

POD analysis of the finite-length square cylinder wake

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ABSTRACT: The near wake of a wall-mounted finite-length square cylinder with $H/d = 7$ is measured with PIV in three spanwise planes, i.e. near cylinder free end, at mid span and near the flat wall. The Reynolds number based on freestream velocity and cylinder width is 9300. Proper orthogonal decomposition (POD) technique is utilized to analyze the experimental data. Under the effects of free-end downwash flow, the near wake is highly three dimensional and drastically different from that of a 2D cylinder. The turbulent kinetic energy distributes over more POD modes relative to 2D cylinder wake. Low-dimensional model is not applicable to reconstruct this 3D flow. Both symmetrical and anti-symmetrical vortices structures occur in the finite-length cylinder wake, which can be identified based on the coefficients of the first two POD modes.

KEYWORDS: Bluff body, Finite-length cylinder, Wake, PIV, POD.

1 INTRODUCTION

The flow around a two-dimensional (2D) cylinder has long been the subject of extensive study. However, in engineering applications, the cylinder like structures are often finite in length (or height), with one end free and other end mounted on the ground, such as aerodynamic forces on high-rise buildings, plume behaviors behind chimneys, etc. Due to the effects of cylinder free end and boundary layer, the flow around a finite-length cylinder differs drastically from that behind a 2D cylinder^[1,2]. The near wake of a wall-mounted finite-length cylinder is characterized by the presence of two pairs of longitudinal tip and base vortices, in addition to the possible spanwise Karman vortex shedding. The interactions between these three types of vortices complicate the flow and make it highly three dimensional. The tip vortices are associated with the downwash flow behind cylinder free end^[3,4]. The base vortices depend on the conditions of the boundary layer where the cylinder mounted. When the boundary layer thickness is negligibly small, the base vortices will not present^[4,5].

The near wake of a finite-length cylinder depends strongly on H/d , where H and d are cylinder characteristic height and width, respectively. When H/d falls below a critical value, vortex shedding changes from antisymmetrical to symmetric^[5-7]. The critical value is 2~6, depending on factors such as the boundary layer thickness and incoming turbulence intensity^[6,8]. Wang & Zhou^[2] measured the near wake of finite-length square cylinder with $H/d = 3, 5$ and 7. They found both symmetrical and anti-symmetrical vortex shedding can be observed throughout cylinder span for all tested H/d . At mid-span of the cylinder with $H/d = 7$, which is larger than the critical value proposed, there are still more than 10% of instantaneous vortices are symmetrically arranged.

The proper orthogonal decomposition (POD) is a quantitative and an unbiased method that has been increasingly applied in conjunction with instantaneous whole-field measurement techniques such as PIV for identifying the coherent structures. The POD technique provides an optimal set of basis function (POD mode) from an ensemble of data to reconstruct the coherent structures as mixtures of POD modes. This method has been successfully used in various types of flows, such as jets, mixing layers, channel flows and cylinder wakes. Detailed information about POD in fluid dynamics may be found in Sirovich^[9].

The present paper aims to reveal the difference between the near wake structure of wall-mounted square cylinder with $H/d = 7$ and 2D one using PIV and POD techniques. From the PIV data, the time-averaged velocities, and other turbulent statistical quantities were obtained. Comparisons are also made in the context of POD mode and mode energy distribution to highlight the three dimensional effects for the wall-mounted finite-length cylinder wake.

2 EXPERIMENTAL DETAILS & POD METHOD

2.1 Experimental setup

All experiments were conducted in a closed-loop low-speed wind tunnel with a 2.4m long test section of 0.6m×0.6m. The flow velocity in the test section was uniform to 0.1% and the longitudinal turbulence intensity was less than 0.4%. A flat plate of 0.5m in width and 1.2m in length was installed in the test section with its upper surface 0.15m from the bottom wall. The leading edge of the flat plate was rounded to avoid flow separation. A square cylinder with a width $d = 20\text{mm}$ was placed at the centerline, 0.3m downstream from the leading edge of the plate, as shown in figure 1. The H/d was 7. All measurements were conducted at a free-stream velocity $U_\infty = 7\text{ms}^{-1}$, corresponding to a Reynolds number $\text{Re}_d \equiv U_\infty d/\nu = 9300$, where ν is the kinematic viscosity of fluid. A LDA was used to document the boundary layer at the cylinder axis. The boundary layer disturbance thickness (δ) was estimated to be 27mm so that $\delta/d \approx 1.35$. Considering the thickness of the boundary layer, it may result in pronounced upwash flow and associated base vortices^[10]. A nominal 2D square cylinder (with $H/d \approx 28$) wake was also measured at the same Re_d for comparison.

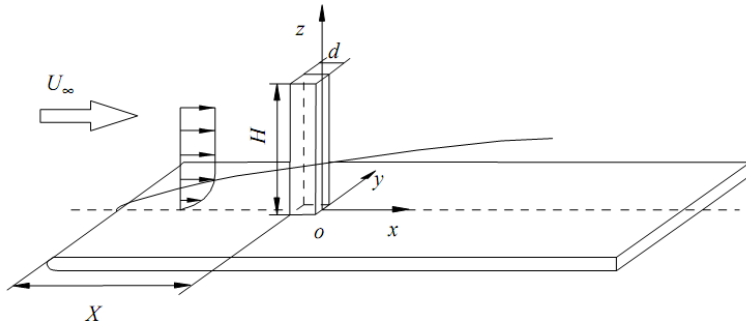


Figure 1. Experimental setup and the definition of coordinate system.

2.2 PIV measurements

A Dantec standard PIV2100 system was used to measure both the instantaneous and time-averaged flow fields. Flow illumination was provided by two NewWave standard pulse laser sources of a 532 nm wavelength, each with a maximum energy output of 120 mJ pulse⁻¹. Particle images were taken using one CCD camera (HiSense type 13, gain×4, double frames, 1280×1024 pixels). Synchronization between image taking and flow illumination was provided by the Dantec FlowMap Processor (PIV2001 type). PIV measurements were performed in three (x , y) planes at $z^* = 1, 3.5$ and 6, i.e. $1d$ above the flat plate, cylinder mid-span and $1d$ lower than the free end. The superscript asterisk in present paper indicates normalization by U_∞ and/or d . The image magnification was 148 $\mu\text{m}/\text{pixel}$. In processing PIV images, an interrogation area of 32×32 pixels was used with a 50% overlap in both directions, producing 78×62 in-plane velocity vectors. For each case, a total of 1500 PIV images were captured to calculate time-averaged velocities and Reynolds stresses and perform POD analysis.

2.3 POD method

The snapshot POD method, proposed by Sirovich^[9], was employed to analysis the PIV data. By this method, each instantaneous PIV data is considered a snapshot of the flow field. The present snapshot POD follows the procedure outlined by Meyer *et al.*^[11]. The first step is to calculate the mean velocity field, which can be considered the zero mode of POD. The rest of analysis works on the fluctuation parts of the velocity components (u_i^n, v_i^n), where u and v denote the fluctuation velocity in streamwise and lateral directions. Index n runs through the N snapshots, and i runs through the M positions of velocity vectors in a given snapshot.

All fluctuating velocity components from N snapshots are arranged in a matrix U as:

$$U = [u^1, u^2 \dots u^N] = \begin{bmatrix} u_1^1 & u_1^2 & \dots & u_1^N \\ \vdots & \vdots & \vdots & \vdots \\ u_M^1 & u_M^2 & \dots & u_M^N \\ v_1^1 & v_1^2 & \dots & v_1^N \\ \vdots & \vdots & \vdots & \vdots \\ v_M^1 & v_M^2 & \dots & v_M^N \end{bmatrix} \quad (1)$$

The autocovariance matrix C is created as $C = U^T U$. A set of N eigenvalues λ^i and corresponding set of eigenvectors A^i which satisfy $CA^i = \lambda^i A^i$ are evaluated. The eigenvalues are ordered by decreasing value as $\lambda^1 > \lambda^2 > \dots > \lambda^N > 0$. The normalized POD modes φ^i are constructed from the projection of the eigenvectors A^i as:

$$\varphi^i = \frac{\sum_{n=1}^N A_n^i u^n}{\left\| \sum_{n=1}^N A_n^i u^n \right\|}, \quad i = 1, 2, \dots, N \quad (2)$$

Each snapshot can be expanded in a series of the POD modes with expansion coefficients a_i for each φ^i . The coefficients a_i also called POD coefficients, are determined by projecting the fluctuating part of the velocity field onto the POD modes $a^n = \Psi^T u^n$, where $\Psi = [\varphi^1, \varphi^2 \dots \varphi^N]$. Then, the expansion of the fluctuation part of a snapshot n is obtained from:

$$u^n = \sum_{i=1}^N a_i^n \varphi^i \quad (3)$$

Readers may refer to Sirovich^[9] and Meyer *et al.*^[11] for more details of the POD method.

3 RESULTS & DISCUSSIONS

Figure 2 presents the contours of $\overline{U^*}$, $\overline{u_{rms}^*}$, $\overline{v_{rms}^*}$ and $\overline{u^* v^*}$ at $z^* = 6$ and 3.5 for a finite-length cylinder with $H/d = 7$, together with those for a 2D square cylinder at the same Re_d for comparison. The overbar denotes time-average and the subscript *rms* indicates the root-mean-square value of fluctuation velocity. The results at $z^* = 1$ are qualitatively similar to those at $z^* = 3.5$ thus not present here. The streamwise extent of the reverse flow zone at mid-span, indicated by the bold line in figure 2(i), is far larger than that of 2D cylinder (figure 2a). A shorter reversed flow zone implies stronger entrainment of high-speed free-stream fluid into the wake, i.e. stronger vortex shedding. The maximum values of $\overline{u_{rms}^*}$, $\overline{v_{rms}^*}$ and $\overline{u^* v^*}$ at the mid-span (figure 2 j, k & l) are about

one-half of those for a 2D cylinder at the same Re_d (figure b, c & d), which also suggests that the vortex shedding is less organized in a finite-length cylinder wake. Near the free end ($z^*=6$), all the contours appear to converge towards the centerline with increasing x^* due to downwash flow from the free end, which differ not only quantitatively but qualitatively from those at mid-span. Under the effect of free-end downwash flow, the reverse flow zone at $z^*=6$ shrinks significantly relative to that at mid-span, while still far larger than that of 2D cylinder.

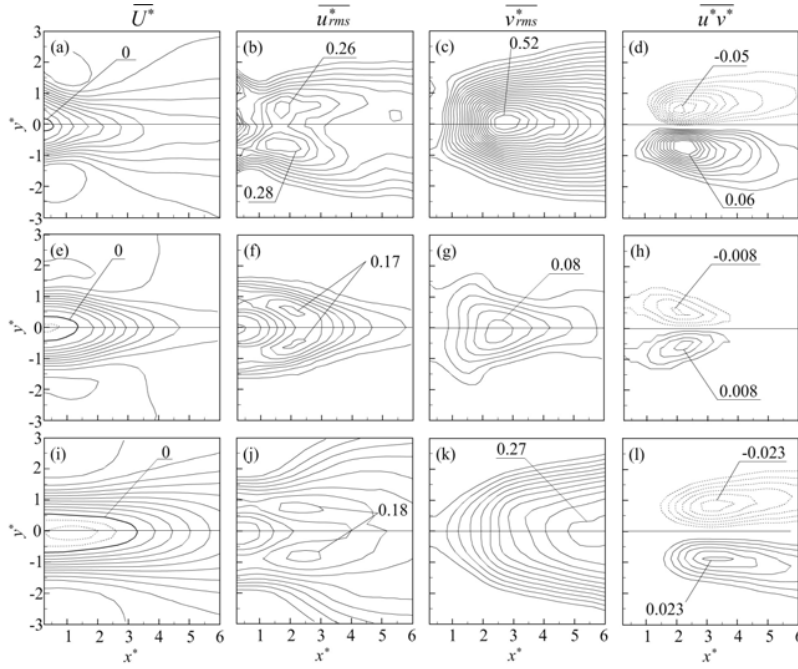


Figure 2. The contours of $\overline{U^*}$, $\overline{u_{rms}^*}$, $\overline{v_{rms}^*}$ and $\overline{u^* v^*}$. (a-d) 2D square cylinder; (e-h) $z^*=6$, for $H/d=7$; (i-l) $z^*=3.5$, for $H/d=7$. (a, e & i) Contour interval = 0.1; (b, f & j) 0.02, 0.02, 0.02; (c, g & k) 0.02, 0.01, 0.02; (d, h & l) 0.005, 0.002, 0.004.

To reveal the three-dimensionality of vortex structures for finite-length cylinder, POD analysis was applied in spanwise planes of different z^* . Since the sample size or number of snapshots necessary to obtain converged POD analysis was about 1000^[12], the 1500 PIV records at each spanwise position for present experiments are sufficient. POD analysis was also applied for a 2D square cylinder wake at the same Re_d for comparison.

Figure 3 shows the first 3 modes of u^* , v^* and ω_z^* for a 2D square cylinder wake. The contour structures of Mode1 and 2 are highly correlated. Particularly, Mode 2 can be regarded as a streamwise shift of Mode 1, with the phase difference of about 1/4 shedding circle. These observations are similar to those in a 2D square cylinder wake^[13]. For Mode1 and 2, the contours of ω_z^* with positive and negative values present alternating pattern. This pattern inherently resembles the alternating Karman vortex structure. The pattern of Mode 3 is drastically different from those of the first two modes, as shown in figure 3. The contours of $\phi_{w_z}^*$ are somewhat symmetrical about the wake centerline. However, the strength of Mode3 is far weaker than Mode 1 and 2, with the maximum of $\phi_{w_z}^*$ less than 5% of those for the first two modes, which may suggest that its contribution to the total fluctuation energy is limited.

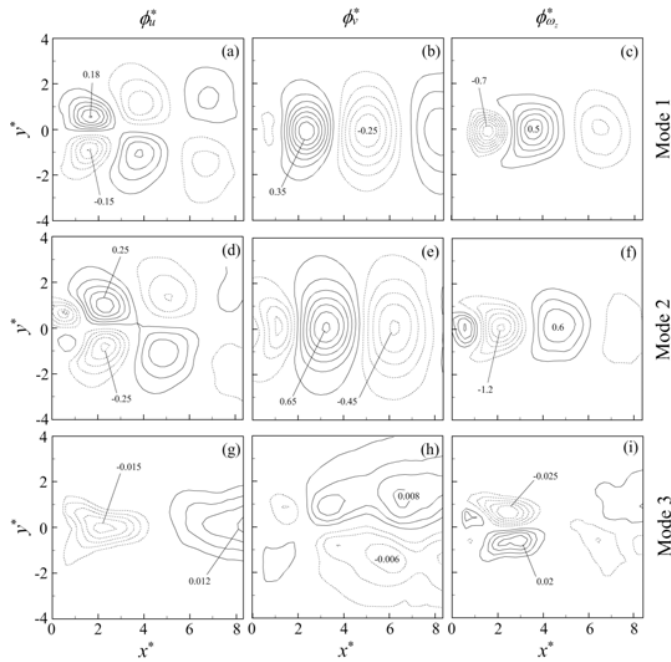


Figure 3. Contours of the first three POD modes in a 2D square cylinder wake: ϕ_u^* , ϕ_v^* and $\phi_{w_z}^*$. (a) Contour interval = 0.03, (b) 0.05, (c) 0.1, (d) 0.05, (e) 0.1, (f) 0.2, (g) 0.003, (h) 0.002 and (i) 0.005.

Figures 4 & 5 present the first three modes in the finite-length cylinder wake at $z^* = 6$ and 3.5. The results at $z^* = 1$ are qualitatively similar to those at $z^* = 3.5$, thus not present here. At $z^* = 6$, i.e. $1d$ lower than the free end, Wang & Zhou^[2] found over 55% spanwise vortices are symmetrical or approximately symmetrical. They ascribed this observation to the free-end downwash flow, which tends to change vortex shedding from anti-symmetrical to symmetrical. For a wall-mounted finite-length circular cylinder ($H/d = 7$, $Re_d = 1060$), Okamoto & Sunabashiri^[5] suggested the anti-symmetrical vortex shedding vanishes at $0.5d$ lower than the free end. For the present finite-length square cylinder wake, Wang & Zhou^[2] suggested that about 13% of vortices at $1d$ lower than the free end are anti-symmetrical. Drastically different from that of 2D cylinder wake, the contours of $\phi_{w_z}^*$ of Mode 1 is symmetrical about the centerline, as shown in figure 4(c). Although $\phi_{w_z}^*$ of Mode 2 is also approximately symmetrical about the centerline, its structure does not correlate with Mode1. The contours of $\phi_{w_z}^*$ for the first two modes converge towards the centerline, accord with the behavior of time-averaged quantities as shown in figure 2(e-h). For Mode3, the structure becomes more complicated and less organized.

At $z^* = 3.5$, i.e. the cylinder mid-span, the positive and negative $\phi_{w_z}^*$ contours for the first two modes occur alternately along centerline (figure 5c & f), generally similar to those of 2D cylinder wake. However, a close look at the topology may find that the contours are stretched in transverse direction and the local extreme values occur symmetrically on both sides of the centerline (figure 5c & f), which are far different from those of 2D cylinder wake (figure 3 c & f). The transverse stretched $\phi_{w_z}^*$ contours of the first two energetic modes are ascribed to the free-end downwash flow, which tends to entrain fluid from near the free end and separate the shear layers from both sides of the cylinder. The structure of Mode3 for finite-length cylinder (figure 5g-i) is drastically different from that of 2D cylinder. It is interesting that, a close resemblance

can be observed between Mode 1 at $z^* = 6$ (figure 4a-c) and Mode 3 at $z^* = 3.5$ (figure 5g-i) except the opposite sign.

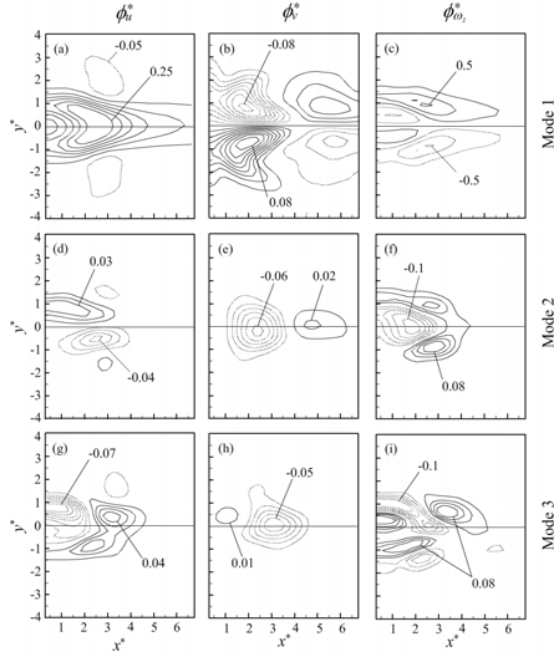


Figure 4. Contours of the first three POD modes of ϕ_u^* , ϕ_v^* and $\phi_{w_z}^*$ at $z^* = 6$ for $H/d = 7$ (a) Contour interval = 0.05, (b) 0.01, (c) 0.2, (d) 0.01, (e) 0.01, (f) 0.02, (g) 0.01, (h) 0.01 and (i) 0.02.

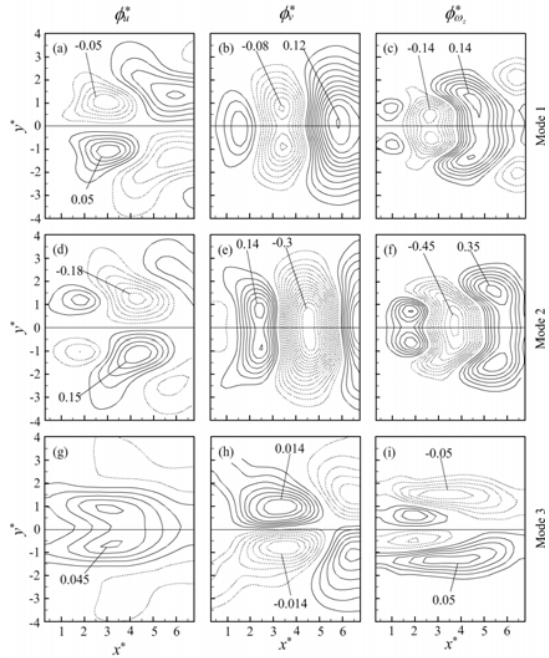


Figure 5. Contours of the first three POD modes of ϕ_u^* , ϕ_v^* and $\phi_{w_z}^*$ at $z^* = 3.5$ for $H/d = 7$ (a) Contour interval = 0.01, (b) 0.02, (c) 0.02, (d) 0.03, (e) 0.02, (f) 0.05, (g) 0.01, (h) 0.002 and (i) 0.01.

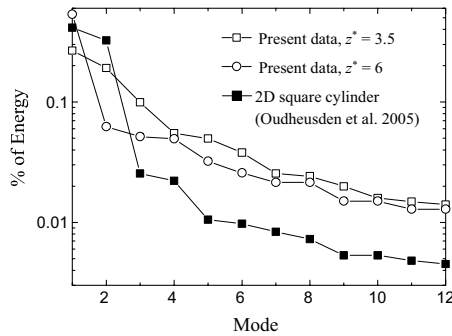


Figure 6. Fractional contribution to the turbulent kinetic energy by different mode.

The fractional contributions to the turbulent kinetic energy (TKE) from different modes are illustrated in figure 6. For the 2D square cylinder at $Re_d \approx 10000$ ^[13], the fractional contribution reduces quickly with increasing mode number. Over 73% (41.3% + 32.4%) TKE is from the first two modes, suggesting the near wake is dominated by the alternating Karman vortex shedding. The TKE from the symmetrical Mode 3 diminishes remarkably to about 2.5%. For the present finite-length cylinder, at $z^* = 3.5$, the first two modes account for 45.9% (the first and second modes contributes 26.7% and 19.2%, respectively) of total TKE, significantly smaller than that for 2D cylinder wake. Contrary to the reduced contribution of the first two modes, the TKE contributions from higher modes are larger than the corresponding values for 2D cylinder. Particularly, the TKE from the symmetrical Mode 3 is 10% at $z^* = 3.5$, which suggests that symmetrical vortex shedding is still remarkable. This observation accord with the finding proposed by Wang & Zhou^[2] that about 10% vortex shedding is symmetrically arranged at cylinder mid-span. The TKE contributions from Mode 1 and 2 are close to each other for 2D cylinder wake, which is also observed at the mid-span for $H/d = 7$. This observation may because the first two modes are highly correlated. However, the TKE contribution at $z^* = 6$ is completely different. First, the TKE from the symmetrical Mode 1 is over 54%, which is far larger than that at mid-span. Second, the TKE from Mode 2 is about only 1/10 of that from Mode 1, suggesting the near wake at $z^* = 6$ is dominated by Mode 1 which represent the effects of free-end downwash flow, as shown in figure 4(a-c). Figure 6 also suggests that, the TKE distributes over more modes for the finite-length cylinder wake than for 2D cylinder wake. For example, for 2D cylinder wake, the first two modes account for over 70% of total TKE, while at mid-span of $H/d = 7$, it needs 6 modes to contribute 70% of total TKE.

In view of the small TKE contribution from Mode 3 and other higher modes in 2D cylinder wake, the instantaneous velocity field can be reconstructed with a low-order flow model incorporating only the time-averaged and the first two POD modes^[13] via:

$$u_{LOM} = \bar{U} + a_1 \phi^1 + a_2 \phi^2 \quad (4)$$

$$a_1 = \sqrt{2\lambda_1} \sin \theta, \quad a_2 = \sqrt{2\lambda_2} \cos \theta \quad (5)$$

where u_{LOM} is the instantaneous velocity field and θ is the vortex-shedding phase angle (Durai et al. 1988). The coefficient a_1 and a_2 are not independent, but forms an ellipse in the (a_1, a_2) plane. The cross plot of a_1 and a_2 , normalized by their own eigenvalues, is shown in figure 7. For the 2D square cylinder wake, as shown in figure 7(a), the cross plot of normalized (a_1, a_2) approximately forms a circle, confirms the relation shown in Eqs. (4 & 5). As shown in figure 7(a), the observed scatter of the data point around the theoretical circle can be an indication of cycle-to-cycle variations of the vortex shedding process, induced by small scale fluctuations or turbu-

lence^[13]. This scatter reduces with decreasing Re_d . Ma & Karniadakis^[14] found the cross plot of (a_1, a_2) forms nearly an ideal circle and not scattering presents at $Re_d = 185$.

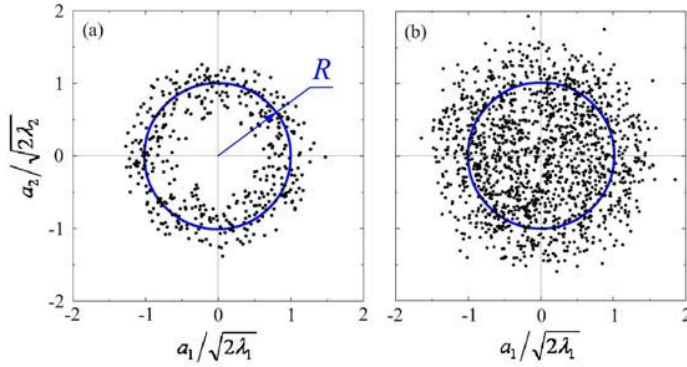


Figure 7. Snapshot mode coefficient correlation (a) 2D cylinder wake, (b) at mid-span for $H/d = 7$.

For the present finite-length cylinder, at $z^* = 3.5$ (figure 7b), the points distribute more dispersedly in (a_1, a_2) plan relative to that of 2D cylinder wake. Considering the Re_d is the same for figure 7(a & b), the dispersion in figure 7(b) may be ascribed to the different vortex shedding mechanism in a finite-length cylinder wake. As shown in figure 6, at $z^* = 3.5$, since the first two modes accounts for less than half of total TKE, the low-order model may be not applicable. It is worth mentioning that, each point in figure 7 indicates a snapshot. For the points near the origin point (as shown in figure 7b), the TKE contributions from the first two modes are quite limited, because of the small absolute values of a_1 and a_2 . For theses snapshots, the main TKE are from Mode 3 and other higher modes, which are less organized. On the other hand, for the points away from the origin point, most TKE are from the first two modes.

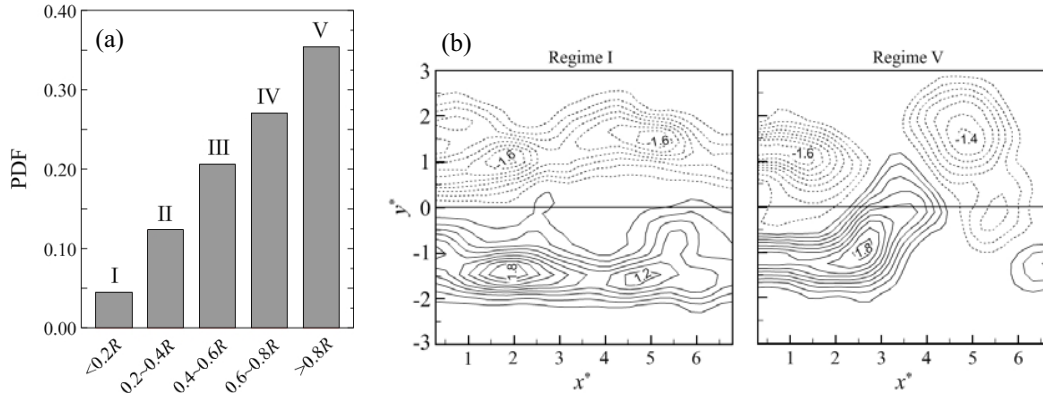


Figure 8. PDF of different flow regimes and typical instantaneous vorticity contours for Regime I and V. (a) PDF distribution; (b) normalized vorticity contours for Regime I and Regime V, contour interval = 0.2

Since the vortices in mode 1 and mode 2 are stagger arranged, different from those in mode 3 and other higher modes, it may be possible to use the distance between the snapshot point and origin point to classify the instantaneous flow status. Five regimes are classified based on this distance, i.e. $<0.2R$, $0.2\sim0.4R$, $0.4\sim0.6R$, $0.6\sim0.8R$ and $>0.8R$. The probability density function for these five regimes determined from 1500 instantaneous snapshots is shown in figure 8(a). Figure 8(b) presents two typical instantaneous vorticity contours for Regime I and V, respectively. For Regime I, the most TKE are from mode 3 and other higher modes; while for Regime V, most TKE are from the first two modes. Since the flow structure of the first two modes is different from that of the other higher modes, as shown in figure 5, the corresponding flow structures for Regimes I and V are also different (figure 8b). For Regime I, the shear layers from both

sides of the cylinder do not roll into successive vortices and the near wake is symmetrical about the wake centerline. For Regime V, the near wake is dominated by stagger arranged vortices, similar to those observed in 2D cylinder wake.

4 CONCLUSIONS

The near wake of a wall-mounted finite-length cylinder has been measured with PIV and analyzed using POD technique. The cylinder aspect ratio $H/d = 7$. PIV measurements were conducted in spanwise planes at $z^* = 1, 3.5$ and 6. The following conclusions may be drawn from the analysis of the data:

- (1) The flow depends strongly on the spanwise position, the contours of $\overline{U^*}$, $\overline{u_{rms}^*}$, $\overline{v_{rms}^*}$ and $\overline{u^* v^*}$ converge towards wake centerline near the free end ($z^* = 6$), which is significantly different from those at mid span. Under the effects of free-end downwash flow, the streamwise extend of reverse flow zone at most spanwise positions of present tested finite-length cylinder is larger than that for 2D cylinder, and the magnitudes of $\overline{u_{rms}^*}$, $\overline{v_{rms}^*}$ and $\overline{u^* v^*}$ are significantly suppressed, especially near the free end.
- (2) At the mid span of $H/d = 7$, although the vortical structures of the first two POD modes are staggered arranged along wake centerline and highly correlated with each other, somewhat similar to those in 2D cylinder wake, the vorticity contours are stretched in lateral direction with the local extreme values occur symmetrically on both sides of centerline. Near the free end, i.e., $z^* = 6$, the structure of first POD mode is completely different from that of higher modes.
- (3) For finite-length cylinder wake, the TKE distributes over mode POD modes relatively to that of 2D cylinder wake. Instantaneous vortex shedding patterns can be classified based on the coefficients of the first two POD modes. Both symmetrical and anti-symmetrical vortex shedding occurs in the near wake at $z^* = 3.5$.

5 ACKNOWLEDGEMENTS

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