

A conditional analysis of roller vortices within the atmospheric log layer

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ABSTRACT: The scope of this work is a characterization of the fluctuations and near-ground roller vortices within the turbulent atmospheric boundary layer (ABL) at heights of approximately 5% of the ABL thickness. This height is comparable to a y^+ value of approximately 100. To evaluate the roller vortices, a conditional-analysis technique is applied on a data set acquired on January 5th, 2012. This study begins with an evaluation of the ABL stability and thickness (δ). Subsequently statistical descriptors of the turbulent fluctuations are compared to those measured by Klebanoff [1] over a flat plate. The ABL data set is demonstrated to be neutral with a thickness of approximately 1000m. The statistical descriptors of the turbulent fluctuations of the data are found to qualitatively agree with values measured over a flat plate. From the conditional analysis, it is found that the centre heights of the roller vortices follow a Rayleigh distribution with a mean and standard deviation of 0.056δ and 0.027δ , respectively. However, the roller vortices concentrated around 0.05δ tend to have near-zero strength, while vortices above or below increase in strength. The concentration of near-zero strength vortices at 0.05δ is believed to be caused by the omission of the viscous core in the theoretical model.

KEYWORDS: Atmospheric turbulence, coherent structures, conditional analysis.

1 INTRODUCTION

Understanding turbulent fluctuations and coherent structures in the lower log layer of the atmosphere is valuable for several applications such as wind-turbine control, managing gusts during the takeoff and landing of aircraft, as well as wind loadings on large civil structures. Earlier research into atmospheric turbulence has generally focused on the time-averaged characteristics of the velocity fluctuations. For example, as demonstrated in Kaimal [2], the power spectra of the velocity fluctuations and their cross-correlations follow normalized curves that are functions of the Richardson number. Similar relations involving the fluctuation spectra have been determined and are well documented in various texts such as Lumley and Panofsky [3] and Kaimal and Finnigan [4].

The drawback of quantifying events in this manner is that no information regarding short-term coherent structures within the wind field is obtained. For this reason, methods such as Conditional Analysis and Proper Orthogonal Decomposition (POD) have recently been used to

investigate coherent structures directly. In meteorological studies, POD has provided significant results; see Wilson [5] and Lin et al. [6]. However, meteorological studies focus primarily on heights beyond 0.2δ . Furthermore, the coherent structures that are studied are on the order of δ , which are not of interest in the current study. In contrast, studies on canonical turbulent boundary layers (TBL) have investigated turbulent coherent structures on orders much smaller than the TBL thickness. For example, the properties of hairpin and horseshoe vortices are well documented [7] and have been measured experimentally by Schroeder et al. [8] and Adrian [9]. It has been shown that at 5% of the TBL thickness, i.e. the lower log layer, canonical TBLs are populated by both streak vortices and hairpin vortices; see Robinson [7] and Smith and Walker [10].

Given the breadth of research performed on canonical TBLs, demonstrating that ABLs share similarities with canonical TBLs would be extremely valuable for future wind-tunnel modelling. Scarabino et al. [11] attempted to demonstrate the similarities between ABLs and canonical TBLs by showing that both share similar coherent structures. Using data collected by Sterling et al. [12], velocity fluctuations of extreme gust events were classified into three groups and were then ensemble averaged. One of the ensemble groups exhibited vortex-like flow, which led Scarabino et al. [11] to fit a hairpin vortex model onto the ensemble average. In this regard the study had limited success: Although the hairpin-vortex model accounted for sharp changes in streamwise velocity, it failed to capture the simultaneous vertical fluctuations properly.

It is believed that the shortcomings of the hairpin-vortex model in Scarabino et al. [11] were due to an oversimplification since it was assumed that a gust event could be represented by a simple hairpin vortex. In reality, there are a wide range of structures within TBLs, thus resulting in much more complicated flow fields. It is argued that these coherent structures must be studied first in terms of their strength and orientation prior to investigating how they contribute to gust events. Thus, the current study takes a different approach than that in Scarabino et al. [11]. Rather than identifying gust events within the data and attempting to fit a certain coherent structure to each, events that meet specific criteria suggestive of a hairpin vortex are identified instead. Velocity fields are then determined for each specific event and compared to experimental data. In this way the height and strength of hairpin vortices within the lower log region of the ABL can be determined, statistically analysed and compared to those within canonical TBLs.

Since the structure of hairpin vortices is complex, it is assumed that such vortices can be simplified as two-dimensional rollers instead. This assumption is based on the following observations: (1) It is noted in Robinson [7] that the lifespan of transverse vortices is much longer than the lifespan of streamwise vortices; and (2) through experiments performed by Head and Bandyopadhyay [13] at $0 \leq Re_\theta \leq 10000$, it was found that the legs of hairpin vortices are around 100 wall units (y^+) apart, which in the ABL translates to approximately 50m.

The current study compares properties of ABLs to those of canonical TBLs. First various statistical descriptors of turbulent fluctuations measured by a wind mast are compared to measurements taken over a flat plate. Subsequently a conditional analysis is used to identify hairpin-vortex events. The occurrence-frequency distributions of normalized circulation ($\Gamma/U_0\delta$) and normalized vortex-centre height (a/δ) are discussed, as well as the correlation between normalized circulation and normalized height. By assaying the reasonableness of the aforementioned results, the robustness of the model is then evaluated.

2 EXPERIMENTAL METHODS

For this investigation, a 2-hour long data set was acquired on January 5th, 2012 from a 50m wind mast erected on university land. The area is primarily rural. It is generally flat with a shallow depression of 5m depth upwind of the mast. A schematic of the wind mast, with the approximate positioning of its sensors, is provided in Figure 1a. A photograph depicting the mast and the surrounding terrain is shown in Figure 1b. The five cupped anemometers (CA) and wind vanes (WV), as well as the two-component ultrasonic anemometer, are all used to determine the mean wind speed and direction. Coherent structures are determined using the measurements taken by the two three-component ultrasonic anemometers (3CUS). The 3CUS sensors are located at 40m and 50m heights.

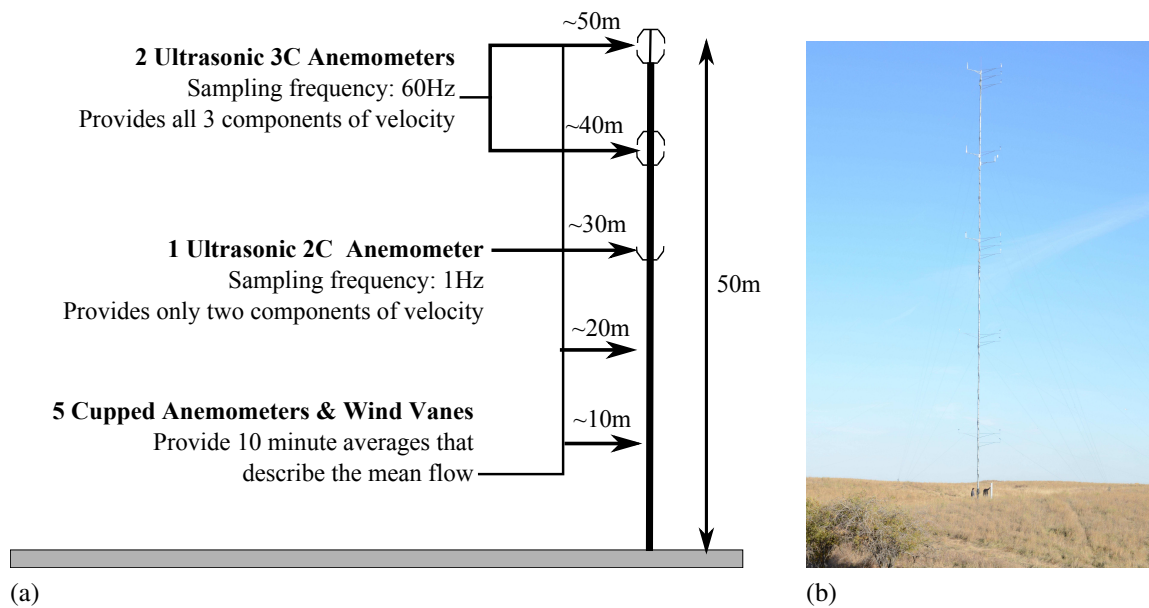


Figure 1: (a) A schematic of the wind mast with the approximate positioning of sensors indicated. (b) A photograph of the mast and surrounding terrain.

During the experiment the mean wind velocity profile varied considerably. The data was therefore split into 10-minute long sets during which the mean velocity remained fairly constant. The reference frames of the two 3CUSs were rotated so that the x-, y-, and z-axes were respectively oriented into the streamwise, vertical and lateral wind directions as determined by the WVs. This was performed independently for each 10-minute data set. Upon performing this rotation, the x-axis mean velocities measured by the 3CUS sensors were nearly equal to the velocity measured by the CA of corresponding height (mean discrepancy of $\pm 0.7\text{m/s}$). The other two components measured essentially zero mean velocities (mean discrepancy of $\pm 0.5\text{m/s}$). The mean values of all three components of the 3CUS sensors were subtracted from the respective component for each 10-minute data set. This modified data now represented the wind fluctuations in the longitudinal, vertical and lateral directions.

A condition suggestive of hairpin vortex impingement will now be presented. It is argued that a hairpin vortex be represented as an infinitely long potential vortex being convected towards the mast, as shown in Figure 2. The circulation of the vortex is Γ and the vortex centre sits at an arbitrary height $y = a$. Also the vortex is convected by the mean local velocity U_a

at the vortex-centre height. To account for ground effect, two counter-rotating vortices are positioned at heights $+a$ and $-a$. Figure 2 also shows the mast with the heights of the top and bottom ultrasonic sensors labelled as h_T and h_B , respectively. If Taylor's hypothesis is assumed to hold, then the vortex's shape should not change as it convects. This allows b to be set equal to $-U_a(t - t_R)$, where t_R is the instant the vortex centre has passed the mast.

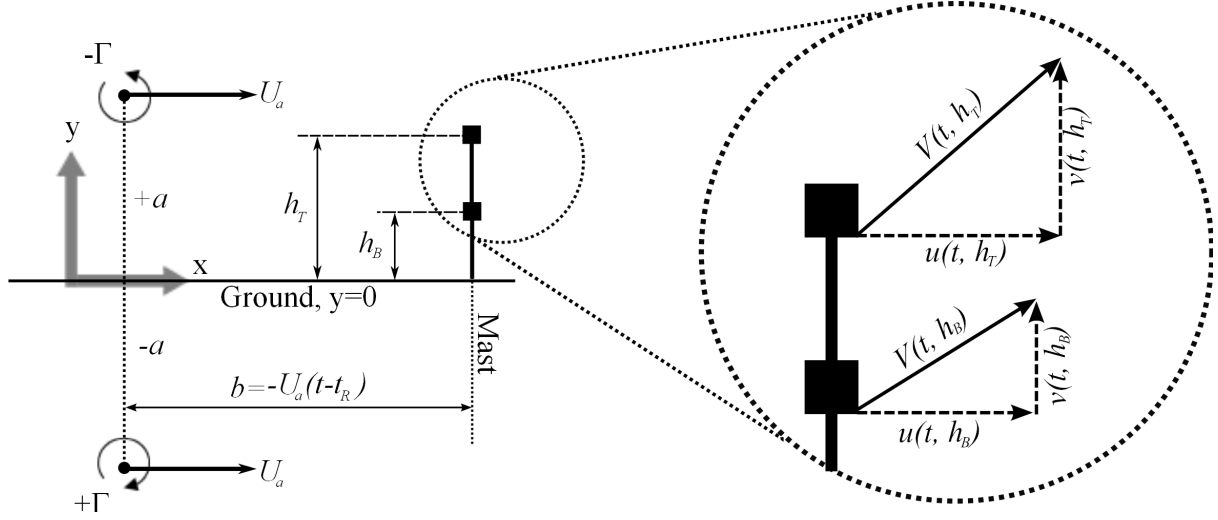


Figure 2: A potential line vortex of circulation Γ at an arbitrary height a being convected towards the mast at U_a (the mean wind velocity at height a). The vortex induces a fluctuation field V , comprised of a streamwise component u and vertical component v . Fluctuations from the mean velocity are measured by the two 3CUS sensors at heights h_T and h_B . A counter rotating vortex at $-a$ has been included to account for ground effect.

The fluctuation field generated by this vortex is given by u and v , which are the longitudinal and vertical fluctuations induced by the vortex, respectively. At the top and bottom 3CUS sensors, the vortex induces fluctuations of:

$$\vec{V}(t, h_T) = u(t, h_T)\hat{i} + v(t, h_T)\hat{j} \quad (1)$$

$$\vec{V}(t, h_B) = u(t, h_B)\hat{i} + v(t, h_B)\hat{j}. \quad (2)$$

If the current vortex is far from other vortices, then the fluctuation field is given by:

$$u(t, y) = +\frac{\Gamma(y - a)}{\pi((U_a(t - t_R))^2 + (y - a)^2)} - \frac{\Gamma(y + a)}{\pi((U_a(t - t_R))^2 + (y + a)^2)} \quad (3)$$

$$v(t, y) = +\frac{\Gamma U(t - t_R)}{\pi((U_a(t - t_R))^2 + (y - a)^2)} - \frac{\Gamma U(t - t_R)}{\pi((U_a(t - t_R))^2 + (y + a)^2)}. \quad (4)$$

The vortex-induced fluctuations at the sensors can be determined by replacing y with h_T and h_B in equations (3) and (4). If corresponding components are divided by one another, followed by setting t to t_R , equations (5) and (6) are determined:

$$\frac{u(0, h_T)}{u(0, h_B)} = \frac{a^2 - h_B^2}{a^2 - h_T^2} = F \quad (5)$$

$$\frac{v(0, h_T)}{v(0, h_B)} = \frac{h_T}{h_B} \left(\frac{a^2 - h_B^2}{a^2 - h_T^2} \right)^2 = \frac{h_T}{h_B} F^2, \quad (6)$$

where F is only dependent on the heights of the two 3CUS sensors and the vortex-centre height.

It is interesting to note that F is only negative when $h_B < a < h_T$. Also note that regardless of vortex-centre height, equation (6) is never negative, meaning that vertical fluctuations always occur in the same direction.

Equations (5) and (6) define a condition for identifying roller vortices. If a vortex centre passes the mast at time t_R , then the following must hold true at that instance as well:

$$\frac{v(t_R, h_T)}{v(t_R, h_B)} = \frac{h_T}{h_B} \left(\frac{u(t_R, h_T)}{u(t_R, h_B)} \right)^2. \quad (7)$$

By locating a time instance where (7) is satisfied, F is effectively known. From F , the values of a and Γ can be determined, thereby defining equations (3) and (4) completely. Finally equations (3) and (4) can be tested against the data time series in the immediate vicinity of t_R . If a roller in fact passes at t_R , then equations (8) and (9) must be true during the immediate time series:

$$\sigma_T = \overline{|u(t, h_T) - u(t, h_T)_D|} = 0 \quad \sigma_B = \overline{|u(t, h_B) - u(t, h_B)_D|} = 0 \quad (8)$$

$$\nu_T = \overline{|v(t, h_T) - v(t, h_T)_D|} = 0 \quad \nu_B = \overline{|v(t, h_B) - v(t, h_B)_D|} = 0, \quad (9)$$

where subscript D indicates fluctuations measured in the data set. Equations (8) and (9) will be evaluated over approximately $t_R \pm \frac{1}{2}T$, where T is the integral time scale of the fluctuations as determined from the data set. T is determined by integrating the autocorrelation functions of the turbulent fluctuations. It was determined that T for this data set is on the order of 1s. The details of how this was determined are not included in the current study. If equations (8) and (9) are nearly equal to 0, then the roller model predicts fluctuations of similar value to those in the data. Rollers must track fluctuation trends as well. For this reason, all roller events are manually checked to ensure that the immediate fluctuation trends are captured.

3 RESULTS & DISCUSSION

This section describes the ABL data set by evaluating important descriptors such as the stability, the thickness and the Reynolds number. Afterwards, the results of the conditional analysis are evaluated.

3.1 Description of the ABL Data Set

The stability of the ABL may be determined by analysing the mean velocity profile. In a neutral ABL, the mean velocity profile fits well onto the curve given by Kaimal and Finnigan [4]:

$$U(y) = \frac{u_*}{k} \ln \frac{y}{y_0}, \quad (10)$$

where y is the vertical distance from the ground, u_* is the friction velocity, k is the von Karman constant (approximately 0.4) and y_0 is the friction length. Both u_* and y_0 are determined from the curve fit. Figure 3a shows the mean velocity measurements taken by the cupped anemometers for the first 10-minute data set. The solid curve represents equation (10) fitted onto the mean velocity measurements. The friction velocities and roughness lengths for all 11 sets are provided in Figure 3b. The R^2 values are provided also, which are a measure of the quality of the fit. All the R^2 values for the 10-minute time sets exceed 0.97, indicating that equation (10) is a good descriptor of the mean velocity. Thus the ABL is presumably neutral. Also, the fact that the wind-velocity profiles are well described by a logarithmic fit indicates that the mast sits within the log region of the ABL.

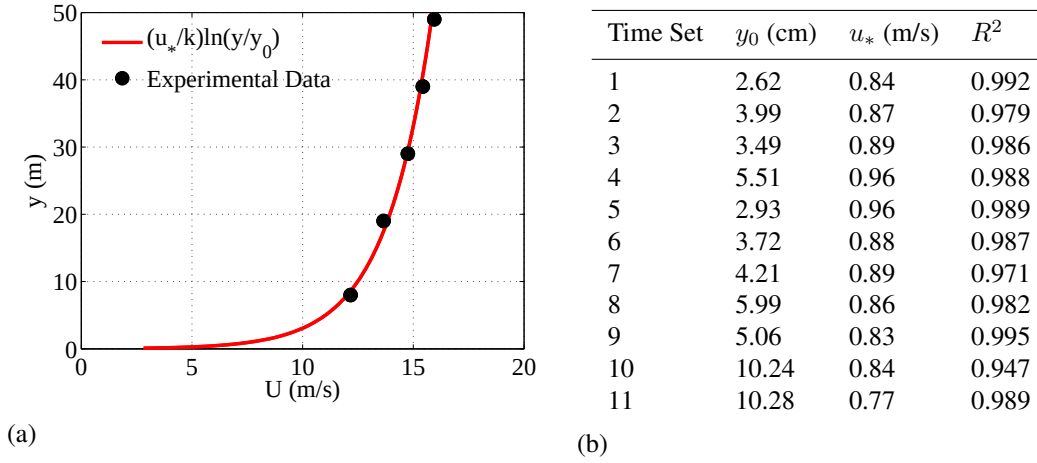


Figure 3: (a) The mean velocity profile for the first 10-minute set (14:50 to 15:00). (b) Roughness lengths, friction velocities and R^2 values for each 10-minute set as approximated by fitting the data to (10).

There are various methods to approximate the ABL thickness. For example, Blackadar and Tennekes [14] provides an approximation for δ of neutral ABLs based on friction velocity and Coriolis effects. The current study determines δ by comparing the ABL data to data collected from canonical TBLs over a flat plate. Cheng [15] states that the velocity profiles of TBL flow over a flat plate is classically represented as:

$$\frac{U}{U_0} = \left(\frac{y}{\delta}\right)^{\frac{1}{m}} \text{ where } m \approx \ln \frac{y_0}{0.2\delta}. \quad (11)$$

This provides an iterative method for determining the value of δ : Its value is first assumed; then the profile provided by equation (11) is compared to the experimentally measured profile normalized by U_0 . A value of 1000m for δ provides reasonable agreement between equation (11) and the experimental data, as shown in Figure 4. The error bars in Figure 4 represent a single standard deviation in the observed results.

Figure 4 also shows various statistical descriptors of the turbulent fluctuations as measured by the 3CUS sensors (observed flow), as well as those measured in a canonical TBL over a flat plate by Klebanoff [1]. The statistical descriptors of the fluctuations for the observed flow and TBL flow share the same order of magnitude. However, the statistical descriptors for the observed flow and TBL flow often differ by approximately one standard deviation. The discrepancy between the observed flow and TBL flow may be due to the difference in Reynolds number. The TBL-flow data was collected at a Reynolds number $Re_x = 4.2. \times 10^6$ while the Reynolds number for this ABL data is approximately $Re_x = 4.3. \times 10^{15}$.

3.2 Results of the Conditional Analysis

The conditional analysis was performed on all eleven 10-minute data sets. 774 events were identified in which the roller model accurately tracked the fluctuations within an acceptable degree of precision. It was often found, however, that precision had to be compromised for the sake of accuracy. Table 1 quantifies the precision error of the roller model by providing the mean values of σ_T , σ_B , ν_T and ν_B . On average, the 50m 3CUS sensor was less precise than its 40m counterpart. This is likely because the turbulent intensities at 50m are greater than those at 40m.

Figure 5 shows the occurrence-frequency distribution of vortex-centre height a and circu-

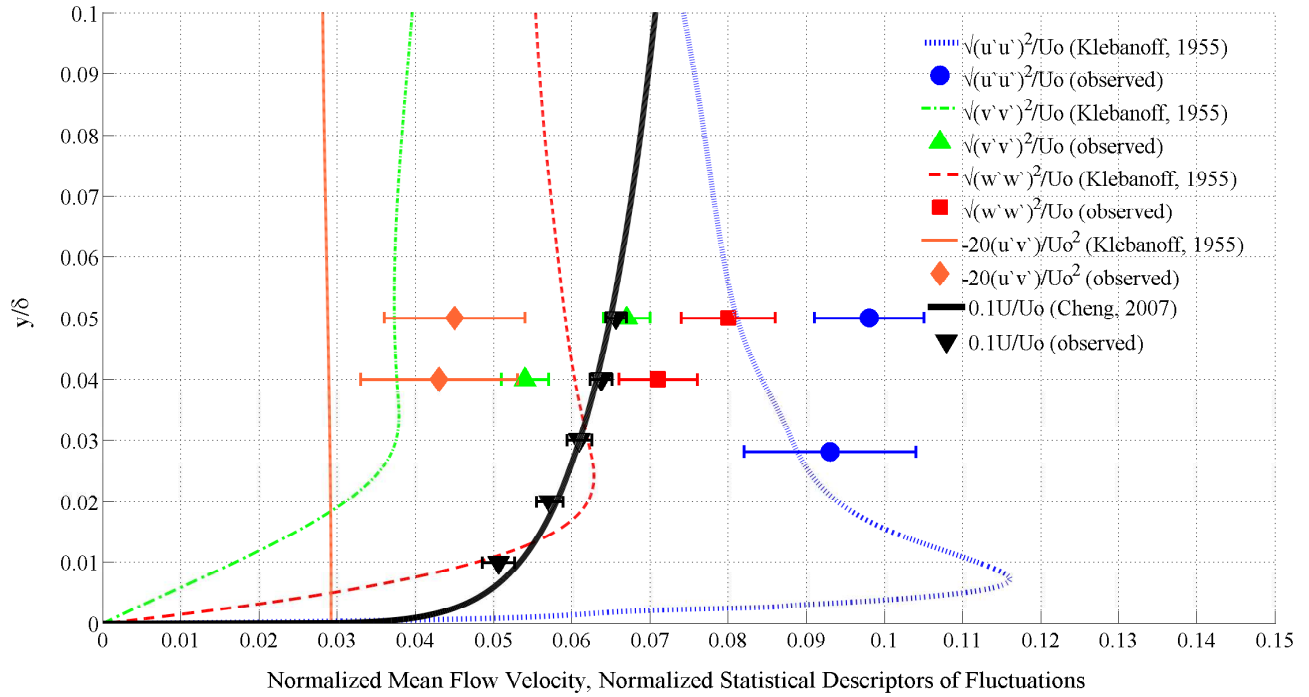


Figure 4: Theoretical and observed values for the velocity profile and its respective turbulent fluctuations.

σ_T (m/s)	σ_B (m/s)	ν_T (m/s)	ν_B (m/s)
0.772	0.561	1.093	0.610

Table 1: The mean precision error of the roller model in tracking the measured fluctuations. The precision error is evaluated for all eleven 10-minute sets. σ and ν refer to precision errors of the streamwise and vertical fluctuation components, respectively. Subscripts T and B refer to the top and bottom 3CUS sensors, respectively.

lation Γ normalized by δ and $U_0\delta$, respectively. The distribution of a resembles a Rayleigh probability distribution. The mean of a is 0.056δ , while the standard deviation of a is 0.027δ . In contrast, Γ appears normally distributed; its mean is $0.000U_0\delta$ while its standard deviation is $0.010U_0\delta$.

It is possible that the near-ground roller vortices have vortex-centre heights that follow the distribution shown in Figure 5a. Various studies performed on canonical TBLs have found similar distributions. From data of a numerical simulation of a canonical boundary layer ($Re_\theta = 670$) by Spalart [16], Robinson [17] found that spanwise vortices also follow a Rayleigh distribution with an average height of approximately 0.2δ . From experiments run at $Re_\theta = 1120$ Smith and Lu [18] also found that the vortex-centre heights follow a Rayleigh distribution with an average height of 0.06δ . Finally, from experiments Wu and Christensen [19] found that over a large Reynolds-number range ($4000 \leq Re_\theta \leq 11000$) the centre height of the spanwise vortices followed a Rayleigh distribution with a mean of approximately 0.06δ .

It seems unlikely that the circulation is normally distributed about 0. Wu and Christensen [19] found that the circulation of spanwise vortices followed a bimodal distribution about 0 ($Re_\theta = 8830$), while in Robinson [17] the circulation distribution was slightly skewed towards prograde vortices. Furthermore results from Wu and Christensen [19] demonstrate that pro-

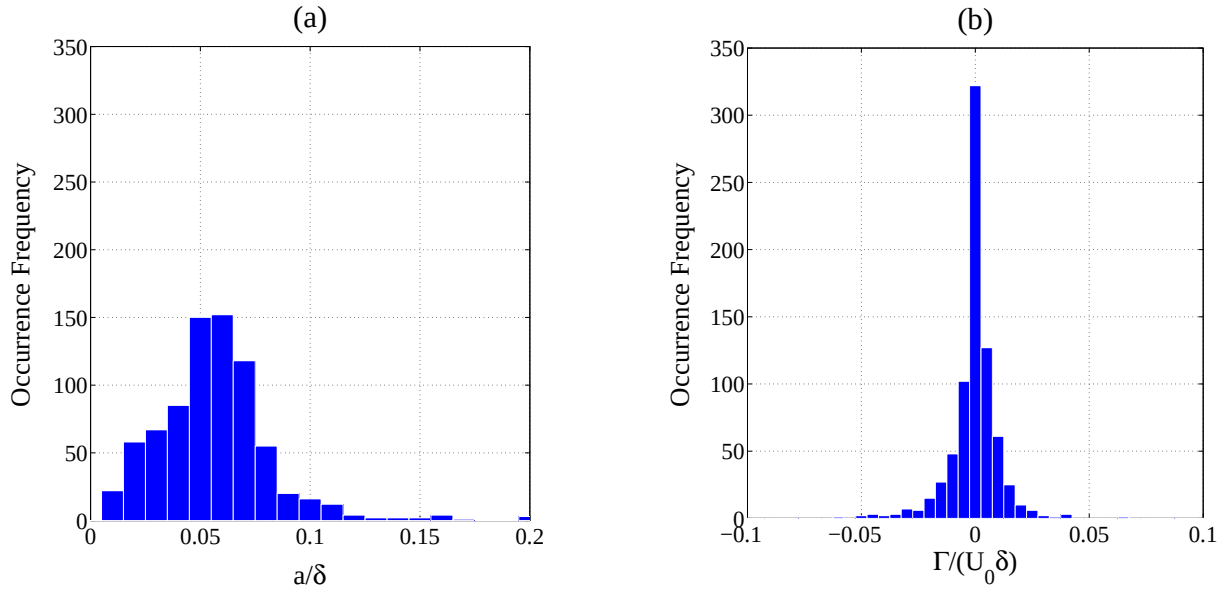


Figure 5: Occurrence-frequency distributions of (a) vortex-centre height a and; (b) circulation Γ for all eleven 10-minute data sets. Γ is normalized by $U_o\delta$, while a is normalized by δ . δ is assumed to equal 1000m, while U_o is determined by evaluating equation (10) at 1000m.

grade vortices are more common near the shearing surface, while retrograde vortices are more common higher up in the boundary layer. Note that a prograde vortex causes a negative stream-wise fluctuation below its centre while a retrograde vortex causes a positive fluctuation below its centre. Thus it would be expected for the circulation distribution in Figure 5b to be shifted to the right.

The zero mean in circulation is likely caused by shortcomings in the conditional model. One significant limitation in the model is the absence of a viscous core for the roller vortices. This may have caused the model to underestimate the circulation magnitude for the roller vortices concentrated at 0.05δ , the height at which the two 3CUS sensors are located. If a vortex-centre is at a similar height as the 3CUS sensors, then the sensors would possibly be within the viscous core. Being inside the viscous core would cause the conditional model to substantially underestimate the circulation. This is because the model does not account for the decrease in velocity inside the viscous core. Figure 6 shows normalized circulation plotted against normalized vortex-centre height, which in fact demonstrates that circulation tends to 0 as the vortex-centre height approaches 0.05δ . This supports the argument that neglecting the viscous core may have skewed the results. However, the circulation distribution determined by the current study may in fact be correct. Results from Wu and Christensen [19] demonstrate that as the Reynolds number is increased the ratio of prograde to retrograde vortices approaches unity.

4 CONCLUSIONS & RECOMMENDATIONS

The current study characterised the fluctuations and coherent structures within the atmospheric log layer. Hairpin-vortex structures were identified using a conditional analysis that modelled hairpin-vortices as infinitely long roller vortices convected towards the mast. For simplicity the roller vortices were assumed to be irrotational. It was found that the ABL data set qual-

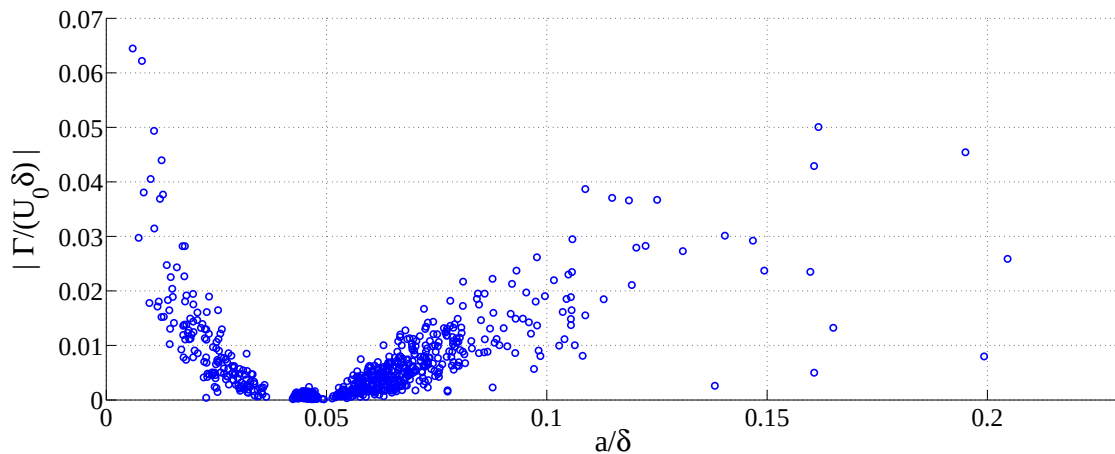


Figure 6: The magnitude of circulation Γ plotted against vortex-centre height a for all eleven 10-minute data sets. Γ and a are normalized as they were in Figure 5.

itatively agreed with statistical descriptors of TBL flow. The conditional analysis found that the occurrence frequency of vortex height followed a Rayleigh distribution, with a mean and standard distribution of 0.056δ and 0.027δ , respectively. The circulation followed a normal distribution with a mean and standard deviation of $0.000U_0\delta$ and $0.010U_0\delta$, respectively, with the bulk of the rollers measured around 0.05δ having circulations near zero. Despite the mean circulation of zero, the height distribution determined by the conditional analysis agreed with previous studies. The conditional model should be updated to account for the viscous core so as to accurately evaluate the vortex rollers when in proximity to the sensors.

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