{N\text{-mode}}\omega_{N\text{-mode}}), [\bar{M}]^{-1}[\bar{K}] = [\Lambda_\omega] = \text{diag}(\omega_1^2 \cdots \omega_{N\text{-mode}}^2)$ .

For equation (4), the modal coordinate vector can be obtained by Duhamel integral equation in time domain. Because it is time consuming in time domain, so, we solve equation (4) by frequency domain method. Fourier transformation is defined as

$$\bar{\xi}(\omega) = \int_{-\infty}^{+\infty} \xi(t) e^{-i\omega t} dt \quad (5)$$

Where,  $i = \sqrt{-1}$  is imaginary unit.

Equation (4) is represented in frequency domain as

$$(-\omega^2[I] + i\omega[\Lambda_c] + [\Lambda_\omega])\{\bar{\xi}(\omega)\} = [\Phi]^T\{\bar{F}(\omega)\} \quad (6)$$

Define  $[H(\omega)] = -\omega^2[I] + i\omega[\Lambda_c] + [\Lambda_\omega]$  as frequency response function matrix. Then the response produced by dynamic loading can be obtained in frequency domain as

$$\{\bar{\xi}(\omega)\} = [H(\omega)]^{-1}[\Phi]^T\{\bar{F}(\omega)\} \quad (7)$$

Wind loading is one type of random dynamic loadings, so the wind induced responses should be represented in the form of RMS. According to equation (7), PSD matrix of the modal coordinate can be represented as

$$[S_{\xi\xi}(\omega)] = [H(\omega)]^{-1} [\Phi]^T [S_{FF}(\omega)] [\Phi] [H(\omega)]^{-1*T} \quad (8)$$

Where,  $[H(\omega)]^{-1*T}$  is complex conjugate transpose of matrix  $[H(\omega)]^{-1}$ .

The PSD of the displacement responses can be obtained from equation (2)

$$[S_{qq}] = [\Phi] [S_{\xi_i \xi_j}] [\Phi]^T \quad (9)$$

So the covariance matrix of the displacement response can be expressed as

$$[\sigma_{qq}^2] = \left[ \int_0^{+\infty} S_{qq}(\omega) d\omega \right] \quad (10)$$

Based on the load-responses relationship, displacement responses, the key dynamic responses of the structure, can be obtained as

$$\{\hat{R}\} = [I_q] \{F_R^{eq}\} = g \{\sigma_q\} = g \left\{ \sqrt{\int_0^{+\infty} S_{qq}(\omega) d\omega} \right\} \quad (11)$$

Where,  $\{\hat{R}\}$  is the dynamic displacement vector;  $\{F_R^{eq}\}$  is the ESWLs vector;  $[I_q]$  is the influence line matrix. In the calculation, we hypothesize the ESWL on the constraint note whose displacement response is zero, is zero, and the constraint notes are not included in equation (11). Then the ESWLs of the structure can be represented as

$$\{F_{R_i}^{eq}\} = g [I_q]^{-1} \left\{ \sqrt{\int_0^{+\infty} S_{q_i q_i}(\omega) d\omega} \right\} \quad (12)$$

Where,  $g$  is the peak factor, and can take  $g = 3.5$  in most cases.  $[I_q]^{-1}$  is the flexibility matrix.

Equation (12) shows that the ESWLs have clear physical significance. If the mean wind loading is not considered, the ESWLs gotten from equation (12) are the only accurate ESWLs based on the displacement responses.

## 2. ACCURATE ESWLs OF LONG-SPAN BRIDGES BASED ON THE DISPLACEMENT RESPONSES

For long-span bridges, with the increase of the span, the structure stiffness becomes less and less smaller, and buffeting responses induced by wind load can not be ignored. Based on the analysis in part 1, we take the bridge deck as an example to calculate the ESWLs based on the displacement responses. The approach also applies to the bridge tower.

There are mainly three component forces on the bridge deck: lift force, drag force, and twist moment (Figure 1). The buffeting responses are calculated through coupled flutter and buffeting method<sup>[9][10][11][12]</sup>. Self-excited forces are estimated by Scanlan's calculation model<sup>[13]</sup> which contains 18 flutter derivatives. Buffeting forces are estimated by Davenport's buffeting calculation model<sup>[13]</sup>. For convenience, aerodynamic admittances are not considered in the buffeting calculation.

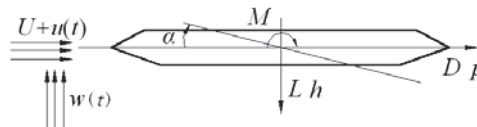


**Figure 1. Freedoms and wind forces in the nodes of the bridge deck**

The frequency response function can be represented as

$$[E_{ij}] = \left( -K^2 [I] + iK \left( [\Lambda_c] - \frac{\rho(2B^4)K}{2} [A_{ij}] \right) + \left( [\Lambda_K] - \frac{(2B^4)K^2}{2} [B_{ij}] \right) \right) \quad (13)$$

Where,  $K = \frac{\omega B}{U}$  is the reduced frequency;  $\frac{\rho(2B^4)K}{2} [A_{ij}]$  and  $\frac{(2B^4)K^2}{2} [B_{ij}]$  are aerodynamic damping and aerodynamic stiffness induced by self-excited forces. Detailed derivatives can be found in reference [11].  $\omega$  is the natural circular frequency;  $B$  is the width of the bridge deck;  $U$  is the mean wind speed;  $\rho$  is air density.

Matrix  $[E]$  contains the influence of self-excited forces, so it is called aerodynamic frequency response function<sup>[14]</sup>. After getting the PSD of buffeting forces  $[S_{b_{b_j}}(\omega)]$ , the PSD of the modal coordinate can be obtained by equation (8)

$$[S_{\xi_i \xi_j}] = \left( \frac{\rho B^4}{2U} \right)^2 [E_{ij}]^{-1} [S_{b_{b_j}}(\omega)] ([E_{ij}]^{-1})^{*T} \quad (14)$$

So the PSD of the displacement responses in vertical, horizontal, and twist can be expressed as

$$\begin{bmatrix} [S_{h_{h_j}}] & [S_{p_i p_j}] & [S_{\alpha_i \alpha_j}] \end{bmatrix} = \begin{bmatrix} [Bh] & [S_{\xi_i \xi_j}] & [Bh]^T & [Bp] & [S_{\xi_i \xi_j}] & [Bp]^T & [\alpha] & [S_{\xi_i \xi_j}] & [\alpha]^T \end{bmatrix} \quad (15)$$

For long-span bridges, the influence line functions in vertical, horizontal, and twist i.e.  $[I_h]$ ,  $[I_p]$ ,  $[I_\alpha]$ , are independent. The ESWLs in vertical, horizontal, and twist can be obtained through equation (12)

$$\{F_{L1}^{eq} \quad \dots \quad F_{LN-node}^{eq}\}^T = g[I_h]^{-1} \left\{ \sqrt{\int_0^{+\infty} S_{h_{h_1}}(K) dK} \quad \dots \quad \sqrt{\int_0^{+\infty} S_{h_{N-node} h_{N-node}}(K) dK} \right\}^T \quad (16-a)$$

$$\{F_{D1}^{eq} \quad \dots \quad F_{DN-node}^{eq}\}^T = g[I_p]^{-1} \left\{ \sqrt{\int_0^{+\infty} S_{p_i p_i}(K) dK} \quad \dots \quad \sqrt{\int_0^{+\infty} S_{p_{N-node} p_{N-node}}(K) dK} \right\}^T \quad (16-b)$$

$$\{F_{M1}^{eq} \quad \dots \quad F_{MN-node}^{eq}\}^T = g[I_\alpha]^{-1} \left\{ \sqrt{\int_0^{+\infty} S_{\alpha_1 \alpha_1}(K) dK} \quad \dots \quad \sqrt{\int_0^{+\infty} S_{\alpha_{N-node} \alpha_{N-node}}(K) dK} \right\}^T \quad (16-c)$$

### 3. EXAMPLE OF ONE SIMPLE BRIDGE WITH IDEAL THIN PLATE SECTION

Due to the aerodynamic forces of the ideal thin plate having theory expression, we take a simple bridge with ideal thin plate section as an example to verify the validity and effectiveness of the method. The length of the simple bridge is  $L = 300\text{m}$ , and the width of the bridge deck is  $B = 40\text{m}$ . The stiffness of the bridge in vertical, horizontal, and twist are separately  $EI_z = 2.1 \times 10^6 \text{ MPa} \cdot \text{m}^4$ ,  $EI_y = 1.8 \times 10^7 \text{ MPa} \cdot \text{m}^4$ ,  $GI_t = 4.1 \times 10^5 \text{ MPa} \cdot \text{m}^4$ . The mass per unit length is  $m_{beam} = 20,000 \text{ kg/m}$ ; the mass moment of inertia per unit length is  $I_m = 4.5 \times 10^6 \text{ kg} \cdot \text{m}^2/\text{m}$ ; the air density is  $\rho = 1.225 \text{ kg/m}^3$ ; the modal damping ratios are assumed to 0.005.

The finite element model of the simple bridge is established by ANSYS software. The bridge deck is simulated by Beam 4 element, and the total bridge is divided into 30 finite elements. In the analysis, the elastic modulus is  $E = 2.06 \times 10^{11} \text{ Pa}$ , the shear modulus  $G = 0.79 \times 10^{11} \text{ Pa}$ . In the finite element model, there are relationships as follows:

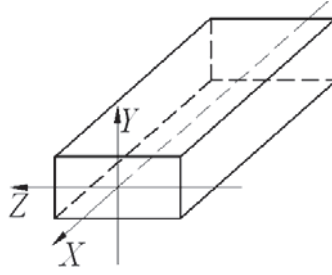


Figure 2. Directions of Beam 4 element

$$I_m = \int_A r^2 dm = \int_A r^2 d(A\rho) = \rho \int_A (y^2 + z^2) dA = \rho \left( \int_A y^2 dA + \int_A z^2 dA \right) = \rho (I_z + I_y) \quad (17)$$

From equation (17), we can get following equation

$$A_{beam} = m_{beam} / \rho = m_{beam} / \left( I_m / (I_z + I_y) \right) = m_{beam} (I_z + I_y) / I_m \quad (18)$$

Where,  $I_y$ ,  $I_z$  are the inertia moment around Y and Z axis separately.  $A_{beam}$  is the section area of the bridge deck.

The boundary conditions of the simple bridge finite element model are as follows: For the left node, constrain UX, UY, UZ and RotX; For the right node, constrain UY, UZ and RotX. (X is the along bridge deck direction; Y is the vertical direction, and Z is the cross bridge deck direction.) The top 50 modes of the simple bridge are gotten using modal analysis. In the modal analysis, the mass matrix is consistent matrix, and the damping is not considered. The top 10 modes are shown in Table 1.

Table 1. Natural frequency and mode shape of simple bridge with ideal thin plate section

| Mode No. | Frequency(Hz) | Mode shape | Mode No. | Frequency (Hz) | Mode shape |
|----------|---------------|------------|----------|----------------|------------|
| 1        | 0.1786        | 1-S-V      | 6        | 1.5154         | 2-S-T      |
| 2        | 0.5033        | 1-S-T      | 7        | 1.5912         | 2-S-V      |
| 3        | 0.5179        | 1-S-L      | 8        | 1.7614         | AD         |
| 4        | 0.7117        | 1-AS-V     | 9        | 2.0076         | 1-AS-L     |
| 5        | 1.0080        | 1-AS-T     | 10       | 2.0270         | 2-AS-T     |

V: vertical bending; T: torsion; L: lateral bending; S: symmetrical; AS: antisymmetrical; A: axial deformation

The self-excited forces of the bridge deck with ideal thin plate section can be expressed by Theodorson function:  $C(k)=F(k)+iG(k)$ , and can also be represented using R. T. Jones's approximate expressions<sup>[15]</sup>.

$$\begin{cases} F(k) = 1 - \frac{0.165}{1 + \left(\frac{0.0455}{k}\right)^2} - \frac{0.335}{1 + \left(\frac{0.03}{k}\right)^2} \\ G(k) = -\frac{0.165 \times 0.0455 / k}{1 + \left(\frac{0.0455}{k}\right)^2} - \frac{0.335 \times 0.03 / k}{1 + \left(\frac{0.03}{k}\right)^2} \end{cases} \quad (19)$$

Where,  $k = \omega b / U = K / 2$  is reduced frequency. Flutter derivatives of the ideal thin plate section bridge deck, i.e.  $H_i^*$ ,  $A_i^*$  ( $i=1\sim 4$ ), can be obtained from equation (19).

$$\begin{cases} 4H_1^* = \frac{-2\pi F}{k}; & 8H_2^* = \frac{-\pi}{k} \left( 1 + F + \frac{2G}{k} \right) \\ 8H_3^* = \frac{-2\pi}{k^2} \left( F - \frac{kG}{2} \right); & 4H_4^* = 2\pi \left( \frac{1}{2} + \frac{G}{k} \right) \\ 8A_1^* = \frac{\pi F}{k}; & 16A_2^* = \frac{\pi}{k} \left( \frac{G}{k} + \frac{F-1}{2} \right) \\ 16A_3^* = \frac{\pi}{k^2} \left( F - \frac{kG}{2} + \frac{k^2}{8} \right); & 8A_4^* = \frac{-\pi G}{k} \end{cases} \quad (20)$$

The flutter derivatives obtained from equation (20) are shown in Figure 3.

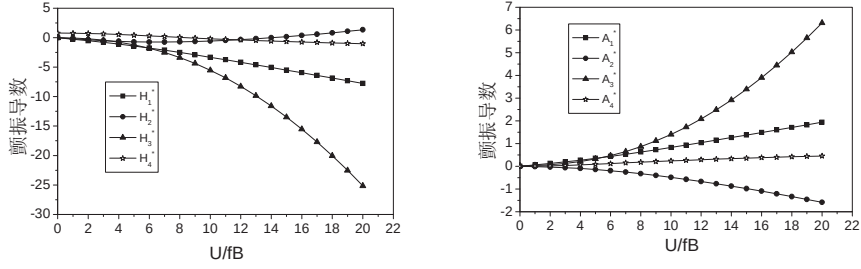


图3 理想平板断面的颤振导数

Figure 3. Flutter derivatives of the ideal thin plate section bridge deck

In order to calculate the buffeting responses, the aerodynamic parameters are taken the same values as reference [12]. The aerodynamic force coefficients in 0 wind attack angle are  $C_L = 0.128$ ,  $C_D = 0.0697$ ,  $C_M = -0.0074$ ,  $dC_L/d\alpha = -5.5577$ ,  $dC_D/d\alpha = 0.0$  and  $dC_M/d\alpha = 1.2662$ . The height of the bridge deck above the ground is  $z=60\text{m}$ . The roughness length is  $z_0=0.01\text{m}$ <sup>[16]</sup>.

The Kaimal spectra<sup>[13]</sup> is chosen as the power spectra of the fluctuating wind in horizontal direction:

$$\frac{nS_{uu}(n)}{u_*^2} = \frac{200f}{(1+50f)^{5/3}} \quad (21)$$

Where,  $f = \frac{nz}{U(z)}$  is the Monin (or similarity) coordinate;  $z$  is the height of the bridge deck above ground;

$U(z)$  is the meaning wind speed at the height of  $z$ .  $u_* = \frac{KU(z)}{\ln(\frac{z}{z_0})}$  is the friction velocity, a function of the

surface roughness.  $K \approx 0.4$  is the Karman constant.  $z_0$  is the roughness length.  $n$  is the frequency of the fluctuating wind (Unit: Hz).

The Lumley – Panofsky spectra<sup>[13]</sup> is chosen as the power spectra of the fluctuating wind in vertical direction:

$$\frac{nS_{ww}(n)}{u_*^2} = \frac{3.36f}{1+10f^{5/3}} \quad (21)$$

The cross spectra can be expressed as

$$S_{uw}(n) = C_{uw}(n) + iQ_{uw}(n) \quad (22)$$

Where,  $C_{uw}(n)$  is the cospectrum, choosing an empirical formula which is suitable for engineering applications;  $Q_{uw}(n)$  is the quadrature spectrum which has not quantitative assessment, and be ignored in the calculation.

$$\frac{nC_{uw}(n)}{u_*^2} = \frac{14f}{1+9.6f^{2.4}} \quad (23)$$

The correlation in spanwise of the fluctuating wind can be expressed in conventional form<sup>[13]</sup> as

$$S(x_i, x_j, n) = S(n) e^{\frac{\lambda n}{U} |x_i - x_j|} \quad (24)$$

Where,  $7 \leq \lambda \leq 21$ , in the calculation  $\lambda = 7$  considering the safety factor;  $x_i, x_j$  are the coordinate in the spanwise;  $|x_i - x_j|$  is the length between  $x_i$  and  $x_j$ .

The maximal displacement responses of the simple bridge had all occurred in the mid-span, and the RMS of the displacement responses including 50 modes are shown in figure 4.

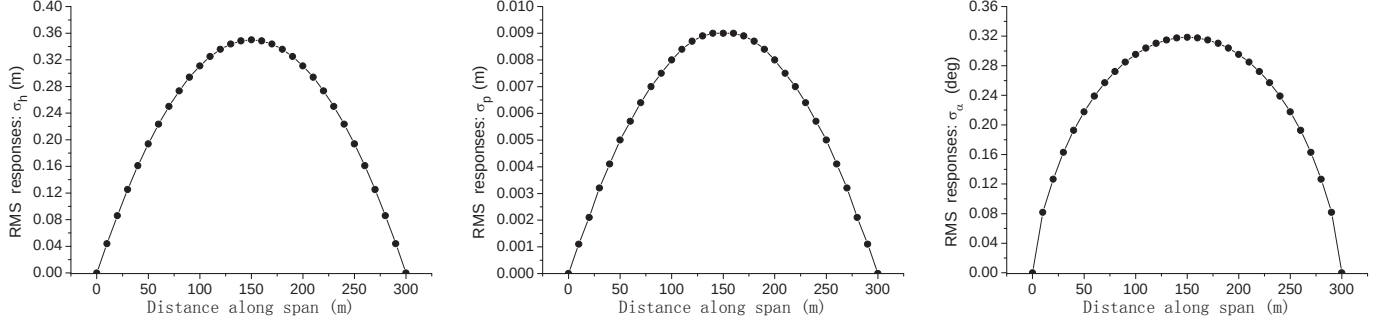


Figure 4. RMS of responses of the simple bridge with ideal thin plate section ( 50 modes )

According to equation (16), the accurate ESWLs of the simple bridge with ideal thin plate section in vertical, horizontal, and twist are shown in Figure 5 (exclude the mean wind load).

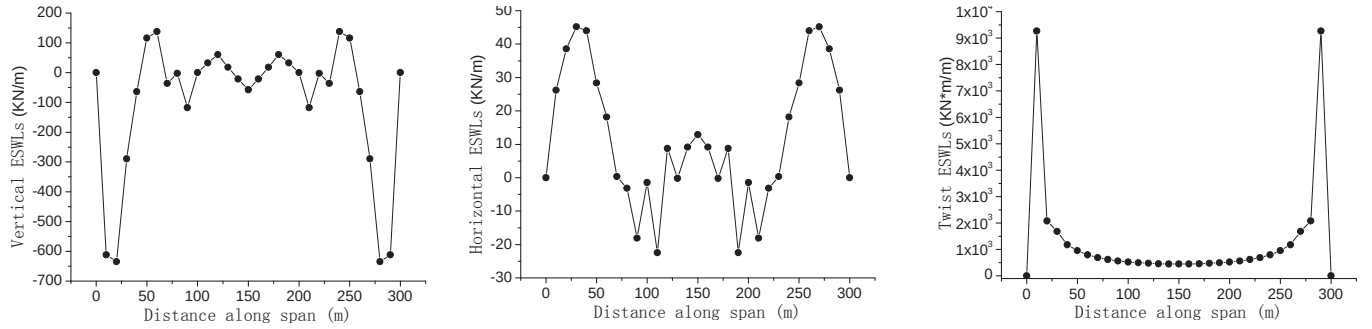


Figure 5. ESWLs of simple bridge with ideal thin plate section

In order to verify the validity of the method, we take the mid-span as an example to check the results. The influence lines of the mid-span are shown in figure 6. Put the ESWLs calculated by the method in this paper on the simple bridge, then the displacement responses can be obtained through static calculation. The results are shown in equation (24).

$$\hat{R}_h = 1.225m = 3.5\sigma_h; \quad \hat{R}_p = 0.0316m = 3.5\sigma_p; \quad \hat{R}_\alpha = 1.115m = 3.5\sigma_\alpha; \quad (24)$$

Equation (24) shows that the method proposed in this paper according to get equivalent static wind load on the structure, the structure of the displacement response and according to the theories of random vibration dynamic response of the structure of the get extreme value are exactly the same, so as to prove the correctness and effectiveness of this method.

The results show that the displacement responses calculated by ESWLs by method in this paper are the same as that calculated by the coupled flutter and buffeting analysis method. So the ESWL method in this paper is validity and effectiveness.

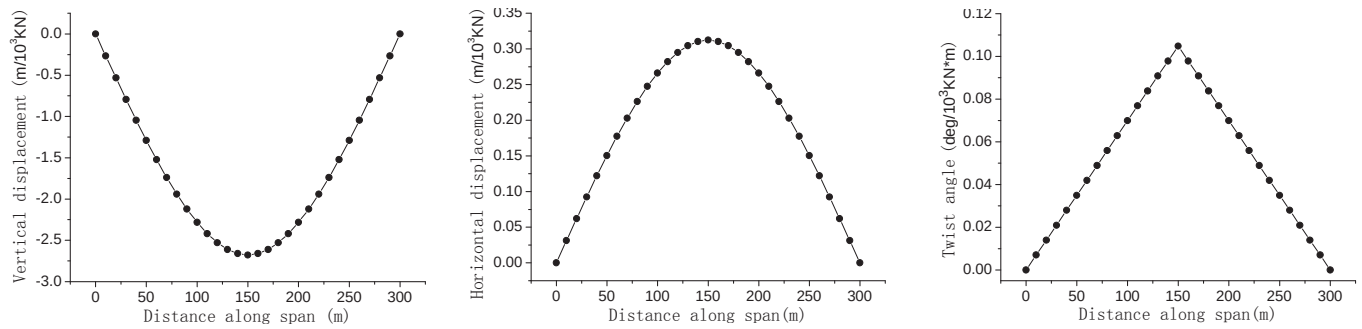


**Table 6. Midspan influence line of the simple bridge with ideal thin plate section**

## 4. CONCLUSION

According to the characteristics of the long-span bridge, one accurate ESWL method based on the displacement responses is proposed in this paper. This paper firstly calculates the mode and influence matrix using finite element method, then getting the PSD matrix of the buffeting forces by coupled flutter and buffeting vibration method. According to random vibration and structure dynamics, the ESWLs of long-span bridges in vertical, horizontal and torsional can be expressed by the mode matrix, influence matrix, aerodynamic mechanical admittance matrix, and PSD matrix of the buffeting forces. According to the load-response relationship, the ESWLs calculated through the method above are the only accurate solution which based on the displacements of the long-span bridge.

## 5. ACKNOWLEDGEMENT

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