

Probability evaluation of flutter failure for long-span bridges

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ABSTRACT: Previous researches found that the moment method is on the unsafe side for structural reliability evaluation compared with the result using Monte Carlo (MC) method for low failure probability events such as flutter instability. In this paper a quadratically asymptotic integral method based on the direct calculation of probability density was proposed in probabilistic evaluation of four parameters of flutter failure mode which was compared with MC method. The fourteen domestic long-span bridges were utilized to validate the proposed algorithm. The results showed that the flutter failure probability based on the proposed method is more reliable and safer than that based on MC method. Of course, this method proposed here involves the calculation about the first-order and second-order derivatives of logarithm of probability density function (PDF) for random variables so that it is not fit in those situations such as unknown PDF of random variables or very complex derivatives.

KEYWORDS: Long-span bridge, Flutter vibration, Flutter failure, Probability evaluation, Monte Carlo method, Quadratically asymptotic integral method.

1 INTRODUCTION

Probability and statistics design method is more reasonable than empirical coefficient design method used in the estimation of flutter failure probability for long-span bridges. Since most of these parameters involved in the prediction are physically uncertain variables and/or subjectively assumed values because of a lack of complete knowledge, it would make more sense to carry out a probabilistic reliability analysis to determine a probability of the bridge flutter failure. Over the years various reliability theories have been proposed for the study of failure probabilities of complex systems, since in many cases an exact numerical calculation is not possible. The first paper, in which approximate methods were proposed, is Freudenthal (A.N. Freudenthal, 1956). The basic idea in this article is to approximate the probability content of a failure region by linearizing the boundary of the region at a point, where the “local” failure probability is maximal. This method is described there only for normal and lognormal random variables, but in principle it is applicable for arbitrary distributions with continuous probability densities.

Hasofer and Lind (A.M. Hasofer and N.C. Lind, 1974) proposed for the case of standardized independent random variables a so-called reliability index β for the failure domains. This index β is the distance of the failure domain to the origin and is not an estimate for the failure probability. But, in consequence of that paper, many papers were published, where approximations for the failure probability were derived by linearizing the failure boundary at the points with minimal distance to the origin, giving $\Phi(-\beta)$ as approximation in the case of normal random variables. Later, also quadratic approximations for the boundary were proposed (B. Fiessler, H.J. Neumann and R. Rackwitz, 1979). All these methods were based on heuristic arguments and no clear mathematical justification was given.

When these approximation methods can not provide ideal probability density distribution, the failure probability of structure can be calculated by directly integral of PDF of random variables in failure margin. Then Breitung (K. Breitung, 1984) showed with methods of asymptotic analysis that for certain methods a mathematical basis can be found. He proved that the asymptotic relative error of quadratic approximations converges to zero, i.e. these approximations are asymptotically exact and the problem of convergence would not be easy to occur.

In the present paper, a reliability analysis model is formulated by considering four random variables, which are mutually independent, and the safety margin which is also a random variable and depends only on the stochastic nature of these variables. Though the probability of failure from the safety margin may be efficiently determined by some approximate reliability methods or by applying simulation techniques such as the MC method, the quadratically asymptotic integral method is applied here and extensively explained through some sample calculations of 14 long-span bridges in China compared with MC method.

2 FLUTTER FAILURE EVALUATION METHODS

2.1 Limit state function of flutter failure

For the sake of comparing and analysing, a flutter reliability model established by Prof. Ge Yaojun was utilized in this study. The corresponding limit state function is represented by the critical flutter speed U_{cr} minus the extreme wind speed U_c (Y.J. Ge, H.F. Xiang, H. Tanaka, 2000)

$$f(U_{cr}, U_c) = U_{cr} - U_c \quad (1)$$

the probability model of the critical flutter speed U_{cr} and the extreme wind speed U_c can be expressed respectively as

$$U_{cr} = C_w U_f \quad (2)$$

$$U_c = G_s U_b \quad (3)$$

where U_f = basic flutter speed which is experimentally determined with some uncertainties of the structural properties; C_w = wind conversion factor from a scaled model to the prototype structure; U_b = basic wind speed at the bridge deck location; and G_s = gust speed factor for taking account of the influence of wind gust speed and its horizontal correlation.

Substituting Eqs.(2) and (3) into Eq.(1), the safety margin Z is now defined as a function which depends on the stochastic nature of a basic variable vector $X = (C_w, U_f, G_s, U_b)$ as

$$Z = f(X) = C_w U_f - G_s U_b \quad (4)$$

where the above four random variables may be described as normal distribution, log-normal distribution, log-distribution and Gumbel distribution respectively. Since f is a non-linear function and the random variables are not all distributed normally, the failure probability can not be gained directly by using PDF of normal distribution. So many algorithms are applied such as a first-order reliability method (FORM) and a second-order reliability method (SORM) etc. Here the probability was calculated by applying both quadratically asymptotic integral method and MC method.

2.2 The MC method

A procedure based on the MC method was implemented to allow for the direct numerical estimation of the probability without first-order reliability approximation. Standard algorithms for MC statistical analysis were considered (C. Bucher, 2009). A standard MC procedure was employed and a large sample of computer generated events (1×10^6) was used to estimate probabilities. A fatal deficiency existing in the MC method is the enormous simulation number when the failure probability of structure is less than 1×10^{-3} . Therefore, the MC method is usually used to validate those proposed methods.

2.3 The quadratically asymptotic integral method

The general procedure of quadratically asymptotic integral method can be resolved into three parts. Firstly, it considers direct integral of probability density function (PDF) in failure region so that non-convergence of calculation can rarely happen. Secondly, it considers direct integral nearby most probable point of structural failure mainly contributes to the evaluation of failure probability. Finally, its direct integral is in fact an asymptotic integral by Taylor series quadratical development for logarithm of PDF of random variables and performance function. The derived quadratic hypersurface is applied in approximating practical failure surface. And then the integral is finished by use of the definition of first-order and second-moment and second-order and second-moment methods.

A likelihood function $h(x)$ of PDF is defined by

$$h(x) = \ln(f(x)) \quad (5)$$

The performance function of structure is defined as $Z = g_x(X)$ and the random variables $X = (X_1, X_2, \dots, X_n)^T$'s joint PDF is defined as $f_x(x)$, then the probability of failure by flutter can be written in the form (K. Breitung, 1984, 1989)

$$p_f = \int_{g_x(x) \leq 0} f_x(x) dx = \int_{g_x(x) \leq 0} \exp[\ln f_x(x)] dx = \int_{g_x(x) \leq 0} \exp[h(x)] dx \quad (6)$$

$h(x)$ is expanded into Taylor series at x^* , which is a point on limit state surface and taken to the quadratic term as

$$\begin{aligned} h(x) &\approx h(x^*) + (x - x^*)^T \nabla h(x^*) + \frac{1}{2} (x - x^*)^T \nabla^2 h(x^*) (x - x^*) \\ &= h(x^*) + \frac{1}{2} \nu^T B \nu - \frac{1}{2} (x - x^* - B\nu)^T B^{-1} (x - x^* - B\nu) \end{aligned} \quad (7)$$

where

$$\nu = \nabla h(x^*) \quad (8)$$

$$B = -[\nabla^2 h(x^*)]^{-1} \quad (9)$$

Substituting Eq. (7) into Eq. (6), the failure probability p_f is now given as

$$p_f \approx f_x(x^*) \exp\left(\frac{\nu^T B \nu}{2}\right) \times \int_{g_x(x) \leq 0} \exp\left[-\frac{1}{2}(x - x^* - B\nu)^T B^{-1}(x - x^* - B\nu)\right] dx \quad (10)$$

Applying the Cholesky decomposition, the matrix B can be defined as $B=AA^T$ and then defining $X-x^*-B\nu=AY$, the failure probability p_f is now defined as

$$p_f \approx \sqrt{\det B} f_x(x^*) \exp\left(\frac{\nu^T B \nu}{2}\right) \int_{g_Y(y) \leq 0} \exp\left(-\frac{y^T y}{2}\right) dy \quad (11)$$

Corresponding performance function Z at x^* can be defined as

$$\begin{aligned} Z_Q &= (AY + B\nu)^T \nabla g_x(x^*) \\ &+ \frac{1}{2} (AY + B\nu)^T \nabla^2 g_x(x^*) (AY + B\nu) \\ &= (Y + A^T \nu)^T A^T \nabla g_x(x^*) \\ &+ \frac{1}{2} (Y + A^T \nu)^T A^T \nabla^2 g_x(x^*) A (Y + A^T \nu) \end{aligned} \quad (12)$$

Let be unit vector

$$\alpha_x = -\frac{A^T \nabla g_x(x^*)}{\|A^T \nabla g_x(x^*)\|} = -\frac{A^T \nu}{\|A^T \nu\|} = -\frac{A^T \nu}{\beta_L} \quad (13)$$

By structuring an orthogonal matrix $H = [H_1, H_2, \dots, H_{n-1}, \alpha_x]$ using α_x , Y space is orthogonally transformed into U space in the form

$$Y = HU \quad (14)$$

Z_Q can be expressed as

$$\begin{aligned} Z_Q &= \|A^T \nabla g_x(x^*)\| \left(\beta_L - U_n - \frac{1}{2} \tilde{U}^T H^T Q H \tilde{U} \right) \\ &\approx \|A^T \nabla g_x(x^*)\| \left[\beta_L - U_n - \frac{1}{2} V^T (H^T Q H)_{n-1} V \right] \end{aligned} \quad (15)$$

where $\tilde{U} = (U_1, U_2, \dots, U_n - \beta_L)^T = (V^T, U_n - \beta_L)^T$.

Substituting Eq. (14) into Eq. (11), the quadratic failure probability p_{fQ} is derived as

$$\begin{aligned} p_{fQ} &= \sqrt{\det B} f_x(x^*) \exp\left(\frac{\nu^T B \nu}{2}\right) \int_{g_U(u) \leq 0} \exp\left(-\frac{u^T u}{2}\right) du \\ &= (2\pi)^{n/2} \sqrt{\det B} f_x(x^*) \exp\left(\frac{\nu^T B \nu}{2}\right) \int_{g_U(u) \leq 0} \varphi_n(u) du \end{aligned} \quad (16)$$

Then, the quadratically asymptotic integral failure probability applying directly the result of SORM in Gauss space is expressed as

$$p_{fQ} = \frac{p_{fL}}{\sqrt{\det[I - \beta_L (H^T QH)_{n-1}]}} \quad (17)$$

where p_{fL} is first order asymptotic integral failure probability (K.M. Breitung, 1994)

$$p_{fL} = (2\pi)^{n/2} \sqrt{\det B} f_x(x^*) \exp\left(-\frac{\nu^T B \nu}{2}\right) \Phi(-\beta_L) \quad (18)$$

3 FLUTTER PROBABILITY FOR 14 LONG-SPAN BRIDGES

The results of the numerical simulations for 14 bridges are summarized in Table 1. In this table the flutter failure probability computed by both the quadratically asymptotic integral method and MC method was compared. The minimum failure probabilities is less than 1×10^{-7} . Most result using the quadratically asymptotic integral method is larger than that using MC method. That is the result using the quadratically asymptotic integral method emphasis on safety compared with that using MC method.

Table 1. Flutter failure probability of 14 Long-span bridges using two methods

Serial number of bridges	Quadratically asymptotic integral		MC		Relative error of failure probability %
	Reliability index β	Failure probability P_f	Reliability index β	Failure probability P_f	
1	3.05	1.16E-03	3.03	1.21E-03	-4.87
2	4.23	1.16E-05	4.24	1.09E-05	6.51
3	2.64	4.09E-03	2.66	3.85E-03	6.30
4	5.12	1.55E-07	5.13	1.42E-07	9.51
5	3.25	5.82E-04	3.25	5.75E-04	1.19
6	2.65	3.97E-03	2.67	3.80E-03	4.43
7	5.32	5.07E-08	5.34	4.56E-08	11.26
8	3.64	1.37E-04	3.66	1.27E-04	8.07
9	3.82	6.57E-05	3.85	6.02E-05	9.06
10	3.09	9.99E-04	3.11	9.31E-04	7.31
11	4.34	7.27E-06	4.36	6.58E-06	10.54
12	3.40	3.39E-04	3.41	3.21E-04	5.63
13	2.59	4.86E-03	2.61	4.58E-03	6.17
14	2.82	2.40E-03	2.84	2.25E-03	6.77

4 CONCLUSIONS

For very small probability case, a quadratically asymptotic integral method has been presented for the consideration of flutter failure of long-span bridges compared with MC method. The proposed method gives asymptotic approximations for failure probabilities without using any transformation of the probability space and any cumulative distribution function of random variable. Some sample calculations using 14 long-span bridges were introduced to demonstrate its effectiveness. It was found that most of failure probabilities using the proposed method were generally larger than that using MC method. The approximation may emphasis on safety compared with that using MC method.

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