

Design wind loads of wire mesh claddings and their fatigue behavior considering structural non-linearities

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ABSTRACT: Cladding elements made of prestressed, permeable wire meshes are for different reasons challenging with respect to their aerodynamic load-response behavior. On the one hand, the aerodynamic admittance is dominated by the flow resistance of the mesh and its individual location on the buildings surface. On the other hand, the wind excited stress range spectra may lead to structural failures caused by fatigue, therefore a proper determination of the expected gust induced vibrations and a subsequent stress analysis of the geometric non-linear structure is necessary. By means of wind tunnel tests and a sophisticated analytical model of the non-linear dynamic structure, a realistic design concept has been derived to allow a closed form structural dimensioning.

KEYWORDS: Permeable Structures, Gust Response, Geometric non-linearity, Fatigue Design

1 INTRODUCTION

Permeable cladding structures are part of modern architectural concepts. In most cases only simplest static load assumptions are considered for their design, i.e. a consideration as a free standing element. However, the complexity of the actual installation position and the thereby caused interaction with a closed inner façade lead to a significantly modified aerodynamic impact. Especially for permeable elements with high porosity, general rules are rare.

Wind affected structures generally have to withstand fluctuating load cycles. For an appraisalment whether the resulting stress range spectra may lead to structural failures during the designed lifetime, standard methods are still rare because the influencing parameter are manifold. A realistic determination of structural stress range spectra provoked by the natural wind, needs profound knowledge about the statistical properties of the wind flow at the considered site, as well as about the dynamic structural behavior. Eventually, the interpretation of the resulting structural responses in terms of stress amplitudes $\Delta\sigma$ and associated frequencies of occurrence $N(\Delta\sigma)_{Life}$ over the expected lifetime are the main task to allow fatigue verifications. The complexity of the topic increases significantly, when the usual linearized considerations are not sufficient anymore. In the present paper the fatigue behavior of geometrically non-linear elements under gusty wind loads is considered.

2 WIRE MESH CLADDINGS

Cladding structures made of wire mesh are part of modern architectural concepts and they are used both as an amendment of conventional closed façades and as sole building closure (e.g. for car parks). In Figure 1 an innovative cladding concept is shown, thereby the permeable wire mesh webs are installed in front of the glazed façade.

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Figure 1. Exemplary Wire Mesh Cladding Structure at the Los Angeles Police Department, USA

2.1 Static Wind Force Description

2.1.1 Simplified Load Considerations

For the calculation of the static wind forces, the aerodynamic characteristics of the used meshes and the influence of the installation position at the building have to be known. In case of multi-layered cladding systems, where the outer skin is made of a permeable mesh structure and the inner one is impermeable, the distance between both is an important additional parameter for the load determination.

Approximative values for the design of mesh elements, a simple accumulation of the flow resistances leads to conservative results. Taking the example of the multi-layered cladding, the flow resistance consists of the mesh resistance and the resistance of the inner barrier. The first effect can be approximated by the Bernoulli Equation under consideration of the pressure loss coefficient of the mesh (1). Hence, the acting local wind speed u_i can be determined to:

$$\frac{\rho}{2} \cdot u_{\infty}^2 = (1 + \zeta_{Mesh}) \cdot \frac{\rho}{2} \cdot u_i^2 \rightarrow u_i = u_{\infty} \cdot \sqrt{\frac{1}{1 + \zeta_{Mesh}}} \quad (1)$$

The computation of local loads can be performed based on the mesh force coefficient and the local flow velocity v_i . Due to the second resistance effect (closed inner wall), generally an additional reduction of the local flow speed can be expected.

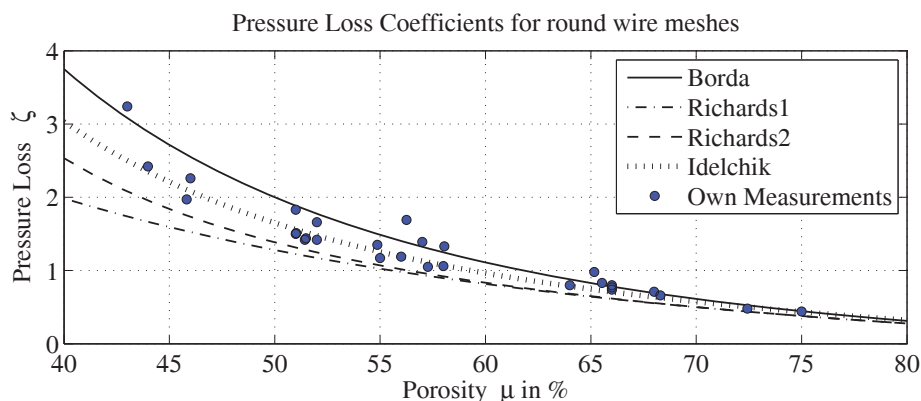


Figure 2. Pressure Loss Coefficients ζ of round Wire Meshes, Comparison of own Measurements and Data from other Authors

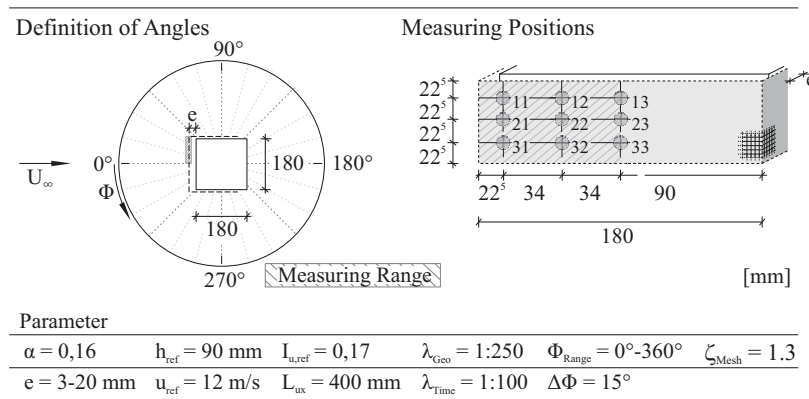


Figure 3. Illustration of the Wind Tunnel Model and Definition of Measurement Parameters, Angles and Measuring Positions

2.1.2 BLWT Investigations

More detailed information about the directional wind load components (due to the mesh resistance as well as due to the inner blockage) can be achieved based on experiments in the boundary layer wind tunnel. With this aim, a model of a building with multi-layered cladding system and variable distance between the cladding layers has been build in a scale of M1:250 (s. Fig. 4). For the measurements, a circular mesh cut-out has been connected to a high precision force balance ($F_{max} = 0.5\text{N}$). The position of the force balance was variable in 9 positions.

In order to allow a statistic analysis of the extreme wind induced forces, for each wind direction 30 independent runs have been performed. For a unique formulation of the location dependent wind load effects but independent of the mesh resistance, a flow parameter α_{local} is introduced, which represents the flow resistance caused by the impermeable inner wall. Generally, the wind load effect is formulated as:

$$\alpha_{local} \cdot c_{f,Mesh} = \frac{1 + \zeta_{Mesh}}{q} \cdot \frac{F_{local}}{A} = \frac{F_{local}}{q_{local} \cdot A} \quad (2)$$

Under consideration of an Extreme Value distribution Type I, the peak values for the locational flow parameter α_{local} can be computed based on the well-known *Cook-Mayne* [3] method, whereat the force coefficient c_f refers to the mean wind load admittance of the mesh. Altogether,

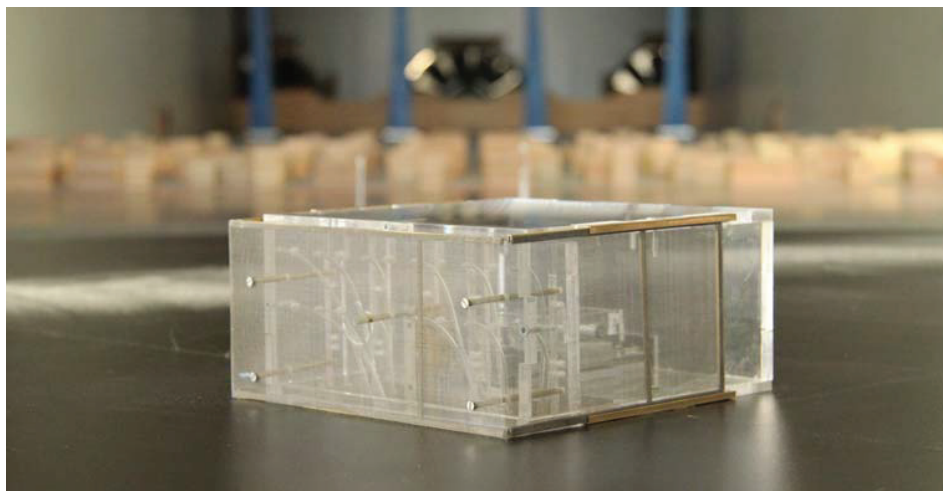


Figure 4. Windtunnel Model of a closed Building with permeable outer Cladding in a scale of 1:250

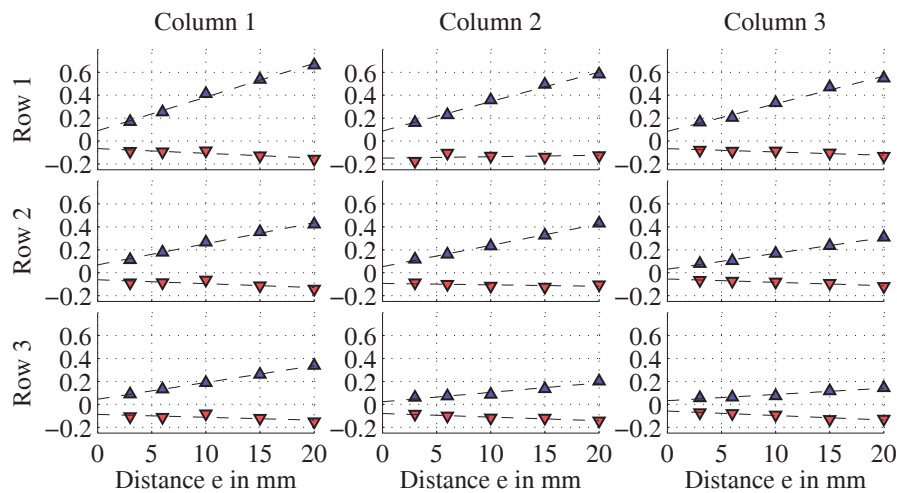


Figure 5. Characteristic Values $\hat{\alpha}$ and $\check{\alpha}$ for a mesh with a Pressure Loss Coefficient $\zeta_{Mesh} = 1.3$ for different Cladding Positions, dependent on the Distance e between Mesh and inner Cladding

the following formulas have been used for the analysis:

$$\hat{\alpha}_{local} = (1 + \zeta_{Mesh}) \cdot \frac{mean(\hat{F}_i) + 0.636 \cdot std(\hat{F}_i)}{q_p(z = H_{OK}) \cdot c_f \cdot A} \quad (3)$$

$$\check{\alpha}_{local} = (1 + \zeta_{Mesh}) \cdot \frac{mean(\check{F}_i) - 0.636 \cdot std(\check{F}_i)}{q_p(z = H_{OK}) \cdot c_f \cdot A} \quad (4)$$

In Figure 6 the results for $\hat{\alpha}$ are plotted, dependent on the wind direction and the distance between the cladding layers in model scale. Obviously, the perpendicular and nearly perpendicular flow towards the mesh ($\Theta \approx 0^\circ - 20^\circ$) is most unfavorable for the positive force in wind direction. With increasing distance between mesh and closed inner façade, as well as with increasing

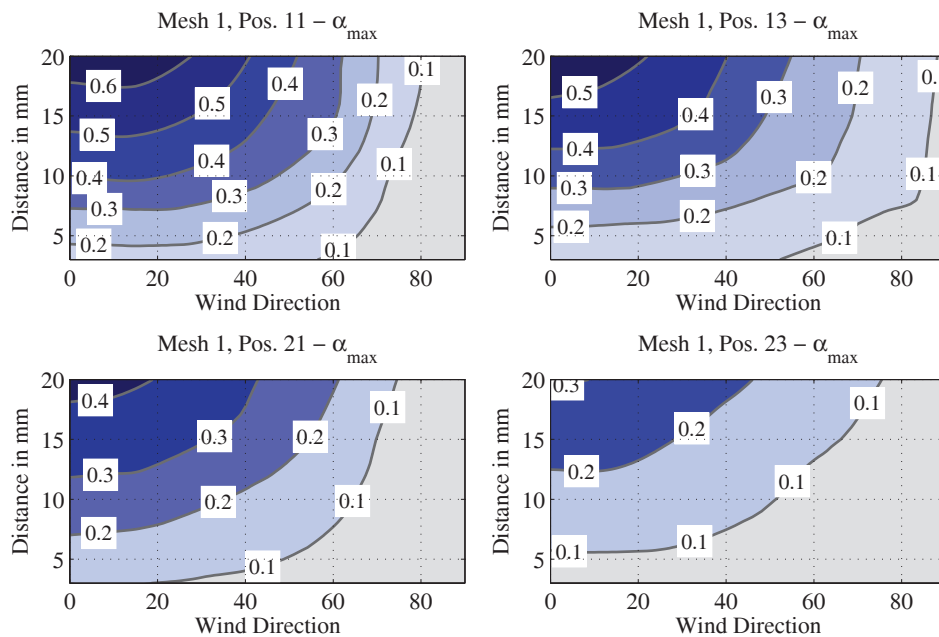


Figure 6. Characteristic Flow Parameter $\hat{\alpha}$ for the building with double-layered Cladding System, dependent on the Layer Distance e and the Direction of Flow Θ , Pressure Loss Coefficient $\zeta_{Mesh} = 1.3$

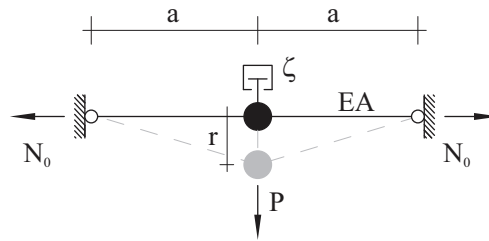


Figure 7. Duffing Oscillator as a 1 DOF Modell for the Dynamic Behaviour of prestressed Cladding Webs

wind direction, the wind positive wind force decreases. Appreciable negative wind forces only occur for wind angles above $\Theta \approx 45^\circ$, but the amplitude is commonly not decisive.

3 GEOMETRIC NONLINEAR STRUCTURAL BEHAVIOUR

The dynamic behavior of prestressed line-like elements can be described by a nonlinear single degree of freedom oscillator. As the resisting force is a polynomial with first and third order terms in r , this kind of oscillator is denoted as *Duffing-Oscillator* (s. Fig. 7). According to the geometric nonlinear kinematic, the deflection r of a Duffing-Oscillator leads to an elastic (additional) strain:

$$\varepsilon = \frac{(a^2 + r^2)^{1/2} - a}{a} = \left[1 + \left(\frac{r}{a} \right)^2 \right]^{1/2} - 1 \quad (5)$$

Due to the increment of strain, the inner force N grows, compared to the prestressing force N_0 :

$$N = N_0 + \varepsilon EA = N_0 + EA \left\{ \left[1 + \left(\frac{r}{a} \right)^2 \right]^{1/2} - 1 \right\} \quad (6)$$

The associated restoring force F_R of the Duffing-Oscillator can be determined by [1]:

$$\begin{aligned} F_R(r) &= 2 \cdot N \sin \phi = 2 \cdot N \frac{r}{(a^2 + r^2)^{1/2}} \\ &= 2 \cdot N_0 \cdot \frac{r}{a} \left[1 + \left(\frac{r}{a} \right)^2 \right]^{-1/2} + 2EA \cdot \frac{r}{a} \left\{ 1 - \left[1 + \left(\frac{r}{a} \right)^2 \right]^{-1/2} \right\} \end{aligned} \quad (7)$$

Obviously, the response behavior from *Duffing-Oscillators* is dominated by the initial force N_0 , the longitudinal elasticity EA and the span-width $2 \cdot a$. The decisive parameter which combines the mentioned values is the degree of nonlinearity δ . It can be expressed as [5]:

$$\delta \approx \frac{EA}{2a^2 N_0} \quad (8)$$

For fatigue verifications, not only the extreme values are relevant, but also the distribution of amplitudes has to be considered. For this aim, it is suggested to make use of advanced stochastic methods. Classical stochastic linearization approaches utilize an optimization of the expected value between the nonlinear and the linear stiffness to minimize the divergence of an equivalent linear system to the original system, in a least-square sense [8]. As a results a single equivalent structural frequency ω_{eq} can be found. This approach generally leads to a correct representation of the spectral energy and the increased structural frequencies, but underestimates the spectral

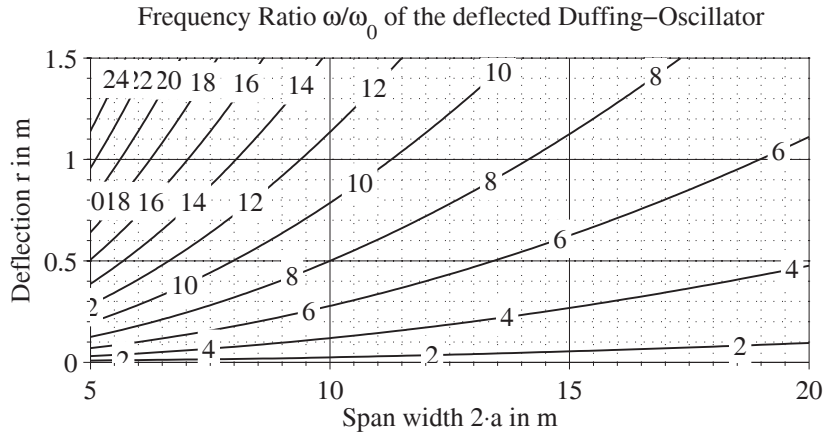


Figure 8. Increase of structural frequency of a Duffing-Oscillator dependent on span-width $2 \cdot a$ and applied deflection r

bandwidth (especially for slightly damped systems) [2]. Therefore *Miles* [7] and *Spanos* [9] used a concept, based on a linear differential equation of motion with a random expression for the amplitude dependent structural frequency (Equivalent Linear System, ELS). Transferring these concepts and extending them to stochastic wind loads with non-zero means leads to good approximations of the spectral system response, as well. Due to the analytic approach, they are advantageous compared to transient methods in case of structures which can be reduced to a single degree of freedom system.

In this paper, the ELS method is presented. The amplitude related structural frequency $\omega(A)$ can be obtained from:

$$\ddot{r} + 2\zeta\omega_0\dot{r} + \omega^2(A) \cdot r = f(t) \quad (9)$$

Hence, the differential equation of motion (9) represents a system of linear single degree of freedom oscillators. The Eigenfrequency ω is dependent on the deflection amplitude A and the expected frequency of its occurrence $P_n(A)$. According to [7], [9], [6] probability distribution can be expressed for a clear white noise excitation as:

$$P(A) = \frac{2\pi\omega_0^2(A + \delta \cdot A)}{\omega(A)} \cdot e^{-\frac{A^2(\delta \cdot A^2 + 2)}{4\sigma_0^2}} \quad \text{and} \quad P_n(A) = \frac{P(A)}{\int_0^\infty P(a)da} \quad (10)$$

A dynamically excited Duffing-Oscillator changes its frequency continuously, the amplitude related detuning can be expressed analytically [1]:

$$\frac{\omega(A)}{\omega_0} = \sqrt{1 + \frac{3}{4}\delta A^2} \quad (11)$$

In Figure 8 exemplarily the increase factors of the structural frequency $\omega(A)/\omega$ are plotted over the system span width and the applied deflection.

The approximate PSD of the structural deflection response $S_{aa}(\omega)$ can now be computed according to:

$$S_{aa}(\omega) = \int_0^\infty \frac{S_{ff}(\omega)}{(\omega^2(A) - \omega^2)^2 + (2\zeta\omega_0\omega)^2} \cdot P_n(A) dA \quad (12)$$

Although equation 10 is only exact for a white noise excitation with zero expectation, it has been used as an approximation here, whereat the variance σ_0^2 of the process has been derived by a

linear spectral computation with the actual force spectrum (based on a *Kaimal* PSD of the wind velocity).

Wind gust effects generally come along with a significant mean load component. The structural vibrations then take place around the resulting average deflection. To incorporate those effects alternatively in the before described computation algorithm, firstly the static displacement r_{stat} has to be computed based on the associated load situation:

$$F_{stat} = 2 \cdot N_0 \cdot \frac{r_{stat}}{a} + (EA - N_0) \cdot \left(\frac{r_{stat}}{a} \right)^3 \quad (13)$$

Due to the static deflection, the structural stiffness increases. Subsequently, all dynamic computations acc. to Equations (11) to (12), may be used with the associated structural characteristics of the modified system (denoted by the index “1”), i.e. the Eigenfrequency ω_1 and the variance σ_1^2 . Consistently, in Equation 12 the varying structural frequency is to be computed for $A_1 = A + r_{stat}$.

4 DETERMINATION OF NONLINEAR RESPONSES WITH STOCHASTIC METHODS

The geometric non-linear behavior can be considered analytically based on the findings of section 3. Typical prestressed cladding constructions show degrees of non-linearity between $\delta = 50$ and $\delta = 100$. In this section, the presented ELS method has been performed for structures with different degrees of non-linearity under varying load cases. As a load scenario, a 10 min gust impact has been considered, whereat for the aerodynamic admittance, different local flow coefficients have been taken into account. All associated properties are listed in Table 1.

As a first result, the power spectral densities of the structural deflections are plotted in Figure 10, as a comparison between the ELS method to conventional time based methods. The spectral results reveal the accuracy of the analytic method with respect to the predicted response behavior. In section 5, the consequence of the method, related to the fatigue prediction is investigated.

5 FATIGUE BEHAVIOUR AND STRUCTURAL INFLUENCE ON THE LIFETIME

Making use of a PSD based cycle count algorithm which is able to interpreted processes with arbitrary bandwidth, namely the *Dirlik*-algorithm [4], the rain-flow cycle count succeeds with frequency based responses. Even in case of nonlinear structures, the Rainflow classification can be performed based on the (with the prior described methods) computed power spectral density $S_{uu}(\omega)$.

The applicability of the presented methodology is illustrated by exemplary computations of flexible cladding elements (membrane structures) with different, realistic mechanical parameters. By means of these results, it is demonstrated how the different parameters influence the expectable damage and the fatigue life respectively.

So far, only the impact of a short-termed extreme wind on the stress range distribution has been considered. For the computation of the expected design life-time under fatigue aspects, the prevalent wind events during the foreseen lifetime have to be taken into account as well.

Table 1. Overview of Aerodynamic and Mechanic Properties of the Exemplary Computations

System	α_{local}	δ	System	α_{local}	δ	System	α_{local}	δ
S01-LC1	0.05	0.4	S02-LC1	0.05	4.0	S03-LC1	0.05	40
S01-LC2	0.5	0.4	S02-LC2	0.5	4.0	S03-LC2	0.5	40
S01-LC3	2.0	0.4	S02-LC3	2.0	4.0	S03-LC3	2.0	40

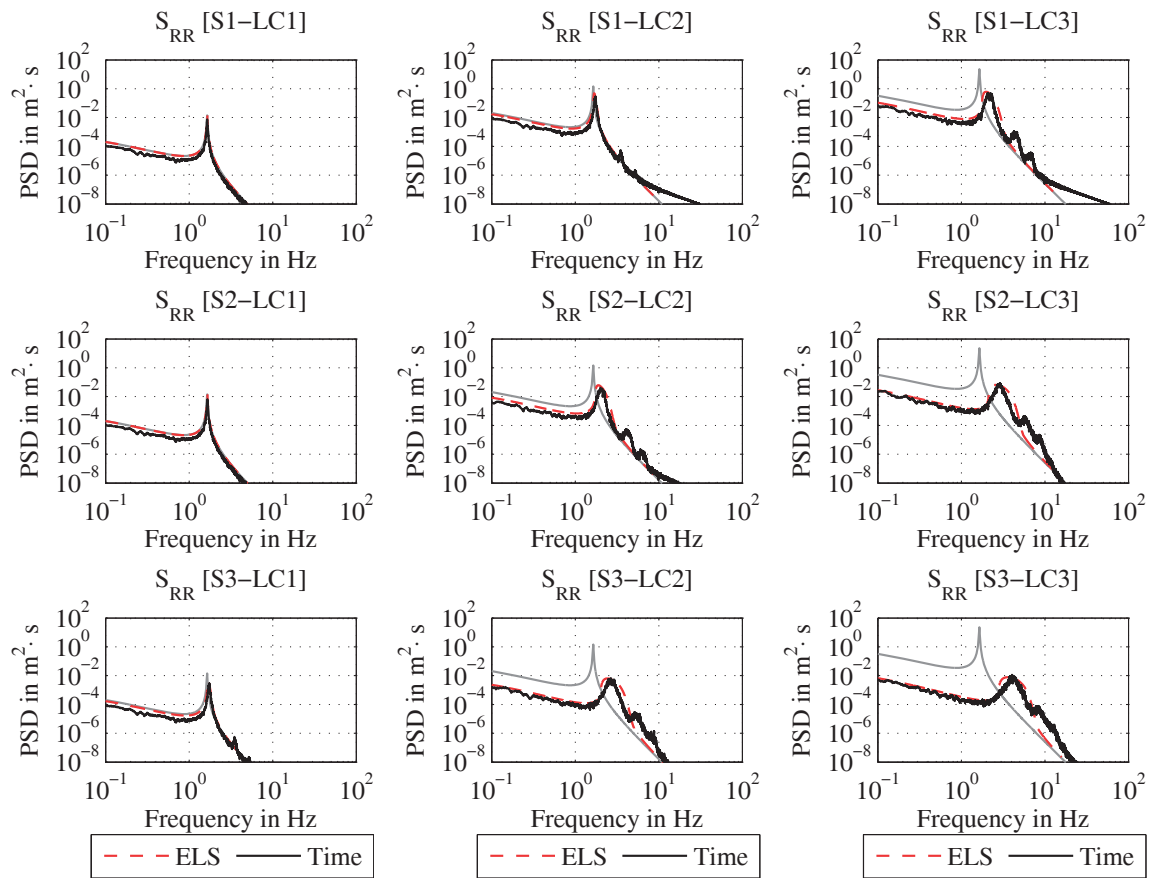


Figure 9. PSD of deflective Response for different Structures with ELS and transient Computation

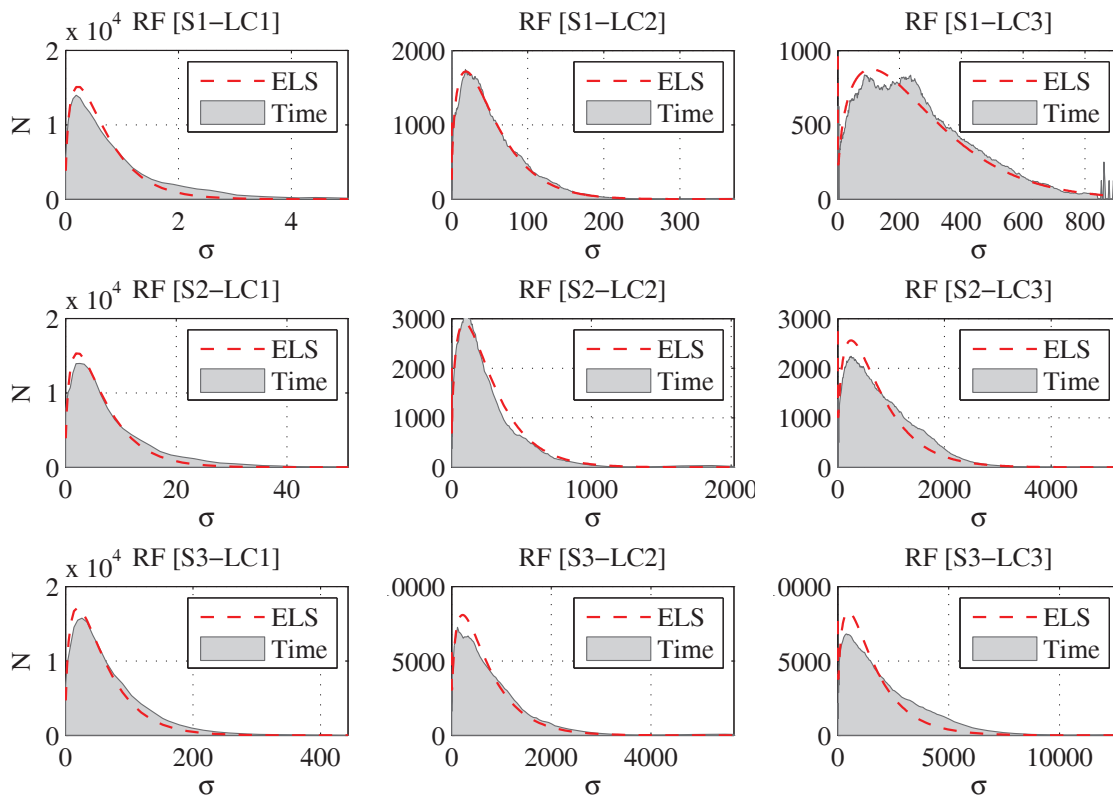


Figure 10. Stress Range Spectra (Rainflow Method) of ELS and transient computed Responses

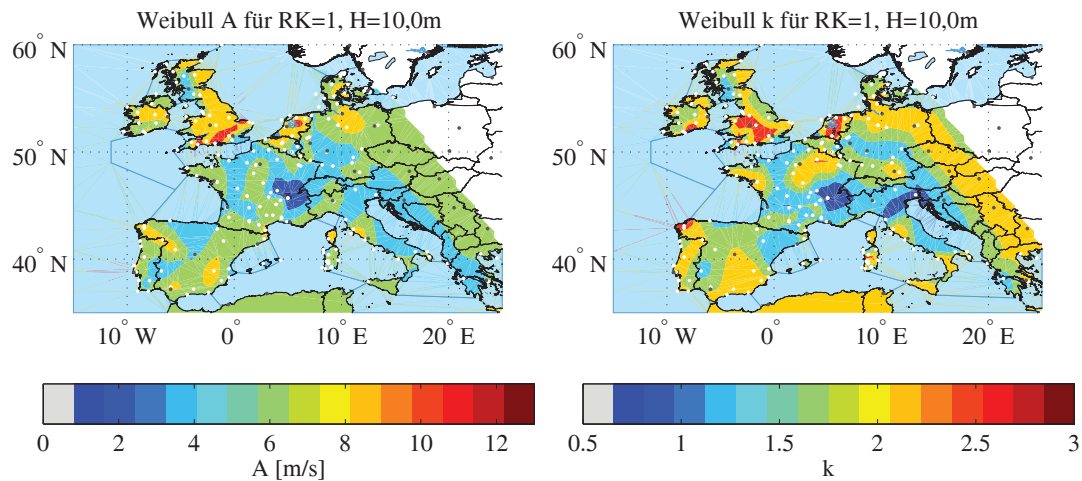


Figure 11. Weibull Parameter for Europe, Roughness Category 1 (Data Source: European Wind Atlas [10])

Merging all before described influence parameters, the stress range spectra can be computed by the following expression:

$$N(\Delta\sigma)_{Life} = \int_0^{\bar{u}} \frac{T_{Life}}{T_{ref}} \cdot f(u) \cdot N(\Delta\sigma, u) du \quad (14)$$

where: T_{Life} is the designed Lifetime, T_{ref} is the average time of the mean wind speed, $f(u)$ is the probability density function of the wind velocity and $N(\Delta\sigma, u)$ is the stress range spectra for a given mean wind speed within the average time period.

The probability density function for the frequency of occurrence of mean wind speeds can be derived based on the *Weibull*-Distribution:

$$f(\bar{u}) = \frac{k}{A} \left(\frac{\bar{u}}{A} \right)^{k-1} \exp \left[- \left(\frac{\bar{u}}{A} \right)^k \right] \quad (15)$$

where: A is the scale and k is the form parameter of the distribution. Site dependent values for these parameters can be taken from e.g. the European Wind Atlas [10]. In Fig. 11, exemplarily the parameters A and k (cumulated over all directions) are plotted in the map of Europe for the roughness category 1 ($z_0 = 0.05m$), for a height of 10m and for an average period of $T = 1$ hour.

Based on the appropriate assumptions for the site dependent extreme wind impact and the probability of exceedance of the prevalent wind speeds, a realistic consideration of fatigue impact is possible. Using the direction related Weibull Parameter, even a directional design is possible.

The fatigue verification now succeeds with common methods, like *Miner*-Rule for a linear damage cumulation with an appropriate s-N-Curve. In most cases the s-N-Curve should be obtained from dynamic fatigue tests, as general notch cases are not classified for crimped wires.

6 CONCLUSIONS

Static wind load assumptions have been derived for permeable cladding structures, installed as an outer skin in front on conventional, closed façades. Based on simplified models of the total pressure drop and the extended Bernoulli equation, conservative loads can be determined. Additional wind tunnel results show a good concordance of the local flow parameter α_{local} . With respect to the structural investigations, it has been shown, that reliable fatigue verifications based on cumulative damage predictions are possible by means of closed form spectral methods,

even for highly nonlinear structures. The chosen method, the equivalent linearization approach, leads to appropriate estimations of the increased structural frequencies of geometrical nonlinear systems under loading, as well as of the obtained damage prognoses. The presented examples emphasize the complexity of the topic and the effects of the various structural and aerodynamic parameters with respect to the expected fatigue life.

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