

Application of nonlinear eddy viscosity model in simulations of flows over bluff body

Jian Zhang ^{a,b}, Qingshan Yang ^b, Q. S. Li ^a

^a *Dept of Civil and Architectural Engineering, City University of Hong Kong, Hong Kong*

^b *School of Civil Engineering, Beijing Jiaotong University, Beijing, 100044, China*

ABSTRACT: Turbulent wind flows around building structures (bluff bodies) are investigated numerically using Reynolds Averaged Navier-Stokes (RANS) turbulence models. A non-linear viscosity assumption model, which can take into account the anisotropy of turbulence with less numerical cost than LES method, is adopted herein as a turbulence treatment method. Model coefficients of the nonlinear terms are adjusted, aiming to model three-dimensional flows over bluff bodies. Compared with the results of the linear eddy viscosity model and experimental measurements, the nonlinear eddy viscosity model yields more reasonable results, especially for the reattachment length and pressure coefficients. It is also noted that the nonlinear eddy viscosity model performs satisfactorily to reproduce complex turbulent flows around bluff bodies.

KEYWORDS: Nonlinear eddy viscosity model, CFD, Bluff body, Wind flow, RANS models

1 INTRODUCTION

Turbulent wind flows around building structures (bluff bodies) have been studied extensively by wind tunnel tests and field measurements. Rapid developments of computer technology in recent years have made it possible to adopt numerical methods, such as Large Eddy Simulation (LES) or Reynolds Averaged Navier-Stokes (RANS) models, to simulate turbulent flows around bluff bodies. Although LES is an effective tool to calculate turbulent flows around civil structures, RANS turbulence models, such as eddy viscosity models or Reynolds Stress Models, are still valid because they require less computer hardware sources than LES. However, it is imperative to overcome the limitations of RANS models. From the viewpoint of computational efficiency and practical application, nonlinear eddy viscosity models are attractive for simulations of flows influenced by anisotropy of turbulence.

The two-equation turbulence models, for instance, the $k - \varepsilon$ and the $k - \omega$ series models, offer significant simplicity and numerical stability in many cases for predictions of flow fields. The Linear Eddy Viscosity Models (LEVMs) construct eddy flow field using solvable turbulence quantities, such as k , ε and ω (Versteeg & Malalasekera, 2007). The Boussinesq hypothesis, together with the standard $k - \varepsilon$ model (Launder & Spalding, 1974), is generally regarded as the conventional linear eddy viscosity concept. It is found that LEVMs can be successfully applied to two-dimensional boundary layers, three-dimensional flow around simple bodies and thin shear flows which can be treated as approximately equilibrium (Hellsten and Bezdard, 2005). This is because the LEVM is only able to provide principle stress component correctly in such flows. Therefore, it is not applicable in flows where anisotropic turbulence eddy fluctuations essentially influence the mean flow. Complex phenomena such as strong three-dimension, separations, recirculations and reattachments are common in wind flows over building structures. It is noted

that the standard LEVMs have severe defects for simulations of complicated flow fields. Consequently, it is required to extend the applicability of the linear eddy viscosity models for capturing anisotropic stress fields.

Recent studies have been mainly focused on the nonlinear relationship between the Reynolds stress and mean flow quantities. The aim of adopting the Nonlinear Eddy Viscosity Model (NLEVM) is to introduce mean flow strain rate (or vorticity) into the Reynolds stress expressions to construct nonlinear stress terms. Quadratic, cubic and fifth order models are commonly used in the NLEVM's, which are different from each another with the degrees of nonlinearity. Craft et al. (1997) and Huang & Rajagopal (1996) developed individual quadratic NLEVM models to simulate two-dimensional turbulent flow over a backward-facing step. Their results showed that the two quadratic NLEVMs successfully predicted the reattachment length of flow over the step. Meanwhile, the quadratic models demonstrated great robustness while CPU cost was not significantly increased. Other quadratic models have been proposed by Speziale (1987) based on the strain rate and by Rubinstein (1990) based on renormalisation-group approach. These models yielded satisfactory results for simple cases.

Jongen et al. (1998) observed that the sensitivity of quadratic models to the rotation is inadequate to predict fully developed channel flow subjected to rapid spanwise rotation. Craft et al. (1995) argued that a cubic Reynolds stress expansion is needed for predicting complex flows with large streamline curvature effects. Craft et al. (1995) adopted a third-order term to sensitize the model to curvature effects to overcome the overestimation of turbulence energy in the impingement area. To capture non-equilibrium flows, Craft et al. (1997) developed a more complicated model, in which the second anisotropy invariant is added in Reynolds stress expansions to form a three-equation of NLEVM. This new model was designed especially for low-Reynolds number transitional and turbulent flows. Apsley & Leschziner (1998) derived a cubic model based on successive iterative approximations to an algebraic Reynolds stress model. They validated the model by considering several flows including a high-lift airfoil flow and separating flow in two-dimensional diffuser. However, they just used the original coefficient-formulas as relationships between the coefficients and recalibrated their values to capture the low-Reynolds number near-wall flow.

Applications of these NLEVMs to complex turbulent flows have displayed inspiring results on flow anisotropy, strong streamline curvature, recirculation and adverse pressure gradient. To evaluate the performance of the NLEVMs in predicting complex turbulent flows fields, this paper presents numerical simulations of wind flow over a bluff body with a quadratic NLEVM in comparison with field measurement data. It should be noted that throughout the numerical simulations, realizability conditions are applied to the NLEVM. It will prove that the NLEVM can be applied to the simulations of turbulent flows with strong anisotropy usually encountered in engineering practices.

2 THE NONLINEAR $k - \varepsilon$ TURBULENCE MODEL

2.1 Basic equations

The basic equations of continuity and momentum for steady incompressible flows are:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0 \\ u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} - \frac{\partial u'_i u'_j}{\partial x_j} + \mu \frac{\partial^2 u_i}{\partial x_j^2} \end{cases} \quad (1)$$

Realizable $k - \varepsilon$ model is adopted in this study as follows:

$$\begin{cases} \frac{\partial(\rho k u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} [\alpha_k \mu_{eff} \frac{\partial k}{\partial x_j}] + G_k - \rho \varepsilon, \\ \frac{\partial(\rho \varepsilon u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} [\alpha_\varepsilon \mu_{eff} \frac{\partial \varepsilon}{\partial x_j}] + \rho C_1 S \varepsilon - \rho C_2 \varepsilon \frac{\varepsilon^2}{k + \sqrt{\nu \varepsilon}} + \rho C_{1\varepsilon} \frac{\varepsilon}{k} C_{3\varepsilon} G_b. \end{cases} \quad (2)$$

where x_i and x_j are the spatial coordinates; u_i and u_j are the averaged wind velocities; $-u'_i u'_j$ is the Reynolds stress; p is the averaged pressure; ρ is the density of fluid; k is the Turbulent Kinetic Energy (TKE); ε is the turbulent energy dissipation rate; ν_t is the turbulent dynamic eddy viscosity, μ is the molecular viscosity and $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $C_1 = 1.44$, $C_2 = 1.92$ are the model constants.

For two-equation turbulence model, Reynolds stress tensors τ_{ij} are expressed by the linear constitutive equation:

$$\tau_{ij} = -\overline{\rho u'_i u'_j} = \mu_t \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \quad \mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad (3)$$

2.2 The quadratic eddy viscosity assumption

Since the standard two-equation model does not take into account the anisotropy of the Reynolds stresses. This shortcoming can be overcome to some extent by introducing a nonlinear constitutive expression as follows:

$$\begin{aligned} \rho u'_i u'_j = & -\mu_t S_{ij} + 2/3 \delta_{ij} k + C_1 \mu_t \frac{k}{\varepsilon} (\Omega_{ik} S_{kj} + \Omega_{jk} S_{ki}) \\ & + C_2 \mu_t \frac{k}{\varepsilon} (S_{ik} S_{kj} - 1/3 S_{kl} S_{lk} \delta_{ij}) + C_3 \mu_t \frac{k}{\varepsilon} (\Omega_{ik} \Omega_{kj} - 1/3 \Omega_{kl} \Omega_{lk} \delta_{ij}) \end{aligned} \quad (4)$$

where $S_{ij} = \partial u_i / \partial x_j + \partial x_j / \partial x_i$ and $\Omega_{ij} = \partial u_i / \partial x_j - \partial x_j / \partial x_i$

For three-dimensional flows around bluff bodies, the coefficients of Reynolds stress terms are needed to be tuned by consideration of the anisotropy (Champagne, 1970). Firstly, Reynolds stress components are extracted and simplified. According to zero-pressure gradient flow over a smooth plate, wind velocity component can be expressed as follows:

$$\begin{cases} u_1 = u_{y0} \left(\frac{y}{y_0} \right)^\alpha \\ u_2 = u_3 = 0 \end{cases} \quad (5)$$

Then, shear deformation invariant $S = \frac{k}{\varepsilon} \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2}$ and vorticity deformation invariant

$\Omega = \frac{k}{\varepsilon} \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2}$ can be simplified as:

$$S = \Omega = M = \frac{k}{\varepsilon} \frac{du_x}{dy} \quad (6)$$

where $M = \max(S, \Omega)$ is deformation tensor invariant.

The normal Reynolds stress components are expressed as follows:

$$\begin{cases} \frac{u'_1 u'_1}{k} = \frac{2}{3} + C_\mu \frac{2C_1 - C_3}{3} M^2 \\ \frac{u'_2 u'_2}{k} = \frac{2}{3} + C_\mu \frac{-C_1 + 2C_3}{3} M^2 \\ \frac{u'_3 u'_3}{k} = \frac{2}{3} + C_\mu \frac{-C_1 - C_3}{3} M^2 \end{cases} \quad (7)$$

In two-dimensional boundary flows, constants $C_1 = 0.4$, $C_2 = 0$, $C_3 = -0.13$ and $C_\mu = 0.09$ vary with anisotropic flow characteristics. To satisfy the above mathematical constraints, damping function f_M is introduced into $C_1 \sim C_3$ as:

$$C_1 = 0.4f_M(M), \quad C_2 = 0, \quad C_3 = -0.13f_M(M) \quad (8)$$

It is well-known that the tensor τ_{ij} is positive, the following inequalities are thus satisfied:

$$\begin{cases} \tau_{ii} \geq 0 \\ |\tau_{ij}| \leq (\tau_{ii}\tau_{jj})^{0.5} \\ \det(\tau_{ij}) \geq 0 \end{cases} \quad (9)$$

We refer to these three properties as realizability conditions. The normal Reynolds stress components can be concluded based on the first realizability condition in equation (9) as follows:

$$\begin{cases} \frac{u'_1 u'_1}{k} = \frac{2}{3} + C_\mu \frac{2C_1 - C_3}{3} M^2 \geq 0 \\ \frac{u'_2 u'_2}{k} = \frac{2}{3} + C_\mu \frac{-C_1 + 2C_3}{3} M^2 \geq 0 \\ \frac{u'_3 u'_3}{k} = \frac{2}{3} + C_\mu \frac{-C_1 - C_3}{3} M^2 \geq 0 \end{cases} \quad (10)$$

From the second expression of equation (10), constant C_μ satisfies:

$$C_\mu \leq \frac{2}{(C_1 - 2C_3)M^2} = \frac{2}{0.66f_M(M)M^2} \quad (11)$$

Based on the realizable $k - \varepsilon$ model, turbulent kinetic energy generation G_k , Reynolds shear stress $-u'_1 u'_2$ and turbulent viscosity μ_t are expressed below:

$$G_k = -u'_1 u'_2 \frac{du_x}{dy}, \quad -u'_1 u'_2 = \nu_t \frac{du_x}{dy}, \quad \nu_t = C_\mu \frac{k^2}{\varepsilon} \quad (12)$$

Reynolds shear stress $-u'_1 u'_2$ can be derived by equations (11) and (12):

$$\frac{u'_1 u'_2}{k} = -C_\mu M \quad (13)$$

Then another C_μ expression is derived as follows:

$$C_\mu \leq \frac{(C_1 + C_3)S + \sqrt{(C_1 + C_3)^2 S^2 + 4[9 + (2C_1^2 + 2C_3^2 - 5C_1 C_3)S^2]}}{9S + (2C_1^2 + 2C_3^2 - 5C_1 C_3)S^3} \quad (14)$$

To make sure C_μ satisfies the realizability condition, C_μ expression is given as follows:

$$C_\mu = \min(0.09, 0.3f_M(M)) \quad (15)$$

Based on the above derivation, the three-dimensional quadratic eddy viscosity model can be expressed as:

$$\left\{ \begin{aligned} \rho u'_i u'_j &= -\mu_t S_{ij} + 2/3 \delta_{ij} k + C_1 \mu_t \frac{k}{\varepsilon} (\Omega_{ik} S_{kj} + \Omega_{jk} S_{ki}) \\ &\quad + C_2 \mu_t \frac{k}{\varepsilon} (S_{ik} S_{kj} - \frac{1}{3} S_{kl} S_{lk} \delta_{ij}) + C_3 \mu_t \frac{k}{\varepsilon} (\Omega_{ik} \Omega_{kj} - \frac{1}{3} \Omega_{kl} \Omega_{lk} \delta_{ij}) \\ C_1 &= 0.4 f_M(M), \quad C_2 = 0, \quad C_3 = -0.13 f_M(M), \\ C_\mu &= \min(0.09, 0.3 f_M(M)), \quad f_M(M) = \frac{1}{1 + 0.01 M^2} \end{aligned} \right. \quad (16)$$

Turbulence modeling, aiming to wind engineering applications, should take the anisotropy of turbulent flow into consideration. It is important to maintain the ease of use and computational stable of the two equation models. Therefore, nonlinear expansions of the Boussinesq hypothesis--the quadratic expressions proposed by Craft et al. (1997)--have been extended in an attempt to account for anisotropic turbulence and curvature related strain effects. Numerical tests of the quadratic model will be carried out in the following section.

3 NUMERICAL DISCRETIZATIONS, MESH SCHEMES AND TEST CASE

3.1 Numerical Discretizations

The commercial package FLUENT has been used to solve the governing equations for mean wind velocities and turbulent quantities. The equations are discretized by the finite volume method on structure grids. The second-order upwind differencing scheme and SIMPLEC algorithm are used for convective terms and the pressure-velocity terms individually. The modified quadratic model is incorporated in FLUENT. The turbulent viscosity μ_t is also modified based on the new expressions.

3.2 Boundary Conditions

For inlet condition, a fully developed velocity profile is adopted, which is expressed by a power law:

$$u_x = u_{y0} \left(\frac{y}{y_0} \right)^\alpha \quad (17)$$

where u_y (m/s) is the mean wind velocity at height y (m), u_{y0} (m/s) is the reference velocity at height y_0 (m). α is the roughness category coefficient.

Turbulence intensity I is defined in AIJ code (Architectural Institute of Japan, 2004):

$$I = \begin{cases} 0.1 \left(\frac{y}{Y_G} \right)^{-\alpha-0.05}, & Y_b < y \leq Y_G; \\ 0.1 \left(\frac{Y_b}{Y_G} \right)^{-\alpha-0.05}, & y \leq Y_b. \end{cases} \quad (18)$$

where y is the height along the fetch; Y_b , Y_G and α are parameters which can be found in AIJ code. The inlet condition of turbulence kinetic energy (TKE) is defined as a function of turbulence intensity I :

$$k = \frac{3}{2} (u_x \cdot I)^2 \quad (19)$$

Dissipation rate ε can be determined by k and turbulence length scale L , and the relationship is:

$$\varepsilon = C_\mu^{3/4} \frac{k^{3/2}}{L} \quad (20)$$

where C_μ is defined as a constant (0.09).

Turbulence length scale L is defined independently of the terrain conditions of the site as:

$$L = \begin{cases} 100 \left(\frac{y}{30} \right)^{0.5}, & 30 < y \leq Y_G; \\ 100, & y \leq 30m. \end{cases} \quad (21)$$

In summary, the inlet boundary conditions of TKE k and dissipation rate (D) ε are listed below:

$$\begin{cases} k(x) = \frac{3}{2} \left(u_0 \left(\frac{y+y_0}{y_0} \right)^\alpha \cdot I \right)^2 \\ \varepsilon(x) = C_\mu^{3/4} \frac{k^{3/2}}{L} \\ I = \begin{cases} 0.1 \left(\frac{y}{Y_G} \right)^{-\alpha-0.05}, & Y_b < y \leq Y_G; \\ 0.1 \left(\frac{Y_b}{Y_G} \right)^{-\alpha-0.05}, & y \leq Y_b. \end{cases} \\ L = \begin{cases} 100 \left(\frac{y}{30} \right)^{0.5}, & 30 < y \leq Y_G; \\ 100, & y \leq 30m. \end{cases} \end{cases} \quad (22)$$

The boundary conditions of the top and bilateral boundaries of the fetch are set as slip boundary conditions, which are expressed as:

$$\begin{cases} u_y = 0, \quad \frac{\partial}{\partial x}(u_x, u_z, k, \varepsilon) = 0, \\ u_z = 0, \quad \frac{\partial}{\partial x}(u_x, u_y, k, \varepsilon) = 0. \end{cases} \quad (23)$$

The outflow face is assumed as fully developed outflow boundary condition, which can be expressed as:

$$\frac{\partial}{\partial x}(u_x, u_y, u_z, k, \varepsilon) = 0 \quad (24)$$

3.3 Numerical model and mesh schemes

To examine the performance of the quadratic eddy viscosity model, numerical simulations of wind flow over a 6m cube are carried out based on the standard realizable $k - \varepsilon$ turbulence model and the quadratic eddy viscosity model. The 6m cube is mounted in atmospheric boundary layer (ABL) at a normal orientation to the incident wind with the computational domain shown in Figure 1. Specific representations of the computational domain and mesh arrangements can be seen in Figure 2.

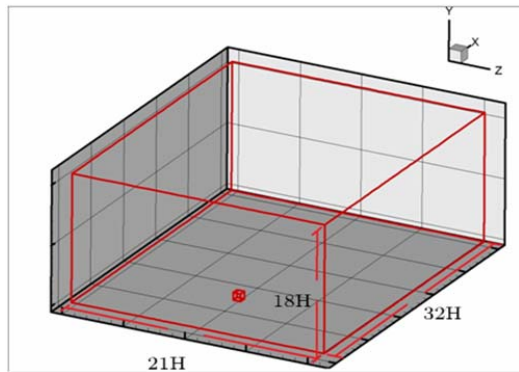


Figure 1. Computational domain around 6m cube.

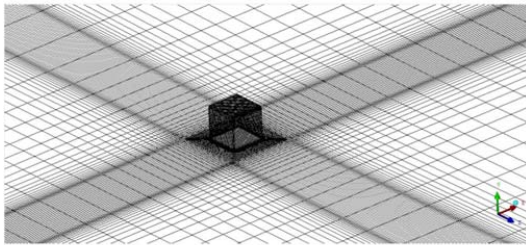


Figure 2. Grids scheme for 6m cube model.

Reynolds number based on the incident wind velocity and the height of the 6m cube model is around 10^7 . To resolve the flow field around the bluff body, the first-layer cell near the cube surface is set as $0.015H_c$, which is appropriate for the realizable $k - \varepsilon$ model. The cells are non-uniformly distributed and the stretching ratio is 1.12 along the surfaces. 32 grids are placed on the windward, roof and leeward surfaces of the cube (z -direction). 40 grids are located along the lateral faces of the cube (y -direction). 30 grids with stretching ratio 1.2 are placed in the wake zone of the computational domain.

4 NUMERICAL RESULTS

The computational results are compared with those by the linear eddy viscosity model (the realizable $k - \varepsilon$ model) and the field measurement results (Richards et al., 2001).

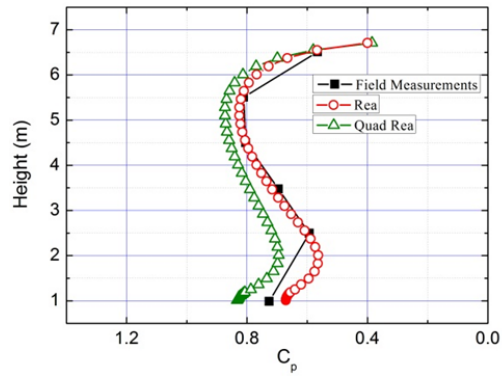


Figure 3. Pressure distributions along the windward face.

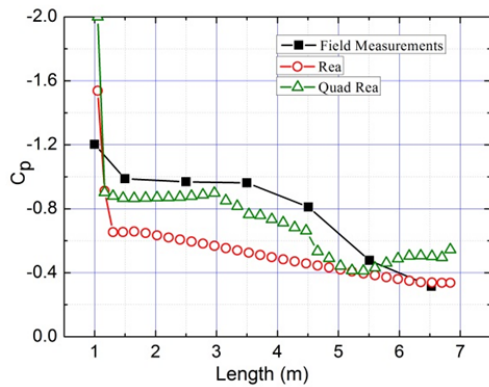


Figure 4. Pressure distributions along the roof face.

Figure 3 shows comparisons of mean pressure coefficient distributions on the windward face. It can be seen that the two turbulence models yield almost same results around the flow stagnation point. The pressure coefficients on the remaining part show relatively similar distribution shapes with differences within approximately 10 percent of the measurement values. It is apparent that the CFD predicted pressure spike position is approximately 0.67m lower than that obtained by measurement results. Figure 4 shows the pressure distributions on the cube roof. The quadratic nonlinear model predicted closer results with the measurements than the linear eddy viscosity model. Both models over predicted the pressure at the front edge of the cube.

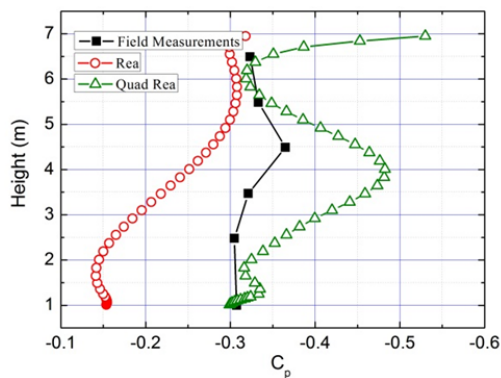


Figure 5. Pressure distributions along the leeward face.

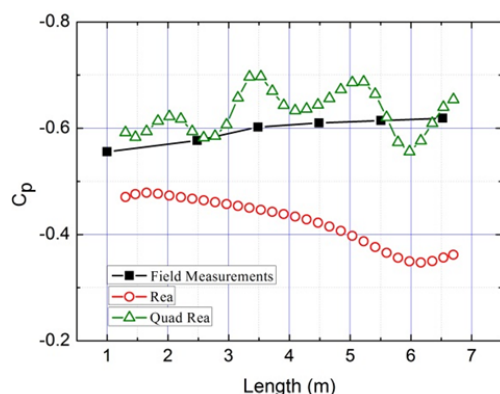


Figure 6. Pressure distributions along the side face.

The pressure distributions on the leeward face are showed in Figure 5. It is apparent that neither of the models could accurately predict the measurement results. The better approximations are obtained by the quadratic model, although the errors are around 45 to 50 percent. The pressure distributions on the side face are displayed in Figures 6. The measurement results on the side face shows a slow increase in negative magnitude as the wind velocity increases with height. The quadratic model predicts more accurate values than the standard linear eddy viscosity model. It has been proved that the size and strength of the roof recirculation zone is reduced as the flow approaching the side walls of the cube. The mass of air flowing over the roof is also reduced. It is therefore clear that the measurement results are affected to a much larger extent by three-dimensional flow phenomenon, due to significant difference between the centre and edge negative pressures. This suggests that the roof recirculation zone in the full-scale measurement contains a much stronger vortex than that predicted by CFD, which causes the pressure difference between the measurements and CFD results.

5 CONCLUSION

The quadratic eddy viscosity model, including new considerations of mathematic constraints, was developed in this paper to predict turbulent flows around bluff bodies. This study shows that adoption of the nonlinear eddy viscosity model yields better results than the standard linear eddy viscosity model, especially for the pressure distributions on the windward, roof and side faces of a 6m cube. Meanwhile, this nonlinear eddy viscosity model has great robustness to calculate turbulence anisotropy and is suitable for numerical simulation of complex flows around building structures. It is indicated that the quadratic NLEV model exhibit its advantages in capturing the pressure variations on a low-rise building, compared with those predicted by the standard two-equation turbulence models.

6 ACKNOWLEDGEMENTS

The work described in this paper is fully supported by grants from National Natural Science Foundation of China (Project No. 90815021 and 50778059) which are gratefully acknowledged.

7 REFERENCES

- 1 ANSYS Fluent 12.1 User Guide[M]. ANSYS Inc, 2010
- 2 Apsley, D. and Leschziner, M., A new low-Reynoldsnumber nonlinear two-equation turbulence model for complex flows. *International Journal of Heat and Fluid Flow*, 1998, 19, 209–222.
- 3 Architectural Institute of Japan, AIJ Recommendations for Loads on Buildings Tokyo, Japan: Architectural Institute of Japan, 2004.
- 4 Champagne FH, Harris VG, Corrsin S. Experiments on nearly homogeneous turbulent shear flow. *Journal of Fluid Mechanics*, 1970; 41:81–139.
- 5 Craft, T.J., Launder, B.E., Suga, K., A non-linear eddy viscosity model including sensitivity to stress anisotropy. In: Tenth Symposium on Turbulent Shear Flows. Pennsylvania State University, University Park, 1995, 23/19–23/24.
- 6 Craft, T., Launder, B., and Suga, K., Prediction of turbulent transitional phenomena with a nonlinear eddyviscosity model. *International Journal of Heat and Fluid Flow*, 1997, 18, 15–28.
- 7 Hellsten, A. and Be'zard, H., Behaviour of nonlinear two-equation turbulence models at the free-stream edges of turbulent flows. In: W. Rodi and M. Mulas, eds. *Engineering Turbulence Modelling and Experiments 6*. Elsevier, 2005.
- 8 Huang, Y.-N., Rajagopal, K.R., On a generalised nonlinear k– ϵ model for turbulence that models relaxation effects. *Theor. Comput. Fluid Dynam*, 1996, 8, 275–288.
- 9 Jongen, T., Machiels, L., and Gatski, T., Predicting noninertial effects with linear and nonlinear eddyviscosity, and algebraic stress models. *Flow, Turbulence and Combustion*, 1998, 60, 215–234.
- 10 Launder B. E., Spalding D. B. *The Numerical Computation of Turbulent Flows*, Comput. Methods Appl. Mech. Eng, 1974, 3: 269-289
- 11 Richards P.J., Hoxey R.P. and Short L.J., Wind pressures on a 6m cube, *Journal of Wind Engineering and Industrial Aerodynamics*, 2001, 89, 1553-1564
- 12 Speziale C. G., On nonlinear K-l and K- ϵ models of turbulence, *Journal of Fluid Mechanics*, 1987, 178 : pp 459-475.