

A state-space model for the wind-induced response of tall buildings with errors in the lateral loading

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ABSTRACT: The dynamic response of a tall building due to turbulent wind is analyzed in this study. A model is proposed to account for errors in the lateral loading, for example emerging from aerodynamic modeling assumptions, used to estimate the load in a wind tunnel test of a scaled model. The buffeting response at full scale is predicted by considering a linear structure with primary bending planes, oriented as the vertical faces, and mean loading orthogonal to one of the faces. The CAARC benchmark building is employed in the simulations. The model is based on a state-space formulation, stochastic calculus and generalized Markov-type state vector with state augmentation.

KEYWORDS: Tall buildings, stationary dynamic response, lateral loading correlation error, stochastic calculus, Reduced Fokker-Plank Equation.

1 INTRODUCTION

Risk-based methods for estimating the wind loading response of tall buildings, accounting for uncertainty in the lateral loading estimation, have been recently proposed (Bashor & Kareem¹, Chen & Huang², Jain *et al.*³, Norton *et al.*⁴). Nevertheless, most methods are developed for ad-hoc problems, such as a specific building or a given structural configuration. A new generalized framework for the simulation and analysis of wind-induced response of tall buildings, in which effects of “errors” in the estimation of the lateral loading are incorporated, is currently under investigation (e.g., Smith & Caracoglia⁵). The ultimate goal of the research is to develop algorithms for performance-based assessment of peak response under severe wind excitation with an application to structural serviceability.

The fundamental hypothesis of the proposed analysis framework requires “error” propagation to be directly incorporated into the dynamics of tall and flexible buildings. Errors in loading estimation can result from aerodynamic modeling assumptions or simplifications used to enable wind tunnel experimentation. A typical example, commonly encountered in the engineering practice during the interpretation of high-frequency force-balance (HFFB) data, is the need for compensation of the idealized mode shapes, assumed in an HFFB wind tunnel test, due to imperfect planarity of lateral modes in the full-scale structure (e.g., Tse *et al.*⁶).

In this paper a preliminary numerical model is presented and discussed. Lateral buffeting loading and stationary response are investigated. Quasi-steady approximation is used to derive the lateral loading model.

Due to parametric random error propagation, the stationary response of the building may deviate from standard second-moment random vibration theory. Therefore, the error-contaminated response is directly estimated from the joint-probability of an augmented state vector, which includes several state variables: generalized modal coordinates, turbulence variables, generalized correlation length of the lateral loading. The proposed model also builds on the results of a stochastic algorithm, proposed in recent years for uncertainty simulation and dynamic analysis of slender bridges (Caracoglia⁷).

2 THEORETICAL BACKGROUND

2.1 Fluctuating wind loading and generalized response of the CAARC building

This section describes the derivation of the state-space model for dynamic response in the time domain. This new model is derived from the standard approach for building aerodynamics in the frequency domain (e.g., Davenport⁸, Melbourne⁹, Kareem¹⁰, Piccardo & Solari¹¹, Kareem¹²). The dynamics of a prototype tall building is studied under the general hypothesis of linear structural behavior. The prototype structure is modeled after the CAARC benchmark building (Melbourne⁹), with rectangular floor-plan of dimensions D_x and D_y and height h . Figure 1 shows a schematic of the building geometry and wind loading; the mean wind direction and mean loading plane is orthogonal to the vertical face of width D_y (Fig. 1b). Figure 1a also coincides with the along-wind loading plane and primary bending plane for the building. Quantities $x(z,t)$ and $y(z,t)$ are used to model the dynamic displacements in the primary bending planes of the structure, as indicated in Figure 1b. Torsional effects are neglected in this study.

The CAARC building mass is assumed as uniformly distributed along the height of the structure with $m(z)$ mass per unit height; the dynamic lateral response in the two primary lateral planes is represented by modal superposition, accounting for the fundamental modes only, as: $x(z,t) = \Phi_{1x}(z) D_x \xi_{1x}(t)$ along wind (Fig. 1b), and $y(z,t) = \Phi_{1y}(z) D_y \xi_{1y}(t)$ in the transverse direction; ξ_{1x} and ξ_{1y} are dimensionless modal coordinates; $\Phi_{1x}(z)$ and $\Phi_{1y}(z)$ are dimensionless mode shapes of the first two lateral modes with frequencies n_{1x} and n_{1y} , respectively. Modal damping ratios are ζ_{1x} and ζ_{1y} with respect to critical value. The plane of bending for mode “1y” is not shown in Figure 1a since it is orthogonal to the elevation plane in the figure.

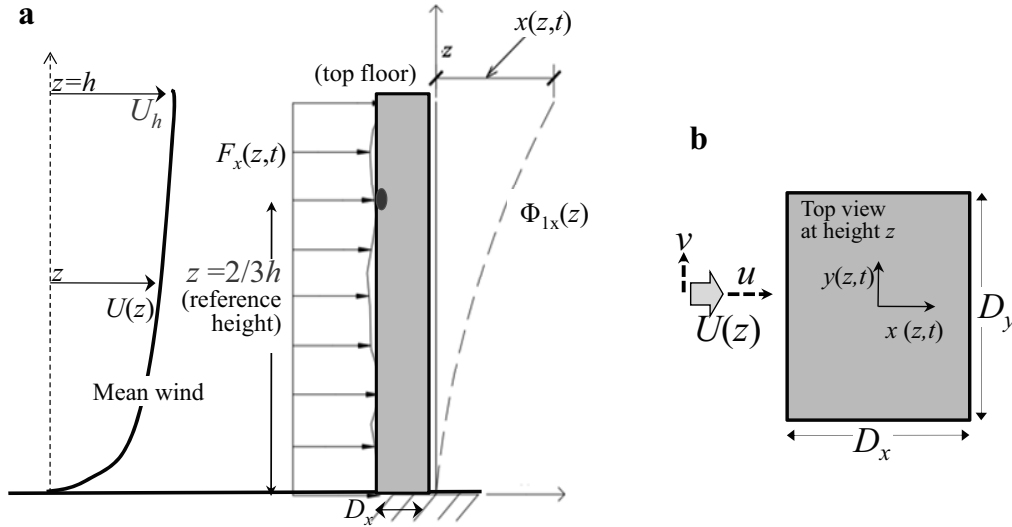


Figure 1. Schematic of the CAARC building with coordinate system and lateral wind loading: (a) lateral elevation view, (b) floor-plan view.

The fluctuating loading model operates in state space by assuming boundary layer wind with mean $U(z)$, stationary turbulence in the two directions, $u(z,t)$, $v(z,t)$ and quasi-steady aerodynamics (Fig. 1b). The along-wind and across-wind buffeting loading per unit height is written in the time domain as (Piccardo & Solari¹¹):

$$F_x(z,t) = \frac{\rho D_x U^2(z)}{2} \left\{ -2C_D \frac{\dot{x}(z,t)}{U(z)} + (\hat{C}_D - C_L) \frac{\dot{y}(z,t)}{U(z)} \right\} + \rho D_x U(z) C_D u(z,t) \quad (1)$$

$$F_y(z,t) = \frac{\rho D_x U^2(z)}{2} \left\{ 2C_L \frac{\dot{x}(z,t)}{U(z)} - (\hat{C}_L + C_D) \frac{\dot{y}(z,t)}{U(z)} \right\} + \frac{\rho D_x U(z)}{2} \left[2C_L u(z,t) + (\hat{C}_L + C_D) v(z,t) \right] \quad (2)$$

In equations above $\rho=1.25 \text{ kg/m}^3$ is the air density. The “dot” symbol designates derivation with respect to time t . The static force coefficients per unit height are independent of z and are normalized with respect to width D_x : drag C_D per unit z and its derivative with respect to mean-wind direction $\hat{C}_D = dC_D / d\alpha$, “lift” (or transverse lateral force) C_L and $\hat{C}_L = dC_L / d\alpha$.

The mean wind velocity is referenced to the value at the top floor $z=h$ (U_h in Fig. 1a) $U(z)=U_h \Delta(z)$, with $\Delta(z)=(z/h)^g$ following a power-law velocity profile (as in Melbourne⁹). If modal superposition is employed, the previous equations can be rewritten as

$$\begin{aligned} F_x(z,t) &= \frac{\rho D_x^2 U_h^2}{2} \Delta(z) \left\{ -2C_D \frac{\Phi_{1x}(z) \dot{\xi}_{1x}}{U_h} + (\hat{C}_D - C_L) \frac{\Phi_{1y}(z) \dot{\xi}_{1y}}{U_h} \right\} + \rho D_x U_h \Delta(z) C_D u(z,t) \\ F_y(z,t) &= \frac{\rho D_x^2 U_h^2}{2} \Delta(z) \left\{ 2C_L \frac{\Phi_{1x}(z) \dot{\xi}_{1x}}{U_h} - (\hat{C}_L + C_D) \frac{\Phi_{1y}(z) \dot{\xi}_{1y}}{U_h} \right\} \\ &\quad + \frac{\rho D_x U(z) \Delta(z)}{2} \left[2C_L u(z,t) + (\hat{C}_L + C_D) v(z,t) \right] \end{aligned} \quad (3, 4)$$

Modal superposition is used in Equations 3 and 4, under the hypothesis that modes “1x” and “1y” are purely planar, with mode shape $\Phi_{1x}(z)$ acting on the z - y plane and $\Phi_{1y}(z)$ on the z - x plane. Generalized forces (Q_{1x} , Q_{1y}) and modal masses (M_{1x} , M_{1y}) of “1x” and “1y” modes are:

$$Q_{1x} = \int_0^h \Phi_{1x}(z) F_x(z,t) dz, \quad Q_{1y} = \int_0^h \Phi_{1y}(z) F_y(z,t) dz \quad (5, 6)$$

$$M_{1x} = \int_0^h [\Phi_{1x}(z)]^2 m(z) dz, \quad M_{1y} = \int_0^h [\Phi_{1y}(z)]^2 m(z) dz \quad (7, 8)$$

If the dynamic response is expressed in terms of time $s=tU_h/D_x$ and dimensionless frequencies $K_{1x}=2\pi n_{1x} D_x/U_h$, $K_{1y}=2\pi n_{1y} D_x/U_h$, the generalized equations of modes “1x” and “1y” are

$$\xi_{1x}'' + 2(\zeta_{1x} + H_{1x,1x} C_D) K_{1x} \xi_{1x}' + H_{1x,1y} (C_L - \hat{C}_D) K_{1y} \xi_{1y}' + K_{1x}^2 \xi_{1x} = \frac{\rho D_x^2 h C_D}{M_{1x}} \int_0^h \Delta(z) \hat{u}(z,s) \Phi_{1x}(z) \frac{dz}{h} \quad (9)$$

$$\begin{aligned} \xi_{1y}'' + [2\zeta_{1y} + H_{1y,1y} (C_D + \hat{C}_L)] K_{1y} \xi_{1y}' - 2C_L H_{1y,1x} K_{1x} \xi_{1x}' \\ + K_{1y}^2 \xi_{1y} = \frac{\rho D_x^2 h}{M_{1y}} \left[C_L \int_0^h \Delta(z) \hat{u}(z,s) \Phi_{1y}(z) \frac{dz}{h} + (C_D + \hat{C}_L) \int_0^h \Delta(z) \hat{v}(z,s) \Phi_{1y}(z) \frac{dz}{h} \right] \end{aligned} \quad (10)$$

In the previous equations the prime symbol designates derivation with respect to s ; also, the turbulence components have been normalized as $\hat{u} = u / U_h$ and $\hat{v} = v / U_h$.

In Equations 10 and 11 fluid-structure coupling terms are introduced. These are derived from the quantities enclosed within braces in Equations 3 and 4; they become, for mode “1x” and “1y” respectively:

$$H_{1x,1x} = \frac{\rho D_x^2 h}{2K_{1x} M_{1x}} \int_0^h \Delta(z) [\Phi_{1x}(z)]^2 \frac{dz}{h}, \quad H_{1x,1y} = \frac{\rho D_x^2 h}{2K_{1y} M_{1x}} \int_0^h \Delta(z) \Phi_{1x}(z) \Phi_{1y}(z) \frac{dz}{h} \quad (11)$$

$$H_{1y,1y} = \frac{\rho D_x^2 h}{2K_{1y} M_{1y}} \int_0^h \Delta(z) [\Phi_{1y}(z)]^2 \frac{dz}{h}, \quad H_{1y,1x} = \frac{\rho D_x^2 h}{2K_{1x} M_{1y}} \int_0^h \Delta(z) \Phi_{1x}(z) \Phi_{1y}(z) \frac{dz}{h} \quad (12)$$

The loading terms on the right-hand side of Equations 9 and 10 are a function of along-wind and across-wind turbulence components at various z ; loads may be simulated by separating the dependency on z and time s after rewriting the partially correlated turbulence-induced terms as equivalent fully correlated lateral loads, acting on a reduced “projected area”, i.e., referenced to a Modal Correlation Length (MCL) in vertical direction. This principle has often been used in wind engineering and is compatible with the assumptions of the Equivalent Wind Spectrum Technique (Piccardo & Solari¹¹, Solari¹³). In the absence of loading errors, three MCLs are required by the model in Equations 9 and 10: $\Lambda_{1x,u}$, $\Lambda_{1y,u}$, $\Lambda_{1y,v}$. These quantities are dimensionless, normalized to the height h , and are computed by least-squares from:

$$[\Lambda_{1r,u}]^2 = \int_0^h \int_0^h \Delta(z) \Delta(v) \Phi_{1r}(z) \Phi_{1r}(v) \exp \left[-\frac{2c_{z,u} n_{1r}}{U_h} \frac{|z-v|}{\Delta(z) + \Delta(v)} \right] \frac{dz dv}{h^2} \quad (13)$$

$$[\Lambda_{1y,v}]^2 = \int_0^h \int_0^h \Delta(z) \Delta(v) \Phi_{1y}(z) \Phi_{1y}(v) \exp \left[-\frac{2c_{z,v} n_{1y}}{U_h} \frac{|z-v|}{\Delta(z) + \Delta(v)} \right] \frac{dz dv}{h^2} \quad (14)$$

Equation 13 is expressed in generic form with mode index “ r ” equal to x or y . For example, the square root of Equation 13, with $r=x$, is $0 < \Lambda_{1x,u} \leq 1.0$, i.e., the MCL of the $1x$ -mode lateral loading (predominantly along-wind). In both Equation 13 and 14 the spatial coherence coefficients, $c_{z,u}=10$ and $c_{z,v}=6.67$, are defined in accordance with standard approaches. If Equations 13 and 14 are utilized, Equations 9 and 10 can be simplified as

$$\xi_{1x}'' + 2(\zeta_{1x} + H_{1x,1x} C_D) K_{1x} \xi_{1x}' + K_{1y} H_{1x,1y} (C_L - \hat{C}_D) \xi_{1y}' + K_{1x}^2 \xi_{1x} = \frac{\rho D_x^2 h C_D}{M_{1x}} \Lambda_{1x,u} \hat{u}_{0.6h}(s) \quad (15)$$

$$\begin{aligned} \xi_{1y}'' + [2\zeta_{1y} + H_{1y,1y} (C_D + \hat{C}_L)] K_{1y} \xi_{1y}' - 2C_L H_{1y,1x} K_{1x} \xi_{1x}' \\ + K_{1y}^2 \xi_{1y} = \frac{\rho D_x^2 h}{M_{1y}} [C_L \Lambda_{1y,u} \hat{u}_{0.6h}(s) + (C_D + \hat{C}_L) \Lambda_{1y,v} \hat{v}_{0.6h}(s)] \end{aligned} \quad (16)$$

In Equations 15 and 16 the turbulence components are referenced to $z=0.6h \approx 2/3h$ (Solari¹³) as $\hat{u}_{0.6h}(s) = \hat{u}(z=0.6, s)$ and $\hat{v}_{0.6h}(s) = \hat{v}(z=0.6, s)$, as also schematically depicted in Figure 1a.

2.2 Stochastic dynamics and state-space model

The dynamic system of equations in the time domain $s=tU_h/D_x$ is transformed into a stochastic differential system of the Itô-type (Grigoriu¹⁴) to simulate effects of errors in the lateral loading.

In this study, random perturbation is applied to the MCL of the $1x$ -mode lateral loading, $\Lambda_{1x,u}$, in Equation 13 (with $r=x$), whereas quantities $\Lambda_{1y,u}$ and $\Lambda_{1y,v}$ are considered marginally affected by errors and, therefore, they are two deterministic constants. This perturbation is simulated by

replacing the deterministic quantity $\Lambda_{1x,u}$ in Equation 15 by $(\Lambda_{1x,u} + \lambda_u)$; λ_u is a zero-mean dimensionless random variable. The quantity $\Lambda_{1x,u}$, which is estimated through Equation 13, can be interpreted as the mean value of the MCL with λ_u being a measure of its error. As a result, while Equation 16 is still deterministic, Equation 15 is rewritten as

$$\xi''_{1x} + 2(\zeta_{1x} + H_{1x,1x} C_D) K_{1x} \xi'_{1x} + K_{1y} H_{1x,1y} (C_L - \hat{C}_D) \xi'_{1y} + K_{1x}^2 \xi_{1x} = \frac{\rho D_x^2 h C_D}{M_{1x}} (\Lambda_{1x,u} + \lambda_u) \hat{u}_{0.6h}(s) \quad (17)$$

The stochastic differential system is subsequently derived from Equations 16 and 17 in state-space form by simulating the time-dependent turbulence components $\hat{u}_{0.6h}$ and $\hat{v}_{0.6h}$ as the outputs of two auto-regressive filters as a function of s , applied to a scalar Wiener process of unit variance $W(s)$. This hypothesis enables the subsequent derivation of an Itô-type system:

$$d\hat{u}_{0.6h}(s) = -G_{1u} \hat{u}_{0.6h}(s) ds + G_{2u} dW(s) \quad (18)$$

$$d\hat{v}_{0.6h}(s) = -G_{1v} \hat{v}_{0.6h}(s) ds + G_{2v} dW(s) \quad (19)$$

In Equations 18 and 19, G_{1u} , G_{2u} , G_{1v} and G_{2v} are deterministic constants, which can be indirectly inferred from the power spectral density of turbulence (e.g., Caracoglia⁷) and are evaluated at $z=0.6h$. Equations 18 and 19 are written in differential form in accordance with Itô's differentiation rule.

Equations 18 and 19 are subsequently combined with Equations 16 and 17 and re-written in state-space form after augmentation (Grigoriu¹⁴) to obtain a stochastic differential system as a function of state vector $\mathbf{Z}(s)$. This vector is composed of physical states (generalized coordinates ξ_{1x} , ξ_{1y} and their derivatives), wind turbulence states ($\hat{u}_{0.6h}$ and $\hat{v}_{0.6h}$) and random variable λ_u . The stochastic differential equation, with $\mathbf{Z}(s) = [\xi_{1x}(s), \xi'_{1x}(s), \xi_{1y}(s), \xi'_{1y}(s), \hat{u}_{0.6h}(s), \hat{v}_{0.6h}(s), \lambda_u]^T$ being a column vector and $s = tU_h/D_x$ a dimensionless time, is:

$$d\mathbf{Z}(s) = \mathbf{a}(\mathbf{Z}(s)) ds + \sqrt{2\pi} \mathbf{d} dW(s) \quad (20)$$

In Equation 20, valid for $s \geq 0$, $\mathbf{a}$ is a nonlinear (drift) function of $\mathbf{Z}(s)$ and $\mathbf{d} = [0^{4 \times 1}, G_{2u}, G_{2v}, 0]^T$ is a scalar $n \times 1$ (diffusion) vector collecting some of the coefficients in Equations 18 and 19.

The dimension of the state vector is $n=7$. Initial conditions on $\mathbf{Z}$ at $s=0$ are irrelevant since the steady-state response (as $s \rightarrow +\infty$) is investigated. If the random state vector is relabeled as $\mathbf{Z} = [Z_1, \dots, Z_n]^T$, $\mathbf{a}(\mathbf{Z})$ is

$$\mathbf{a} = \begin{bmatrix} Z_3 \\ Z_4 \\ -2K_{1x}(\zeta_{1x} + H_{1x,1x} C_D)Z_3 - K_{1y} H_{1x,1y} (C_L - \hat{C}_D)Z_4 - K_{1x}^2 Z_1 + \rho D_x^2 h C_D (M_{1x})^{-1} (\Lambda_{1x,u} + Z_7) Z_5 \\ -[2\zeta_{1y} + H_{1y,1y} (C_D + \hat{C}_L)] K_{1y} Z_4 - 2C_L H_{1y,1x} K_{1x} Z_3 - K_{1y}^2 Z_2 + \frac{\rho D_x^2 h}{M_{1y}} [C_L \Lambda_{1y,u} Z_5 + (C_D + \hat{C}_L) \Lambda_{1y,v} Z_6] \\ -G_{1u} Z_5 \\ -G_{1v} Z_6 \\ 0 \end{bmatrix} \quad (21)$$

2.3 Algorithm for assessing the joint-probability density function of the state vector

The stationary joint-probability density function (PDF) of $\mathbf{Z}$, $p(\mathbf{z})$, can be determined from system 20 through the Reduced Fokker-Planck Equation (Grigoriu¹⁴) as $s \rightarrow +\infty$ and with $r=4$:

$$\sum_{j=1}^{n-1} \frac{\partial}{\partial z_j} [a_j(\mathbf{z}) p(\mathbf{z})] - \pi \sum_{j=r+1}^{n-1} \sum_{k=r+1}^{n-1} R_{jk} \frac{\partial^2 p(\mathbf{z})}{\partial z_j \partial z_k} = 0 \quad (22)$$

The dependency on the time variable is irrelevant at steady-state; its explicit designation is omitted in the subsequent derivations. In Equation 22, $\mathbf{R} = \mathbf{d}^T \mathbf{d}$ is a constant matrix with non-zero elements for $j > r$ and $k > r$. For the numerical solution of the previous equation the joint-PDF is first represented in terms of the probability potential (Lin & Cai¹⁵), $\varphi(\mathbf{z})$, as $p(\mathbf{z}) = C_f \exp[\varphi(\mathbf{z}) + \varphi_0]$ with C_f and φ_0 being two integration constants. These must satisfy the condition $\int p(\mathbf{z}) d\mathbf{z} = 1.0$. As a result, Equation 22 becomes

$$\sum_{j=1}^{n-1} \left[a_j(\mathbf{z}) \frac{\partial \varphi}{\partial z_j} + \frac{\partial a_j(\mathbf{z})}{\partial z_j} \right] - \pi \sum_{j=r+1}^{n-1} \sum_{k=r+1}^{n-1} R_{jk} \left[\frac{\partial \varphi}{\partial z_j} \frac{\partial \varphi}{\partial z_k} + \frac{\partial^2 \varphi}{\partial z_j \partial z_k} \right] = 0 \quad (23)$$

A numerical algorithm has been developed to solve Equation 23, by adaptation of the approach proposed by Soize¹⁶. The numerical algorithm employs a Galerkin expansion for $\varphi(\mathbf{z})$: $\varphi(\mathbf{z}) = \sum \kappa_{\beta} h_{\beta}(\mathbf{z})$. This expansion utilizes a basis of mutually-orthogonal Normalized Hermite Polynomials (NHP) $h_{\beta}(z_1, \dots, z_j, \dots, z_n) = h_{\beta_1}(z_1) \times \dots \times h_{\beta_j}(z_j) \times \dots \times h_{\beta_n}(z_n)$ (Soize¹⁶). The degree of each scalar Hermite Polynomial (e.g., $h_{\beta_1}(z_1)$) is designated by its index (e.g., for $\beta_1=2$ $h_{\beta_1}(z_1) = (z_1^2 - 1)/\sqrt{2}$). The vector index $\beta = (\beta_1, \dots, \beta_j, \dots, \beta_n)$ collects the degrees of each individual h_{β_j} function. The degree of each h_{β} is determined from $\sum \beta_j$, whereas a scalar index $l=1$ to $l=d_{\text{NHP}}$ is used to label and group all the β vector indices in ascending order, based on degree and variables $z_1, \dots, z_n$.

The algorithm operates by exploiting the orthogonality of the NHPs with respect to the standard Gaussian vector of independent $\mathbf{z}$. The unknown coefficients of the series, κ_{β} , are found by integration (Galerkin averaging) of Equation 23 with respect to the Gaussian vector over the multi-variable domain $\mathbf{z}$, after substituting $\varphi(\mathbf{z}) = \sum q_{\beta} h_{\beta}(\mathbf{z})$.

If the series $\varphi(\mathbf{z}) = \sum q_{\beta} h_{\beta}(\mathbf{z})$ is truncated to the first d_{NHP} terms (κ_{β}), Equation 23 can be transformed into a non-linear homogeneous algebraic system of d_{NHP} equations. The expansion should preserve the features of the NHPs until the third degree to account for parametric non-Gaussian uncertainty propagation. The generic l -th scalar equation of this non-linear algebraic system is related to the generic vector index γ in the expansion, and is

$$\sum_{\beta=(1, \dots, 0)} \left(E_{\gamma\beta} - \pi E_{\gamma(\beta)^j} \right) \kappa_{\beta} - \pi \sum_{\beta=(1, \dots, 0)} \sum_{\varepsilon=(1, \dots, 0)} \kappa_{\beta} \kappa_{\varepsilon} G_{(\gamma), \beta, \varepsilon} = 0 \quad (24)$$

In the previous equation the index γ varies from $\gamma=(1, \dots, 0)$ ($l=1$) to $\gamma=(0, \dots, 1, 2)$ ($l=116 = d_{\text{NHP}}$); κ_{β} and κ_{ε} denote the generic unknown coefficients. The $E_{\gamma\beta}$ terms coincide with a series of scalar terms, which are evaluated by numerical integration, whereas the scalar $E_{\gamma(\beta)^j}$ and $G_{(\gamma), \beta, \varepsilon}$ can be found in closed form (details omitted for brevity). Equations 23 and 24 highlight the main advantage of the algorithm, which is superior to Monte Carlo simulation for solving Equation 20, since it enables the assessment of the joint-PDF directly in closed form.

3 NUMERICAL SIMULATIONS: CAARC BUILDING

3.1 Description of full-scale building properties

The geometric, dynamic and aerodynamic properties of the CAARC building are derived from Melbourne⁹. Reference geometric dimensions are $D_x = 30.5$ m, $D_y = 45.7$ m and $h = 183$ m (Fig. 1). The mass per unit height is constant with $m(z) = 220,800$ kg/m. The natural frequencies are taken as $n_{1x} = 0.20$ Hz and $n_{1y} = 0.22$ Hz, respectively. Mode shapes are approximately linear with height (Melbourne⁹): $\Phi_{1x}(z) = (z/h)$ and $\Phi_{1y}(z) = (z/h)$. Modal damping ratios are assumed as $\zeta_{1x} = \zeta_{1y} = 0.01$ for both modes. The aerodynamic coefficients per unit elevation are inferred from the original experimental data (Melbourne⁹), as described in Smith & Caracoglia⁵, as: $C_D = 1.0$, $dC_D/d\alpha = -1.1$, $C_L = -0.1$, $dC_L/d\alpha = -2.2$.

Two mean wind scenarios are investigated: $U_h = 20$ m/s (moderate wind) and $U_h = 40$ m/s (high wind) evaluated at mean roof height. The MCL for $U_h = 20$ m/s are $\Lambda_{1x,u} = \Lambda_{1y,u} = 0.17$ and $\Lambda_{1y,v} = 0.21$; those for $U_h = 40$ m/s are $\Lambda_{1x,u} = 0.23$, $\Lambda_{1y,u} = 0.22$ and $\Lambda_{1y,v} = 0.25$. The power spectral densities of both $\hat{u}_{0.6h}$ and $\hat{v}_{0.6h}$ components are used to determine the parameters of Equations 18 and 19; densities are based on the Harris model.

In the case of the along-wind component $\hat{u}_{0.6h}$ (Fig. 1b) the energy of the spectrum has been linked to a dimensionless form of the Harris spectrum, based on the data in Melbourne⁹, evaluated at $z = 0.6h$:

$$S_{\hat{u}_{0.6h}}(K) = \frac{[I_u]^2}{\Delta_u(0.6h)U_h} \frac{960}{(2 + \tilde{f})^{5/6}}, \quad \tilde{f} = \frac{1600K}{2\pi D_x \Delta_u(0.6h)} \quad (25, 26)$$

with reduced frequency $K = 2\pi n D_x / U_h$ and turbulence intensity $I_u = 11\%$ at $z = 0.6h$. For $\hat{v}_{0.6h}$ the same expression as in Equation 25 is used with I_v replacing the quantity enclosed within brackets. However, a reduction of transverse turbulence intensity is postulated: $I_v = 8\%$. The dimensionless parameters of the autoregressive turbulence models, derived for both $U_h = 20$ m/s and $U_h = 40$ m/s, are summarized in Table 1. Details on the derivation of these parameters are omitted for brevity.

Table 1. Parameters used for $\hat{u}_{0.6h}$ and $\hat{v}_{0.6h}$ in Equations 18 and 19.

U_h (m/s)	I_u	G_{1u}	G_{2u}	I_v	G_{1v}	G_{2v}
20	11%	0.356	0.121	8%	0.356	0.091
40	11%	0.294	0.082	8%	0.294	0.009

3.2 Hermite-Polynomial Galerkin expansion

Figure 2 shows the dimensionless scalar coefficients κ_{β} , associated with each h_{β} term of the Galerkin expansion of $\varphi(\mathbf{z})$ and obtained by numerically solving the nonlinear system (24). In this figure the dynamic response of the CAARC building with reference wind speed $U_h = 20$ m/s is analyzed. The NHPs are ordered in ascending order and in accordance with vector index β . As outlined in Section 2.3 the scalar index $1 \leq l \leq 116$, shown on the horizontal axis of the graph, is used to identify each term of the expansion. In the figure the κ_{β} terms are divided by first, second and third degree. For example $l=1$ belongs to $\beta = (1, 0, \dots, 0)$ and $h_{(1,0,0,0,0,0,0)} = z_1$, $l=7$ corresponds to $\beta = (2, 0, \dots, 0)$ and $h_{(2,0,0,0,0,0,0)} = (z_1^2 - 1)/\sqrt{2}$, $l=34$ to $\beta = (3, 0, \dots, 0)$ and $h_{(3,0,0,0,0,0,0)} = (z_1^3 - 3z_1)/\sqrt{6}$.

In Figure 2, two cases are depicted, the difference being related to the numerical integration limits on $\mathbf{z}$, which are used by Galerkin expansion to numerically evaluate $E_{\gamma\beta}$ in Equation 24.

“Effective” integration limits must be pre-selected a priori on a hyper-cube of the whole domain $\mathbf{z}$ in the proximity of the origin and must account for the local features of the polynomial functions defined through the NHP. Preliminary simulations have suggested as acceptable limiting values $|z_j| \leq 0.50$ for each $j=1, \dots, n$ or $|z_j| \leq 0.25$ (Domain “b” in Fig. 2). Since the integration to determine the $E_{\gamma\beta}$ is conducted numerically, selection of integration limits and a suitable number of integration points ($5E+5$) is crucial.

From the analysis of Figure 2 it can be concluded that the main features of the solution are preserved between the two cases: the continuous line is related to limits $|z_j| \leq 0.50$ and the dashed line is obtained with $|z_j| \leq 0.25$.

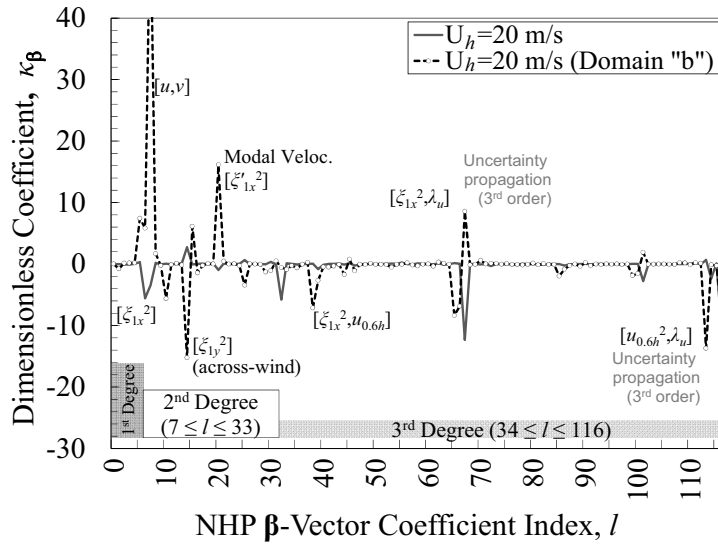


Figure 2. Normalized Hermite-Polynomials of the Galerkin expansion ($U_h = 20$ m/s).

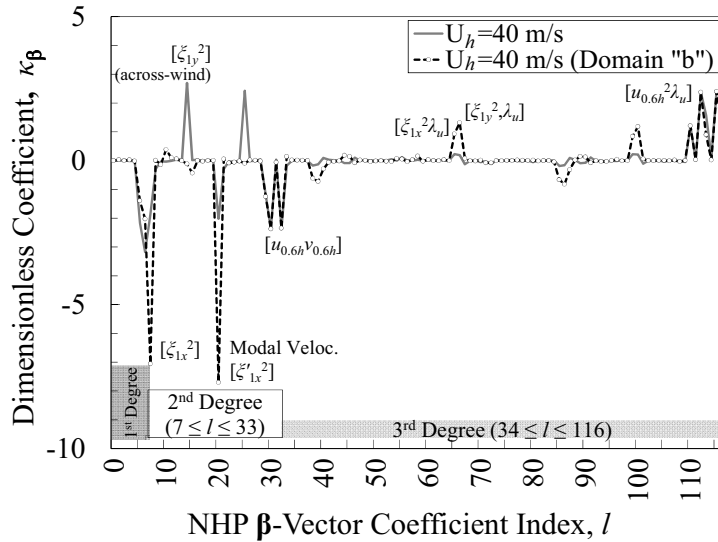


Figure 3. Normalized Hermite-Polynomials of the Galerkin expansion ($U_h = 40$ m/s).

A non-negligible variation in the magnitude of the κ_β parameters can be noticed in a few specific cases (e.g., $l=5$). Nevertheless, Figure 2 suggests that the proposed approach can replicate features typical of parametric uncertainty propagation; this observation is compatible

with the presence of non-negligible 3rd-degree polynomials in the figure, which are necessary to describe the non-Gaussian behavior in the state-space system. Figure 2 also shows that a limited number of κ_{β} terms are predominant in $\varphi(\mathbf{z})=\sum\kappa_{\beta}h_{\beta}(\mathbf{z})$ and, therefore, the joint-PDF; these are highlighted by indicating the corresponding state variables within brackets.

Similarly, Figure 3 depicts the numerical results obtained at large wind speed $U_h=40$ m/s for the same building type. The figure summarizes the results for both integration domains, i.e., limits $|z_j|\leq 0.50$ and $|z_j|\leq 0.25$ (Domain “b”) imposed on each $j=1,...,n$. The vertical axis scale is not the same as in Figure 2.

Figure 3 tends to confirm the results at lower speed, since the same sub-set of relevant κ_{β} is observed; this fact suggests a preservation of the main features in the joint-PDF, which is numerically evaluated through $\varphi(\mathbf{z})=\sum\kappa_{\beta}h_{\beta}(\mathbf{z})$, if the mean wind speed U_h varies. Also, as U_h increases, the absolute value $|\kappa_{\beta}|$ of the corresponding expansion terms (same β) decreases. This remark is compatible with the fact that the overall load variance proportionally grows with U_h and turbulence energy.

3.3 Integration constants of the joint-PDF

From the discussion results in Figure 2 and Figure 3, it was inferred that the results obtained with the integration Domain “b” ($|z_j|\leq 0.25$) are preferable due to a more realistic combination of κ_{β} , possibly more compatible with an actual full-scale scenario. This case is selected and analyzed in more detail in this section. The integration constants of the joint-PDF, $p(\mathbf{z})=C_f\exp[\varphi(\mathbf{z})+\varphi_0]$, C_f and φ_0 , for both $U_h=20$ m/s and $U_h=40$ m/s are computed and shown in Table 2.

The quantities C_f and φ_0 depend on the random properties of λ_u , perturbing the $\Lambda_{1x,u}$. In this study, λ_u has zero mean and standard deviation $SD(\lambda_u)=0.25\Lambda_{1x,u}$ (25% of the mean value) This corresponds to $SD(\lambda_u)=0.042$ for $U_h=20$ m/s ($\Lambda_{1x,u}=0.17$) and $SD(\lambda_u)=0.056$ for $U_h=20$ m/s ($\Lambda_{1x,u}=0.23$).

The choice of employing two constants C_f and φ_0 is dictated by algorithmic efficiency. Since one condition can exclusively be imposed on the probability, $\int p(\mathbf{z})d\mathbf{z}=1.0$, C_f and φ_0 are inherently dependent. In Table 2 φ_0 is initially pre-selected and C_f is subsequently determined. Even though some variability is observed in the table, the choice of an initial $\varphi_0<0$ leads to a more suitable, i.e., a finite value in C_f .

Table 2. Integration constants of $p(\mathbf{z})$, using Domain “b” solution in Figures 2 and 3

U_h (m/s)	Integr. limits	φ_0	C_f	φ_0	C_f	φ_0	C_f
20	$ z_j \leq 0.25$	-1.83E+02	5.72E+00	4.76E+01	5.03E-29	0	8.37E-09
40	$ z_j \leq 0.25$	-7.72E+00	1.50E-01	4.60E-01	7.20E-05	0	8.42E-05

(Note: $j = 1,...,n-1$)

4 CONCLUDING REMARKS

An analytically-based and numerically-implemented algorithm is proposed in this study for the simulation and analysis of modeling errors in the lateral buffeting loads for tall buildings. The model operates in time domain by employing state-space formulation and stochastic calculus techniques. The study shows that, if torsional effects are neglected, the buffeting response of a tall building can be successfully simulated. The paper demonstrates that it is possible to derive the joint probability density function of a generalized state vector, combining modal response with random turbulence and a random variable, which simulates errors in the modeling of the

correlation length of the loads. The algorithm enables the assessment of the joint-PDF directly by Galerkin expansion and is superior to “brute-force” Monte Carlo simulation, which operates by synthetically generating a series of realizations of the time-dependent response.

Buffeting response has been exclusively investigated at this stage. Preliminary results confirm the feasibility of this approach on a 183m building; sensitivity of the solution to the selection of integration domain limits has been noticed and is currently being considered for further investigation.

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