

Non-linear buffeting and flutter analysis of bridges: a frequency domain approach

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ABSTRACT: A frequency domain approach, which is based on the Volterra series expansion, for non-linear bridge aerodynamics is proposed in this paper. The Volterra frequency-response functions (VFRFs) are constructed utilizing the topological assemblage scheme and identified through a full-time-domain non-linear bridge aerodynamic analysis framework. A two-dimensional numerical example of a long-span bridge is presented. The results show good comparison between the time domain simulations and the proposed frequency-domain model for non-linear bridge aerodynamics.

KEYWORDS: Non-linearity; Bridge; Buffeting; Flutter; Volterra series

1 INTRODUCTION

Wind-induced forces on bridges are traditionally represented as the sum of buffeting forces, related to the incoming wind velocity fluctuation, and the self-excited forces, generated by the bridge motion. Both these forces are modelled by linear operators (with memory) whose Frequency Response Functions (FRF) are estimated through specific wind-tunnel tests. On the other hand, when a fluid-structure interaction problem is characterized by high reduced velocity (i.e. the structure time scale is much slower than the fluid time scale) the quasi-steady assumption is invoked and the wind action is represented as a non-linear memory-less transformation of the incoming wind velocity and structural motion, which are often combined in the so-called effective angle of attack. Several experimental experiences suggest that, in some cases, both the mentioned approaches may be inadequate due to the simultaneous presence of both significant nonlinearities and memory effects.

With a specific reference to the case of long-span bridges, some attempts to fill this gap have been made adopting a band-superposition approach in which the quasi-steady model is used to represent the low-frequency part of the forces, while the high-frequency part (both buffeting and self-excited) is modelled through the usual linear models with memory, whose parameters are updated according to the instantaneous low-frequency effective angle of attack [1]. Following this formulation, the high-frequency response is provided by a differential equation whose parameters depend on both time and frequency.

To solve this problem with more efficient computational procedures, Chen & Kareem [2] transformed the problem into a full-time-domain formulation by means of a rational-function approximation and solved the equations of motion in the time domain through an integrate state-space approach. This solution, though mathematically rigorous, consists essentially of a calculation procedure and not a model, thus does not assist in the qualitative assessment of the problem.

In order to circumvent the limitations of the two above approaches, the concept of harmonic-band superposition is revised proposing a full-frequency-domain approach based on the Volterra series expansion of the dynamical systems representing both buffeting and self-excited forces. To this purpose, the governing equations (e.g. [2]) are re-casted into a block-diagram format (Fig. 2) invoking, whenever necessary, polynomial approximations. The synthesis of the result-

ing Volterra system (up to the 3rd order) is carried out adopting the topological assemblage scheme proposed in [3] for scalar systems and generalized in [4] to the case of multi-variate systems.

Section 2 provides some background on Volterra series, with particular reference to its multi-variate form [4]; Section 3 describes the non-linear aeroelastic bridge model assumed as reference; Section 4 shows the synthesis of its Volterra series approximation; Section 5 shows its numerical application for the dynamic analysis of a long-span bridge.

2 VOLTERRA SERIES: THEORETICAL BACKGROUND

Let us consider the nonlinear system represented by the following equation:

$$\mathbf{x}(t) = \mathcal{H}[\mathbf{u}(t)] \quad (1)$$

$\mathbf{u}(t)$ and $\mathbf{x}(t)$ being vectors with size n and m , respectively, representing the input and the output; t is the time. If the operator $\mathcal{H}[\cdot]$ is time-invariant and has finite-memory, its output $\mathbf{x}(t)$ can be expressed, far enough from the initial conditions, through the Volterra series expansion [e.g. 3, 4]:

$$\mathbf{x}(t) = \sum_{j=0}^{\infty} \int_{\boldsymbol{\tau}_j \in \mathbb{R}^j} \mathbf{h}_j(\boldsymbol{\tau}_j) \prod_{r=1}^j \mathbf{u}(t - \tau_r) d\boldsymbol{\tau}_j \quad (2)$$

where $\boldsymbol{\tau}_j = [\tau_1 \dots \tau_j]^T$ is a vector containing the j integration variables; the functions $\mathbf{h}_j$ have values in $\mathbb{R}^{m \times n^j}$ and are called Volterra kernels. The product operator is interpreted as a sequence of Kronecker products, i.e.:

$$\prod_{r=1}^j \mathbf{u}(t - \tau_r) = \mathbf{u}(t - \tau_1) \otimes \dots \otimes \mathbf{u}(t - \tau_j) \quad (3)$$

The 0th-order term of the Volterra series, $\mathbf{h}_0$, is a constant independent of the input; the 1st-order term is the convolution integral typical of the linear dynamical systems, with $\mathbf{h}_1$ being the impulse response function. The higher-order terms are multiple convolutions involving products of the input values for different time delays.

A Volterra system is entirely determined by its constant output and its Volterra kernels. An alternative representation is provided, in the frequency domain, by the Volterra frequency-response functions (VFRF), the multi-dimensional Fourier transforms of the Volterra kernels:

$$\mathbf{H}_j(\boldsymbol{\Omega}_j) = \int_{\boldsymbol{\tau}_j \in \mathbb{R}^j} e^{-i\boldsymbol{\Omega}_j^T \boldsymbol{\tau}_j} \mathbf{h}_j(\boldsymbol{\tau}_j) d\boldsymbol{\tau}_j \quad (4)$$

where $\boldsymbol{\Omega}_j = [\omega_1 \dots \omega_j]^T$ is a vector containing the j circular frequency values corresponding to $\tau_1, \dots, \tau_j$ in the Fourier transform pair. The VFRFs are functions with values in $\mathbb{C}^{m \times n^j}$.

3 NON-LINEAR AEROELASTIC BRIDGE MODEL

In this section the non-linear model for the calculation of the aeroelastic response of bridges described in [2] is briefly recalled. For simplicity, the original model is reduced disregarding the effect of the longitudinal turbulence and of the sway degree of freedom; besides, the aerodynamic load is applied to the bridge deck according to the strip theory. These quite strict hypotheses

could be easily relaxed and are here adopted only because the added formal complexity introduced by a more sophisticated bridge model may hide the conceptual structure of the proposed technique. According to this formulation, the dynamic response of the bridge is provided by the differential equation:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}_L + \mathbf{f}_H + \mathbf{f}_{se} \quad (5)$$

where $\mathbf{x} = [h, \alpha]^T$ is the displacement vector containing the heave displacement h (positive downwards) and the torsional rotation α (positive nose up), as shown in Fig. 1; the response $\mathbf{x}$ is divided into a low-frequency component $\mathbf{x}_L = [h_L, \alpha_L]^T$ and a high-frequency component $\mathbf{x}_H = [h_H, \alpha_H]^T$, separated by the frequency n_c . $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, viscous damping and stiffness matrixes, respectively; $\mathbf{f}_L$, $\mathbf{f}_H$ and $\mathbf{f}_{se}$ are, respectively, the low-frequency force modeled through a quasi-steady non-linear approach, the high-frequency buffeting force and the self-excited force.

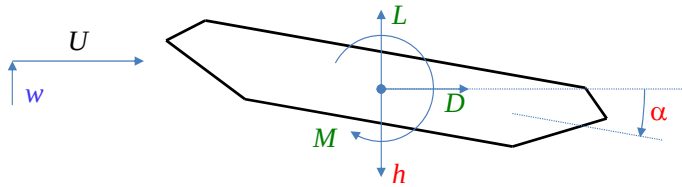


Figure 1. Bridge deck, incoming wind velocity, generalized forces and displacements.

The low-frequency force is defined as:

$$\mathbf{f}_L = \begin{bmatrix} -L \\ M \end{bmatrix} = \frac{1}{2} \rho B l V_r^2 \mathbf{R}(\phi) \mathbf{C}(\alpha_e) \quad (6)$$

where ρ is the air density, B the deck width, l the length of the strip on which the load is applied, V_r is the wind-structure relative velocity expressed as:

$$V_r^2 = U^2 + (w_L + \dot{h}_L + m_1 B \dot{\alpha}_L)^2 \quad (7)$$

where U is the mean wind velocity, w_L is the low-frequency vertical turbulence (i.e. the vertical turbulence, w , low-pass filtered at the frequency $n = n_c$); $m_1 B$ is the leg for the reduction of the apparent velocity field generated by the torsional velocity; $\mathbf{R}$ is a rotation matrix defined as

$$\mathbf{R}(\phi) = \begin{bmatrix} -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

where ϕ is the apparent (low-frequency) wind angle

$$\phi = \arctan \left(\frac{w_L + \dot{h}_L + m_1 B \dot{\alpha}_L}{U} \right) \quad (9)$$

The vector $\mathbf{C} = [C_D, -C_L, BC_M]^T$ contains the aerodynamic static coefficients, estimated for the effective angle of attack $\alpha_e = \phi + \alpha_L$ through static wind tunnel tests.

The high-frequency buffeting force is modeled as:

$$\mathbf{f}_H = \frac{1}{2} \rho U B l \mathcal{B}[w_H; \alpha_e] \quad (10)$$

Where w_H is the high-frequency vertical turbulence (i.e. w high-pass filtered at $n = n_c$) and $\mathcal{B}$ is a linear operator whose Frequency-Response Function (FRF) depends of the low-frequency response through the effective angle of attack α_e and is expressed as

$$\mathbf{B}(\omega; \alpha_e) = \begin{bmatrix} -C_D(\alpha_e) - C'_L(\alpha_e) & 0 \\ 0 & C'_M(\alpha_e) \end{bmatrix} \begin{bmatrix} \chi_{wL}(\omega; \alpha_e) \\ \chi_{wM}(\omega; \alpha_e) \end{bmatrix} \quad (11)$$

where C'_L and C'_M are the prime derivatives of C_L and C_M , respectively; χ_{wL} and χ_{wM} are the admittance functions weighting the effect of the vertical turbulence on the lift force and torsional moment, respectively; $\omega = 2\pi n$ is the circular frequency. Since α_e is variable in time due to w_L and $\mathbf{x}_L$, the operator $\mathcal{B}$ may be interpreted as a time-variant linear operator.

The self-excited forces are defined through the model:

$$\mathbf{f}_{se} = \frac{1}{2} \rho B^2 l \mathcal{A}[\mathbf{x}_H; \alpha_e] \quad (12)$$

where $\mathcal{A}$ is a linear operator whose FRF depends on α_e and is defined as

$$\mathbf{A}(\omega; \alpha_e) = \omega^2 \begin{bmatrix} H_4^*(k; \alpha_e) + i\omega H_1^*(k; \alpha_e) & B(H_3^*(k; \alpha_e) + i\omega H_2^*(k; \alpha_e)) \\ B(A_4^*(k; \alpha_e) + i\omega A_1^*(k; \alpha_e)) & B^2(A_3^*(k; \alpha_e) + i\omega A_2^*(k; \alpha_e)) \end{bmatrix} \quad (13)$$

where $H_j^*(k, \alpha)$ and $A_j^*(k, \alpha)$ are the flutter derivatives estimated at the reduced frequency $k = \omega B/U$ and for a mean angle of attack corresponding to α_e . Likewise the operator $\mathcal{B}$, also $\mathcal{A}$ is linear, but is time-variant because of the dependency on α_e .

4 SYNTHESIS OF A 3RD-ORDER VOLTERRA SYSTEM

In this section, the model described above is synthesized into a 3rd-order Volterra system whose FRFs are calculated adopting the topological assemblage scheme described in [3, 4]. Figure 2 shows a block diagram of the whole system. The input w is separated into w_L and w_H through the low-pass filter $\mathcal{L}$ and the high-pass filters $\mathcal{H}$, whose FRFs are $P_L(\omega)$ and $P_H(\omega)$, respectively. The output $\mathbf{x}$ is generated by the sum $\mathbf{x}_L$ and $\mathbf{x}_H$ obtained, respectively, as the outputs of the low-frequency and high-frequency stages of the system. A channel delivers the effective angle of attack α_e from the low-frequency stage of the system to its high-frequency stage. The rectangular boxes represent operators defined through a constitutive equation; the boxes with rounded corners are operators defined through experimental data; the triangles are gain blocks; $\mathbf{b}_1 = [1 \ 0]$; $\mathbf{b}_2 = [1 \ m_1 B]$. The operator $\mathcal{D}$ represents the left-hand side of Eq. (5) and its FRF is $\mathbf{D}(\omega) = -\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}$. The operators $\mathcal{A}_j$ and $\mathcal{B}_j$ ($j = 0, \dots, 2$) are linear operators whose FRFs, $\mathbf{A}_j$ and $\mathbf{B}_j$, are obtained by a polynomial approximation of $\mathbf{A}$ and $\mathbf{B}$ given by Eq. (11) and (13)

$$\mathbf{A}(\omega; \alpha_e) = \sum_{j=0}^2 \mathbf{A}_j(\omega) \alpha_e^j; \quad \mathbf{B}(\omega; \alpha_e) = \sum_{j=0}^2 \mathbf{B}_j(\omega) \alpha_e^j \quad (14)$$

$\mathbf{A}_j$ and $\mathbf{B}_j$ ($j = 0, \dots, 2$) are the FRFs of the time-invariant linear operators $\mathcal{A}_j$ and $\mathcal{B}_j$ and represented in Figure 2. Let us assume that the low-frequency and the high-frequency stages of the system can be represented by the operators

$$\mathbf{x}_L = \mathcal{L}[w]; \quad \mathbf{x}_H = \mathcal{H}[w] \quad (15)$$

and assume that they can be expanded into convergent Volterra series, at least for the range of input amplitude relevant for the application.

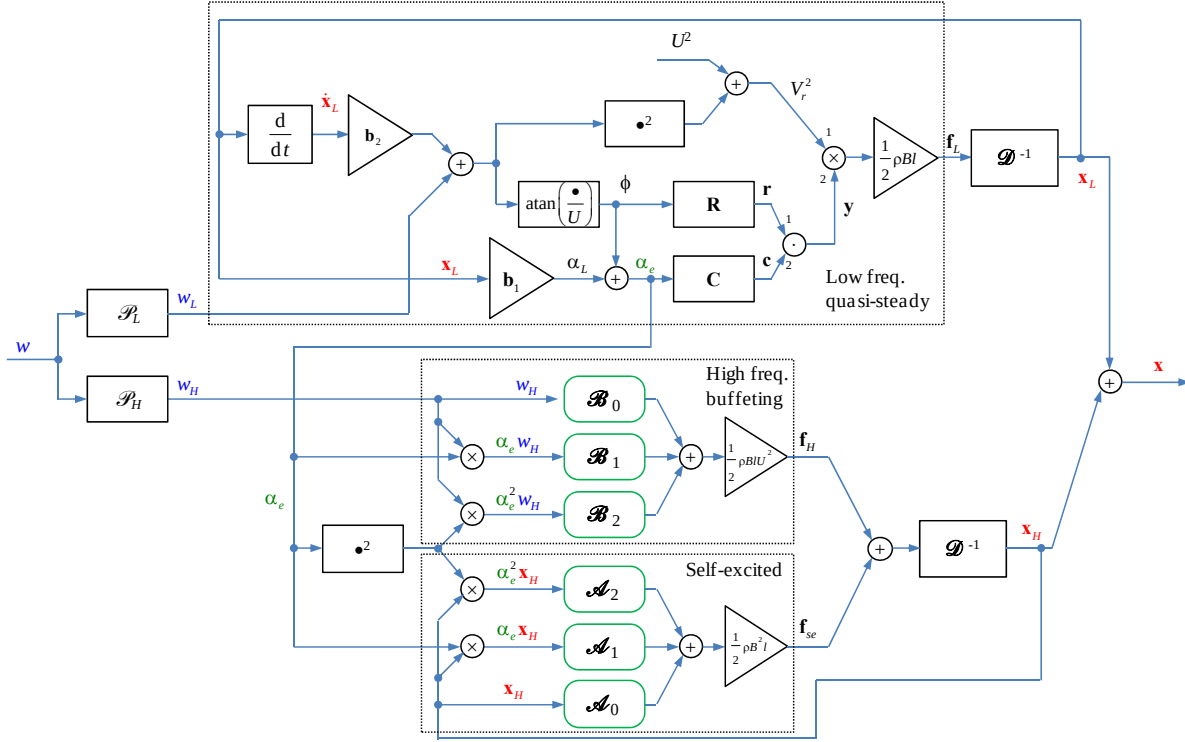


Figure 2. Block diagram of the nonlinear aeroelastic bridge model.

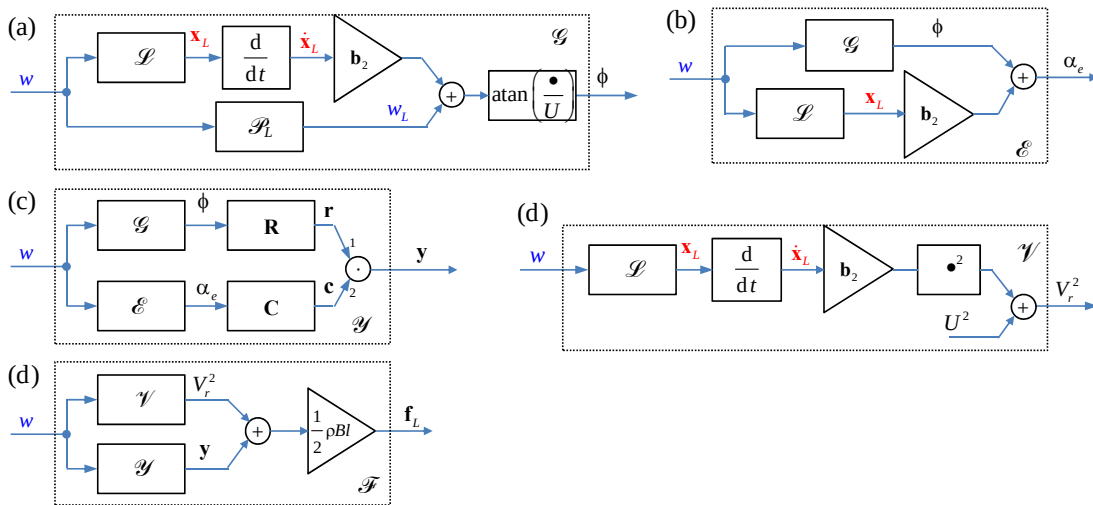


Figure 3. Block diagram of the sub-systems composing the low-frequency stage.

4.1 Low-frequency stage

Due to its topology, the identification of the system must start from the synthesis of the components of $\mathcal{L}$. Let $\mathcal{G}$ be the operator providing ϕ given w , described by the block diagram reported in Figure 3a. Its VFRFs G_j can be expressed as the sum of a direct term $G_j^{(d)}$ that does not depends on $\mathbf{L}_j$ and a feedback term $G_j^{(f)}$ that is linear in $\mathbf{L}_j$ [3, 4].

$$\begin{aligned} G_0 &= G_2^{(d)} = 0 \\ G_1^{(d)}(\omega) &= \frac{1}{U} P_L(\omega) \\ G_3^{(d)}(\Omega_3) &= -\frac{1}{3U^3} \left[\prod_{r=1}^3 P_L(\omega_r) + \prod_{r=1}^3 i \omega_r \mathbf{b}_2 \mathbf{L}(\omega_r) \right] \\ G_j^{(f)}(\Omega_j) &= \frac{1}{U} i \Sigma \Omega_j \mathbf{b}_2 \mathbf{L}_j(\Omega_j) \quad (j=1, 2, 3) \end{aligned} \quad (16)$$

where $\Sigma \Omega_j = \omega_1 + \dots + \omega_j$. Figure 3b shows the block diagram of the operator $\mathcal{E}$ providing the effective angle of attack α_e . Its VFRFs $E_j = E_j^{(d)} + E_j^{(f)}$ are given as:

$$\begin{aligned} E_0 &= \mathbf{b}_1 \mathbf{L}_0 = \alpha_0 \\ E_j^{(d)}(\Omega_j) &= G_j^{(d)}(\Omega_j) \\ E_j^{(f)}(\Omega_j) &= G_j^{(f)}(\Omega_j) + \mathbf{b}_1 \mathbf{L}_j(\Omega_j) \quad (j=1, 2, 3) \end{aligned} \quad (17)$$

where α_0 is the angle of attack obtained for $w = 0$. Figure 3c shows the block diagram of the operator $\mathcal{Y}$ providing the vector $\mathbf{y} = \mathbf{R}(\phi) \mathbf{C}(\alpha_e)$. The matrix $\mathbf{R}$ and the vector $\mathbf{C}$ are approximated by 3rd-order polynomials of ϕ and α_e , respectively.

$$\mathbf{R}(\phi) = \sum_{j=1}^3 \mathbf{R}_j \phi^j; \quad \mathbf{C}(\alpha_e - \alpha_0) = \sum_{j=1}^3 \mathbf{C}_j (\alpha_e - \alpha_0)^j \quad (18)$$

The VFRFs of $\mathcal{Y}$ are then in the form:

$$\begin{aligned} \mathbf{Y}_0 &= \mathbf{R}_0 \mathbf{C}_0 \\ \mathbf{Y}_1^{(d)}(\omega) &= \mathbf{R}_0 \bar{\mathbf{C}}_1^{(d)}(\omega) + \bar{\mathbf{R}}_1^{(d)}(\omega) \mathbf{C}_0 \\ \mathbf{Y}_2^{(d)}(\Omega_2) &= \mathbf{R}_0 \bar{\mathbf{C}}_2^{(d)}(\Omega_2) + \bar{\mathbf{R}}_1(\omega_1) \bar{\mathbf{C}}_1(\omega_2) + \bar{\mathbf{R}}_2^{(d)}(\Omega_2) \mathbf{C}_0 \\ \mathbf{Y}_3^{(d)}(\Omega_3) &= \mathbf{R}_0 \bar{\mathbf{C}}_3^{(d)}(\Omega_3) + \bar{\mathbf{R}}_2(\omega_1, \omega_2) \bar{\mathbf{C}}_1(\omega_3) + \bar{\mathbf{R}}_1(\omega_1) \bar{\mathbf{C}}_2(\omega_2, \omega_3) + \bar{\mathbf{R}}_3^{(d)}(\Omega_3) \mathbf{C}_0 \\ \mathbf{Y}_j^{(f)}(\Omega_j) &= \mathbf{R}_0 \bar{\mathbf{C}}_j^{(f)}(\Omega_j) + \bar{\mathbf{R}}_j^{(f)}(\Omega_j) \mathbf{C}_0 \quad (j=1, 2, 3) \end{aligned} \quad (19)$$

where $\bar{\mathbf{R}}_j = \bar{\mathbf{R}}_j^{(d)} + \bar{\mathbf{R}}_j^{(f)}$ are given as

$$\begin{aligned}
 \bar{\mathbf{R}}_0 &= \mathbf{R}_0 \\
 \bar{\mathbf{R}}_1^{(d)}(\omega) &= \mathbf{R}_1 G_1^{(d)}(\omega) \\
 \bar{\mathbf{R}}_2^{(d)}(\Omega_2) &= \mathbf{R}_2 G_1(\omega_1) G_1(\omega_2) \\
 \bar{\mathbf{R}}_3^{(d)}(\Omega_3) &= \mathbf{R}_1 G_3^{(d)}(\Omega_3) + 2\mathbf{R}_2 G_1(\omega_1) G_2(\omega_2, \omega_3) + \mathbf{R}_3 G_1(\omega_1) G_1(\omega_2) G_1(\omega_3) \\
 \bar{\mathbf{R}}_j^{(f)}(\Omega_j) &= \mathbf{R}_1 G_j^{(f)}(\Omega_j) \quad (j=1, 2, 3)
 \end{aligned} \tag{20}$$

The VFRFs $\bar{\mathbf{C}}_j = \bar{\mathbf{C}}_j^{(d)} + \bar{\mathbf{C}}_j^{(f)}$ can be obtained through Eq. (20) by replacing $\mathbf{R}_j$ with $\mathbf{C}_j$ and G_j with E_j . Figure 3d shows the block diagram of the operator $\mathcal{V}$ providing V_r^2 . Its VFRFs are given as

$$\begin{aligned}
 V_0 &= U^2 \\
 V_1 &= 0 \\
 V_2(\Omega_2) &= P_L(\omega_1) P_L(\omega_2) - \omega_1 \omega_2 \mathbf{b}_2 \mathbf{L}_1(\omega_1) \mathbf{b}_2 \mathbf{L}_1(\omega_2) + 2P_L(\omega_1) \mathbf{b}_2 \mathbf{L}_1(\omega_2) \\
 V_3(\Omega_3) &= 2i P_L(\omega_1) (\omega_2 + \omega_3) \mathbf{b}_2 \mathbf{L}_2(\omega_2, \omega_3) - 2\omega_1 (\omega_2 + \omega_3) \mathbf{b}_2 \mathbf{L}_1(\omega_1) \mathbf{b}_2 \mathbf{L}_2(\omega_2, \omega_3)
 \end{aligned} \tag{21}$$

Figure 3e shows the operator $\mathcal{F}$ providing the low-frequency force $\mathbf{f}_L$. Its VFRFs $\mathbf{F}_j = \mathbf{F}_j^{(d)} + \mathbf{F}_j^{(f)}$ are given as:

$$\begin{aligned}
 \mathbf{F}_0 &= \frac{1}{2} \rho B l V_0 \mathbf{Y}_0 \\
 \mathbf{F}_1^{(d)}(\omega) &= \frac{1}{2} \rho B l V_0 \mathbf{Y}_1^{(d)}(\omega) \\
 \mathbf{F}_2^{(d)}(\Omega_2) &= \frac{1}{2} \rho B l (V_0 \mathbf{Y}_2^{(d)}(\Omega_2) + V_2(\Omega_2) \mathbf{Y}_0) \\
 \mathbf{F}_3^{(d)}(\Omega_3) &= \frac{1}{2} \rho B l (V_0 \mathbf{Y}_3^{(d)}(\Omega_3) + V_2(\omega_1, \omega_2) \mathbf{Y}_1(\omega_3) + V_3(\Omega_3) \mathbf{Y}_0) \\
 \mathbf{F}_j^{(f)}(\Omega_j) &= \frac{1}{2} \rho B l V_0 \mathbf{Y}_j^{(f)}(\Omega_j) \quad (j=1, 2, 3)
 \end{aligned} \tag{22}$$

The VFRFs of the operator $\mathcal{L}$ providing the low-frequency response can be obtained by equating the VFRFs of the operators $\mathcal{D}[\mathcal{L}[\bullet]]$ and $\mathcal{F}$, leading to the equations:

$$\begin{aligned}
 \mathbf{L}_0 &= \mathbf{D}(0)^{-1} \mathbf{F}_0 \\
 \mathbf{D}(\Sigma \Omega_j) \mathbf{L}_j(\Omega_j) - \mathbf{F}_j^{(f)}(\Omega_j) &= \mathbf{F}_j^{(d)}(\Omega_j) \quad (j=1, 2, 3)
 \end{aligned} \tag{23}$$

The 0th-order equation is non-linear due to the dependency of the polynomial approximation of the aerodynamic coefficients from the α_0 (Eq. (18)). The other equations are linear in $\mathbf{L}_j$ ($\mathbf{F}_j^{(f)}$ is linear in $\mathbf{L}_j$) and contain lower-order solutions ($\mathbf{L}_k$ with $k < j$) in the term $\mathbf{F}_j^{(d)}$.

4.2 High-frequency stage

The synthesis of the high-frequency stage of the system requires the identification of the operators $\mathcal{A}$ and $\mathcal{B}$ providing, respectively the high-frequency buffeting force $\mathbf{f}_H$ and the self-excited force $\mathbf{f}_{se}$ (Figure 4). Their VFRFs, respectively, $\bar{\mathbf{A}}_j = \bar{\mathbf{A}}_j^{(d)} + \bar{\mathbf{A}}_j^{(f)}$ and $\bar{\mathbf{B}}_j$ are given as

$$\begin{aligned}
\bar{\mathbf{A}}_0 &= \mathbf{0} \\
\bar{\mathbf{A}}_1^{(d)}(\omega) &= \mathbf{0} \\
\bar{\mathbf{A}}_2^{(d)}(\Omega_2) &= \mathbf{A}_1(\Sigma\Omega_2) \mathbf{H}_1(\omega_1) E_1(\omega_2) \\
\bar{\mathbf{A}}_3^{(d)}(\Omega_3) &= \mathbf{A}_1(\Sigma\Omega_3) [\mathbf{H}_1(\omega_1) E_2(\omega_2, \omega_3) + \mathbf{H}_2(\omega_1, \omega_2) E_1(\omega_3)] + \mathbf{A}_2(\Sigma\Omega_3) \mathbf{H}_1(\omega_1) E_1(\omega_2) E_1(\omega_3) \\
\bar{\mathbf{A}}_j^{(f)}(\Omega_j) &= \mathbf{A}_0(\Sigma\Omega_j) \mathbf{H}_j(\Omega_j) \quad (j = 1, 2, 3)
\end{aligned} \tag{24}$$

$$\begin{aligned}
\bar{\mathbf{B}}_0 &= \mathbf{0} \\
\bar{\mathbf{B}}_1(\omega) &= \mathbf{B}_0(\omega) P_H(\omega) \\
\bar{\mathbf{B}}_2(\Omega_2) &= \mathbf{B}_1(\Sigma\Omega_2) P_H(\omega_1) E_1(\omega_2) \\
\bar{\mathbf{B}}_3(\Omega_3) &= \mathbf{B}_1(\Sigma\Omega_3) P_H(\omega_1) E_2(\omega_2, \omega_3) + \mathbf{B}_2(\Sigma\Omega_3) P_H(\omega_1) E_1(\omega_2) E_1(\omega_3)
\end{aligned} \tag{25}$$

The FRFs $\mathbf{H}_j$ of $\mathcal{H}$ can be obtained by equating, order by order, the VFRFs of the systems $\mathcal{D}[\mathcal{H}[\bullet]]$ and $\mathcal{A} + \bar{\mathcal{B}}$, leading to the equations:

$$\mathbf{D}(\Sigma\Omega_j) \mathbf{H}_j(\Omega_j) - \bar{\mathbf{A}}_j^{(f)}(\Omega_j) = \bar{\mathbf{A}}_j^{(d)}(\Omega_j) + \bar{\mathbf{B}}_j(\Omega_j) \quad (j = 1, 2, 3) \tag{26}$$

which can be solved with respect to $\mathbf{H}_j$ since $\bar{\mathbf{A}}_j^{(f)}$ is linear in $\mathbf{H}_j$. It can be observed that the j^{th} -order high-frequency response is influenced by the low-frequency response up to the order $j-1$ through the VFRFs E_k ($k = 1, \dots, j-1$).

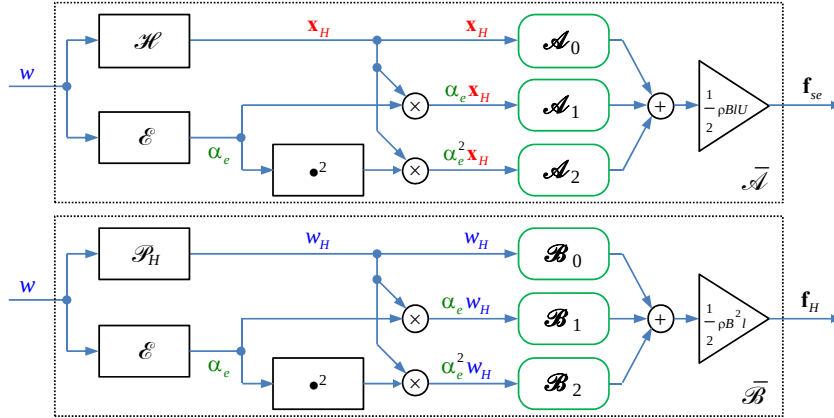


Figure 4. High-frequency stage.

5 NUMERICAL APPLICATION

The accuracy of the Volterra series representation of the non-linear aeroelastic bridge model described in Section 3 is demonstrated through the calculation of the response of a long-span bridge. The static coefficients of the cross section are measured for angle of attack between -10° and 10° with step 1° . These data are approximated through 3rd-order polynomials through a mean-square error minimization procedure [3] (Figure 5a). The admittance function is modeled

through the Sears function and is independent of the angle of attack. The flutter derivatives are estimated for $\alpha = -3^\circ, 0^\circ, 3^\circ$ and are interpolated, for each reduced velocity value, through a 2nd-order polynomial (Figure 5b). The natural frequency of the two considered modes is $n_1 = 0.19$ Hz and $n_2 = 0.53$ Hz. The frequency value dividing low and high-frequency components is $n_c = 0.10$ Hz.

Figure 6 shows the low-frequency response calculated through a Volterra model with order from 1 to 3, compared with the exact non-linear response calculated by a standard ODE solver. The result of the 3rd-order Volterra series appears quite accurate, in particular if compared to the linear model (1st-order Volterra series) which fails in reproducing the torsional response mainly due to its implicit symmetry.

Figure 7 shows the high-frequency response calculated through a Volterra model with order from 1 to 3, compared with the solution obtained by the time-domain procedure proposed in [2]. Also for this response component the 3rd-order Volterra series approximation provides a response very close to the time-domain solution.

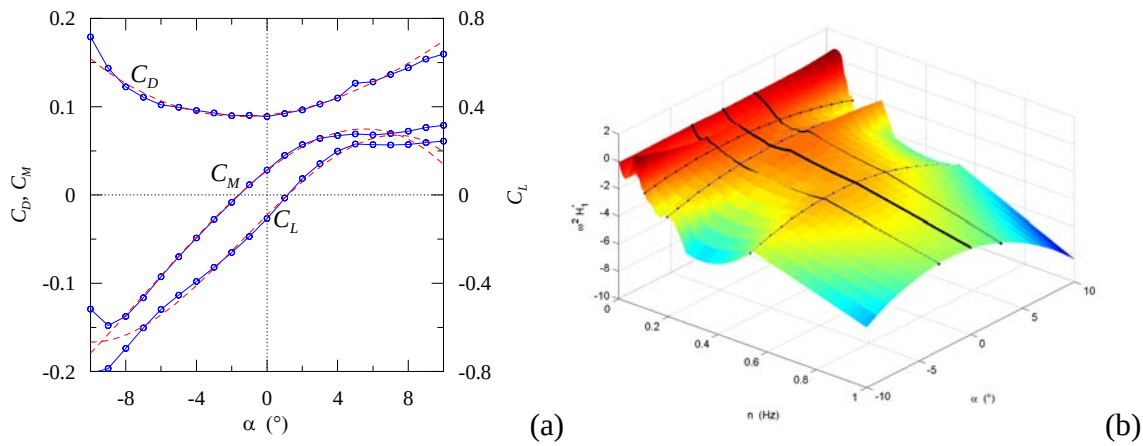


Figure 5. Static aerodynamic coefficients (a); flutter derivative H_1^* function of the mean angle of attack.

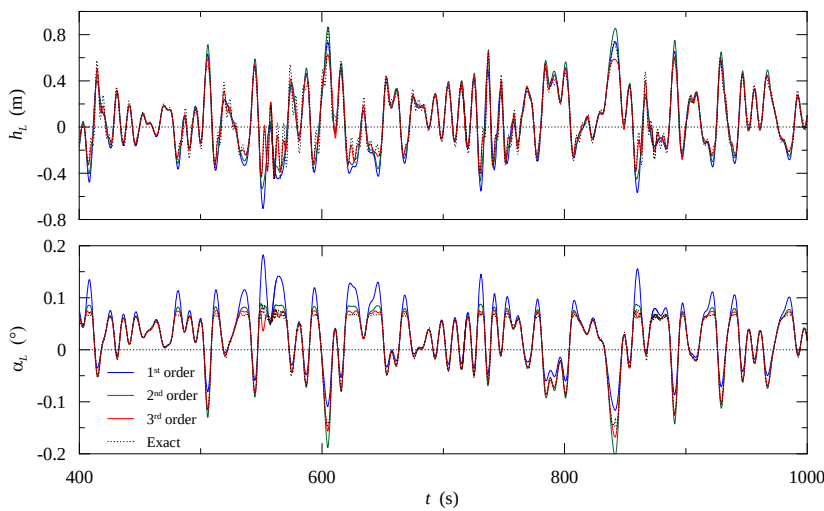


Figure 6. Low-frequency response.

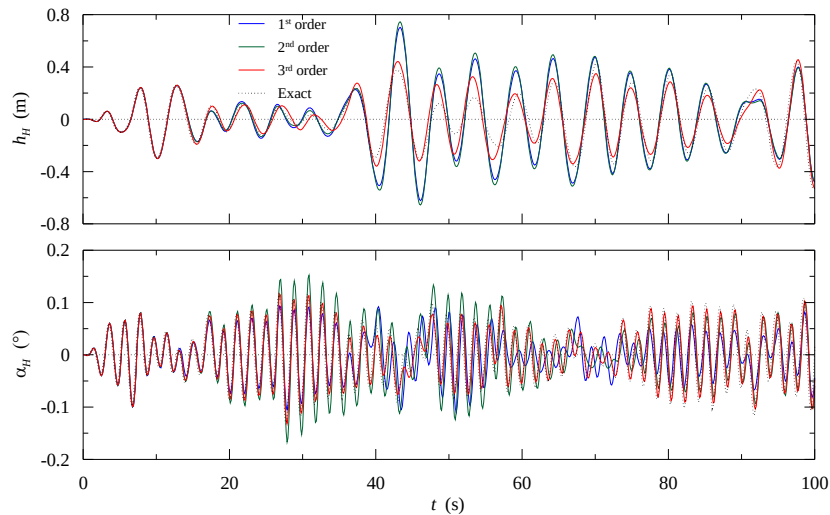


Figure 7. High-frequency response.

6 CONCLUSIONS

The proposed model is essentially the translation of an existing analysis procedure to the frequency domain, and therefore it inherits all its conceptual advantages and limitations (in particular, it is based on the hypothesis of small high-frequency oscillation of the bridge around a slow, possibly large, variation of the effective angle of attack). The added value of this new formulation is mostly related to its potential for deriving qualitative interpretations through this model. Besides, the functional structure of the proposed model may serve as a scaffolding to construct a fully-nonlinear model based on the Volterra series representation, to be identified from ad-hoc experimental procedures.

From a computational point of view, the implementation of the model in the framework of the Volterra series enables the formulation of a very efficient frequency-domain solution based on the concept of Associated Linear Equations (ALE). Accordingly, the model defined herein can be implemented through six linear frequency-domain equations that can be conveniently solved in a cascade manner.

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