

Analytical study on lateral-torsional buckling of long-span suspension bridge

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ABSTRACT: Not only aerodynamic instability such as flutter but also aerostatic instability such as lateral-torsional buckling would be concerns of wind-resistant design of super long-span suspension bridges. Authors' past analytical study on lateral-torsional buckling proved the difficulty of distinction of lateral-torsional buckling from divergence, both of which are divergent response phenomena. This study tried to numerically identify lateral-torsional buckling of a long-span suspension bridge with considering geometric and aerodynamic non-linearity under wind loading.

KEYWORDS: Lateral-torsional buckling, suspension bridge, geometric nonlinearity, aerodynamic nonlinearity

1 INTRODUCTION

As the main span length of a suspension bridge becomes much longer, it was pointed out (Nakamura, et al., 1998) that not only aerodynamic instability such as flutter but also aerostatic instability such as lateral-torsional buckling would be concerns of wind-resistant design. Hirai (1942) of the University of Tokyo of those days firstly proposed a calculation formula of the critical wind speed of lateral-torsional buckling of a suspension bridge. This formula included some simplifications such as only consideration of force equilibrium of a wind-loaded girder and exclusion of geometric nonlinearity. Since then, analytical studies (Nazmy, et al., 1990, Boonyapinyo, et al., 1993) on lateral-torsional buckling including aerodynamic, geometric and material nonlinearity have been conducted.

Recently, owing to the availability of powerful advanced FEM analysis softwares, sophisticated structural analysis including nonlinearities and wind-structure interactions can be possible. Authors have tried to analyze lateral-torsional buckling of suspension bridges (Yamada, et al., 2009). However difficulty of distinguishing lateral-torsional buckling from divergence was encountered. Because they are both aerostatic instability accompanying sudden large wind-induced deflections at a certain high wind speed.

In this study, a Mode Tracing Flutter Analysis method (Nguyen, et al., 1998), which was originally developed to trace modal parameters of pre-selected particular vibration modes in flutter analysis, was used to identify and distinguish lateral-torsional buckling from divergence. The paper presents the analytical methodology and demonstration results for lateral-torsional buckling of a long-span suspension bridge.

2 CLOSED-FORM SOLUTION OF AEROSTATIC INSTABILITY

2.1 Lateral-torsional buckling

A girder of a suspension bridge deflects largely in an along-wind direction under wind loading. Not only lateral bending moment due to a drag force but also vertical lift and torsional moment are applied to the bridge girder. This may accelerate the occurrence of lateral-torsional buckling of the girder more than only drag force case.

Hirai (1942) firstly proposed a calculation formula of the critical wind speed of lateral-torsional buckling of a suspension bridge girder. A girder is subjected to a drag force laterally about its strong axis, a lift force vertically about its weak axis and a pitching moment about its rotation center, as shown in Figure 1. Equilibriums of all forces for the girder are written in Equation (1).

$$EI \frac{d^4 \eta}{dx^4} - 2H_\omega \frac{d^4 \eta}{dx^4} - 2h_1 \frac{d^2 y}{dx^2} + \frac{d^2}{dx^2} (M\phi) - (S + C_d)pb\phi = 0 \quad (1a)$$

$$M \frac{d^2 \eta}{dx^2} - EC_\omega \frac{d^4 \eta}{dx^4} - \left(GK + \frac{H_\omega b^2}{2} \right) \frac{d^2 \eta}{dx^2} - bh_2 \frac{d^2 y}{dx^2} - S_t pb^2 \phi = 0 \quad (1b)$$

where η and ϕ are a vertical and torsional displacement, respectively, and EI , GK and EC_ω are a bending, torsional and bending-torsion stiffness, respectively. H_ω is a cable tension under self weight, M is lateral bending moment of the girder. C_d , S and S_t are a drag coefficient, slopes of lift and pitching moment coefficients, respectively. In addition, h_1 and h_2 are an increment of a cable horizontal tension by η and ϕ , respectively.

Secondly, Hirai developed an equation of motion with including an inertia term of the girder. He identified the critical wind speed condition by comparing a critical condition of lateral buckling, as shown in Equation (2).

$$V_k^2 = \frac{142.2}{\mu C_D \rho b l^3} \sqrt{EJ \cdot \overline{GK}} \quad (2)$$

$$\text{where } EJ = EI + \frac{l^2}{2\pi^2} H_\omega, \quad \overline{GK} = \frac{4\pi^2}{l^2} EC_\omega + GK + \frac{b^2}{2} H_\omega \quad \text{and} \quad \mu = \sqrt{1 + 0.287 \frac{S + C_D}{C_D}}$$

Equation (2) assumes an asymmetric lateral-torsional buckling mode for a mono span suspension bridge. Therefore, some correction is necessary for a multi span suspension bridge or different boundary conditions such as the transmission of wind load to main cables (JSCE, 1967 & Okauchi, et al., 1967).

2.2 Divergence

Divergence is another aerostatic instability which is negative feedback of torsional deflection due to pitching moment. A critical wind speed for divergence is given in Equatin (3) by equilibrium of pitching moment and torsional restoring force. Although divergence is clearly different from lateral-torsional buckling from the viewpoint of occurrence mechanism, it is very difficult to distiguish between two aerostatic instabilities by numerical prediction. Because both two phenomena suddenly occur at a certain wind speed.

$$U_C = \sqrt{\frac{2k_a}{\rho B^2 S_t}} \quad (3)$$

where k_a is torsional stiffness.

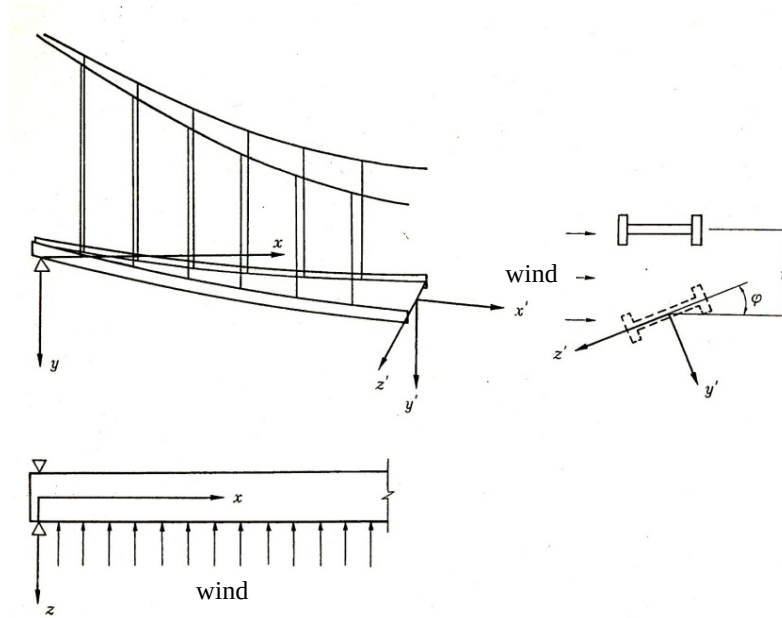


Figure 1. Deflection of suspension bridge girder under wind loading

3 ANALYTICAL FORMULATION OF LATERAL-TORSIONAL BUCKLING

3.1 Mode-trace Method

A mode-trace method, which was developed by Dung (1998), et al, is a flutter analysis method with consideration of large wind-induced deflection of a bridge. It consists of two parts: static and dynamic analyses. The static analysis part solves an equilibrium condition of the bridge structure under wind loading and corresponding large deflection at each analytical wind speed step. The dynamic analysis part uses that equilibrium condition and solves an eigenvalue problem with consideration of aeroelastic wind forces. In particular, the mode-trace method can solve only eigenvalues of interest which are selected in advance. Using this method, particular modes only can be sought with the increase of wind speed.

In this study, the static analysis part is used to find the critical condition of lateral-torsional buckling. In case of a suspension bridge, large wind-induced deflection is caused by wind load. Geometric nonlinearity should be considered. In addition, wind load of a suspension bridge girder is dependent on angle of attack (wind-induced torsional deflection). Wind load should also be updated according to wind-induced torsional deflection obtained at each wind speed.

As described in 2, lateral-torsional buckling and divergence are both static instability that torsional deflection suddenly increases at a certain wind speed. It is difficult to distinguish those two phenomena from wind-induced torsional deflection only. In this study, lateral-torsional buckling is identified by searching a condition that any diagonal term in the stiffness matrix is non positive during conducting the static analysis part of the mode-trace method.

3.2 Benchmark study

In order to validate the identification method of lateral-torsional buckling by diagonal terms, a benchmark study was conducted with using a simple beam subjected to uniform distributed loads as shown in Figure 3. The beam has a 10 m I shape cross section with 300 mm wide, 500 mm high, 11 mm flange thickness and 18mm web thickness. Lateral load which is equivalent to 0.1% span length imperfection was applied. Increasing the uniform distributed load, a critical condition that any diagonal term in the stiffness matrix is non positive was sought.

Figure 4 shows vertical, lateral and torsional deflections in the middle of span versus uniform distributed load intensity. Lateral and torsional deflections suddenly increase at the load intensity of 84 kN/m while vertical deflection increases linearly. At this step, diagonal terms of the stiffness matrix became non positive. It turned out that identification of lateral-torsional buckling by non positive diagonal terms in the stiffness matrix is valid.

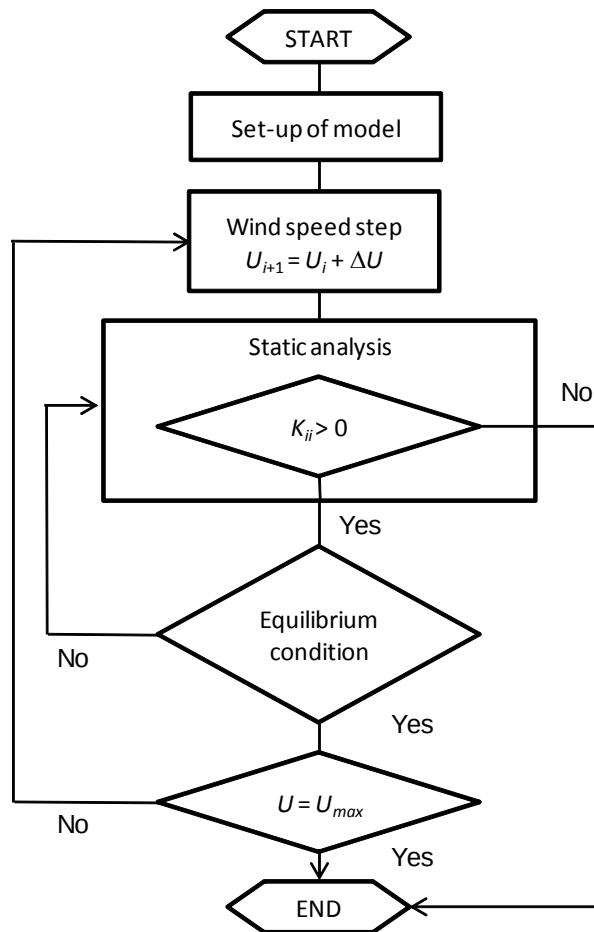


Figure 2. Flowchart of analysis of lateral-torsional buckling

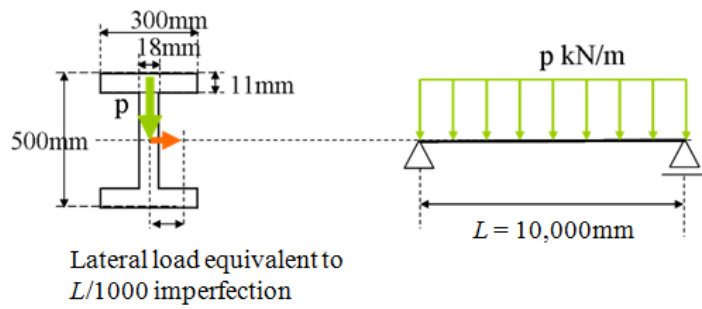
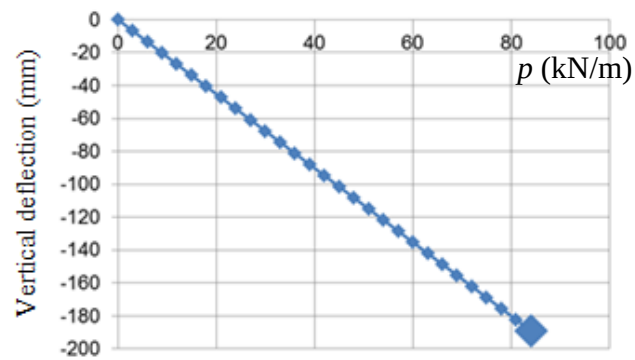
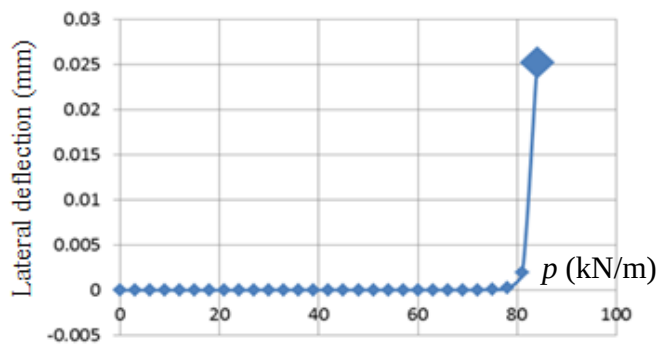


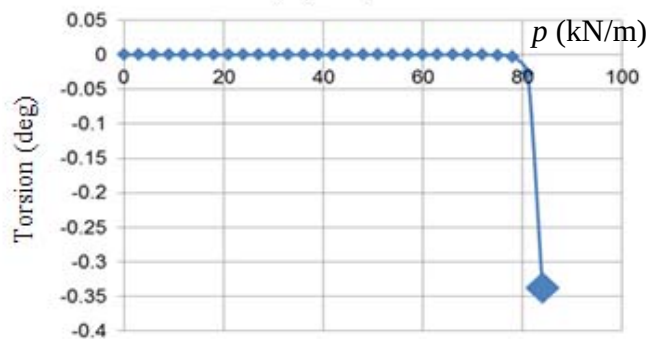
Figure 3. Simple beam for benchmark study



(1) Vertical deflection



(2) Lateral deflection



(3) Torsional deflection

Figure 4. Deflection at span center vs. load intensity

4 LATERAL-TORSIONAL BUCKLING OF AKASHI KAIKYO BRIDGE

4.1 *Original analysis*

Using the mode-trace method, lateral-torsional buckling of the Akashi Kaikyo Bridge was analyzed. It has a truss-stiffened girder with the center span of 1,991 m and two side spans of 960 m. It was modeled by a 3D frame model with lumped masses as shown in Figure 5. Figure 6 shows three component force coefficients vs. angle of attack.

According to the three component force coefficients and an angle of attack at each wind speed step, drag, lift and pitching moment were applied at shear centers of the girder. In addition, drag and lift were applied at main cables, and drag was applied at towers.

Figure 7 shows wind-induced deflections in the middle of the center span of (1) vertical, (2) Lateral and (3) torsion versus wind speed. Vertical and torsional deflections suddenly increase at a wind speed of 140 m/s where a diagonal term of the stiffness matrix is non positive. Because a diagonal term of the stiffness matrix is non positive, this sudden increase of vertical and torsional deflections at a wind speed of 140 m/s must result from the onset of lateral-torsional buckling, which is clearly different from divergence.

4.2 *Parameter analysis*

In order to investigate the contribution of each three component force coefficient to lateral-torsional buckling, parameter studies with different combinations of three component force coefficients were carried out.

Firstly, drag forces were only applied (CD only). Figure 8 and Table 1 show comparisons of wind-induced torsional deflection between drag only case and original case (Figure 7). This case exhibited lateral-torsional buckling at a wind speed of 305 m/s, which is much larger than the original case. This shows that a drag force can cause lateral-torsional buckling due to the structural feature of a suspension bridge that the girder is suspended by hangers. In addition, it can be understood that a lift force and pitching moment can enhance the onset of lateral-torsional buckling because those forces act out of the lateral direction.

Secondly, effects of a lift force were investigated. Figure 9 and Table 1 show comparisons of wind-induced torsional deflection among different lift force coefficient cases. The case that lift force coefficients were decreased to 1/10 exhibited lateral-torsional buckling at a wind speed of 140 m/s. In addition, the 10 times case yielded 125 m/s. From both results, it can be understood that lateral-torsional buckling is not affected much by lift forces.

Finally, effects of a pitching moment were investigated. Figure 10 and Table 1 show comparisons of wind-induced torsional deflection among different pitching moment coefficient cases. From the cases with 10 times and 1/10 pitching moment coefficients, it can be understood that lateral-torsional buckling is largely affected by a pitching moment. Furthermore, only pitching moment cannot cause lateral-torsional buckling easily. It can also be understood that lateral-torsional buckling is an instability phenomenon caused by a multiplication effect of a drag, lift and pitching moment.

Table 1. Analytical conditions of parameter study.

Cases	Critical wind speed (m/s)
Original	140
CD only	305
CL/10	140
10CL	125
CM/10	215
10CM	60
CM only	220
10CM only	70

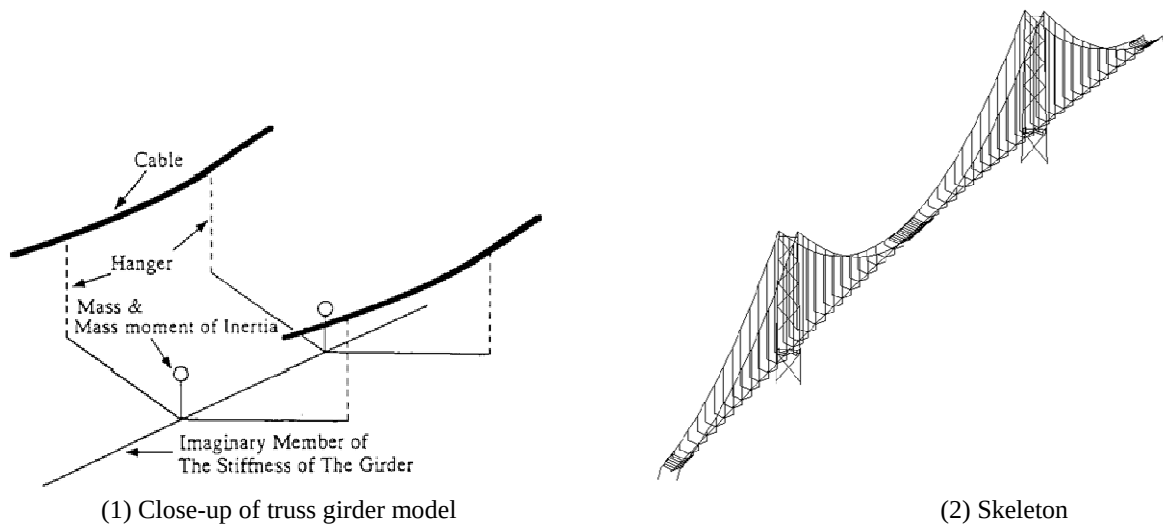


Figure 5. Analytical model of Akashi Kaikyo Bridge

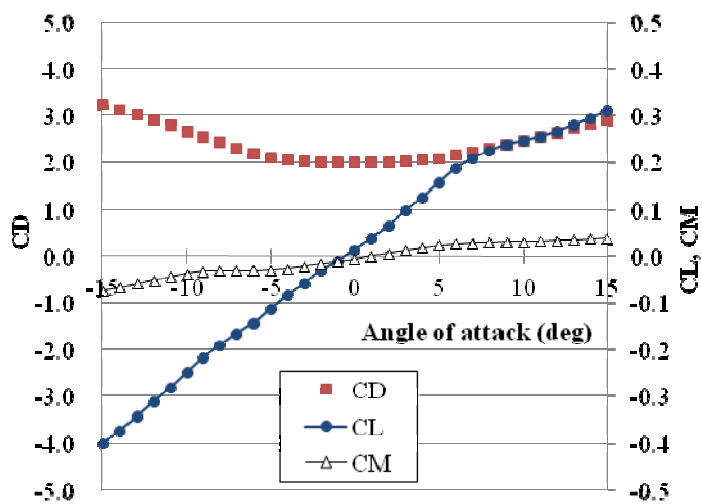
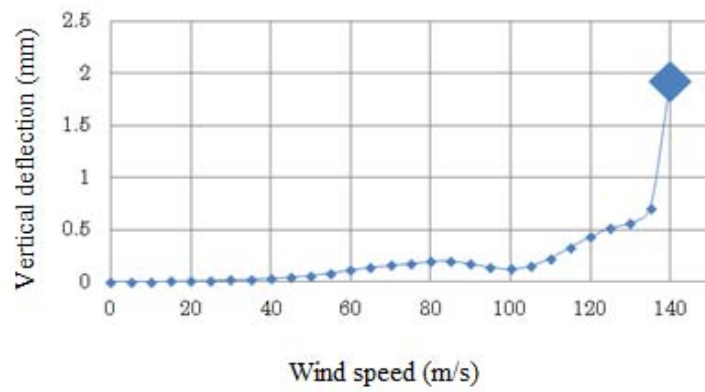
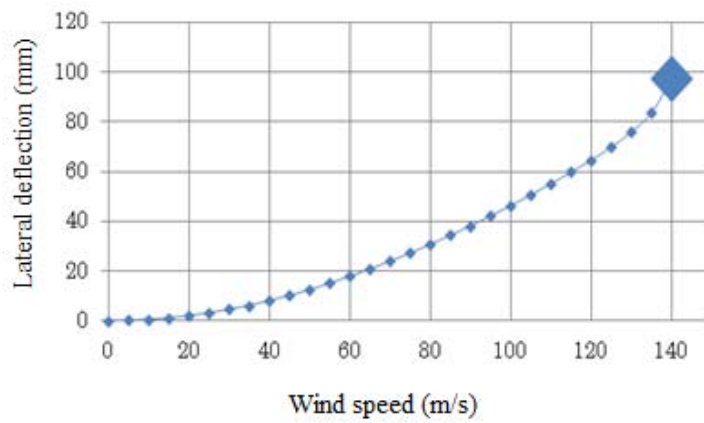


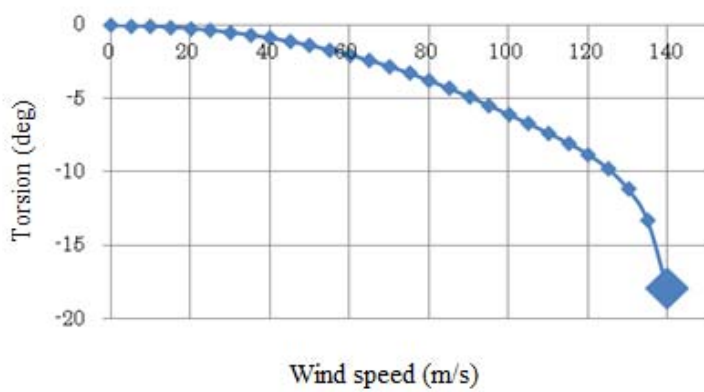
Figure 6. Three component force coefficients



(1) Vertical deflection



(2) Lateral Deflection



(3) Torsional deflection

Figure 7. Deflection at span center vs. wind speed

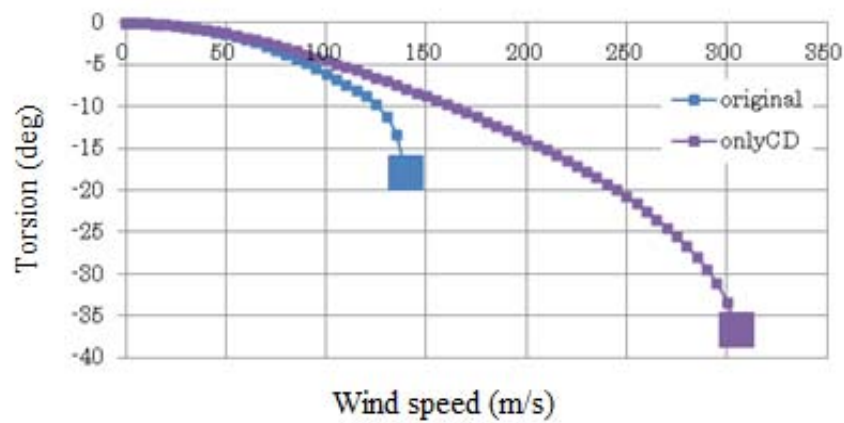


Figure 8. Deflection at span center vs. wind speed for drag only case

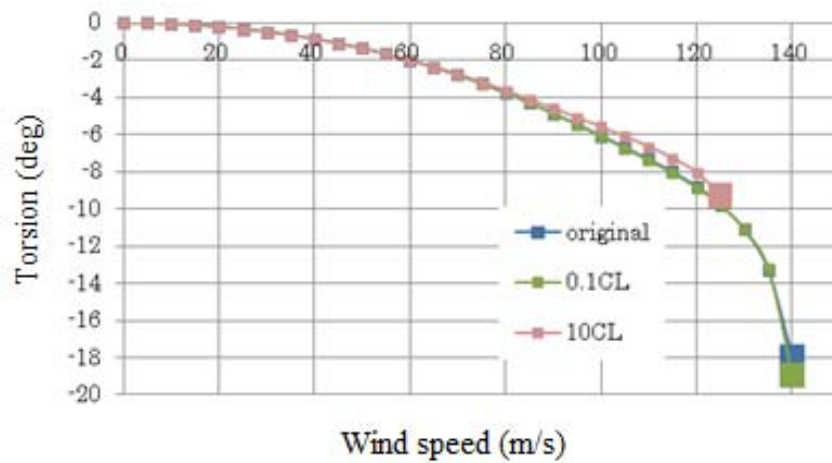


Figure 9. Deflection at span center vs. wind speed for different lift coefficient case

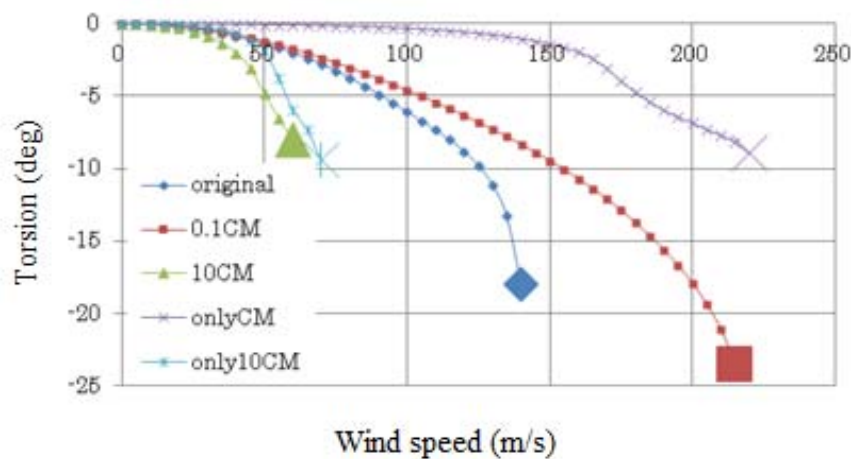


Figure 10. Deflection at span center vs. wind speed for different pitching moment coefficient case

5 CONCLUSIONS

Numerical estimation of lateral-torsional buckling of a long-span suspension bridge with using a Mode Tracing Flutter Analysis method was carried out. Results obtained in this study are as follows:

Onset of lateral-torsional buckling was identified by a non-positive value of diagonal terms in the stiffness matrix incorporating wind-loading effects. A benchmark study for a simple beam and an application for the Akashi Kaikyo Bridge showed that a deflection process with the increase of wind speed well represented a sudden onset of instability where a non-positive value was detected in a diagonal term of the stiffness matrix. However experimental work to validate the estimation method should be done in future.

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