

Two degrees of freedom flow-induced vibrations on a cylinder

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ABSTRACT: A numerical study is performed on the flow-induced vibrations of isolated cylinder that is elastically mounted in two degrees of freedom with varying in-line to transverse natural frequency ratios, f_{nx}/f_{ny} . A characteristic-based-split finite element method is utilized for obtaining the solution of the incompressible flow equations in primitive variables. The Reynolds number based on upstream flow velocity and cylinder diameter, D , is fixed at $Re=150$. The computation is carried out for lower reduced mass of $M_r=2.0$ and for reduced velocities in the range of $U_r=3.0-12.0$, with increment of one; the structural damping ratio is set to zero to maximize the vortex-induced response of the bodies. We mainly focused on the effect of natural frequency ratios on the characteristics of vortex-induced vibration (VIV) responses, including wake frequencies, orbital trajectories, response amplitudes, aerodynamic forces and wake mode patterns.

KEYWORDS: isolated cylinder; two degrees of freedom (2-dof); vortex-induced vibration(VIV); natural frequency ratio; Arbitrary Lagrangian-Eulerian (ALE) formulation; finite element method.

1 INTRODUCTION

Vortex-induced-vibration (VIV) of cylindrical structures is of practical interest to many engineering fields (such as long span cable-stayed building structures, suspension bridges, heat exchangers *etc.*), and has led to extensive fundamental studies in the past decades.

For flexibly mounted circular cylinders immersed in uniform cross-flows, large amplitude transverse motions are excited, when the vortex shedding frequency of the stationary cylinder is sufficiently close to the natural frequency of cylinder vibration. This well-known condition is called ‘lock-in’ or ‘frequency synchronization’, at which the vortex shedding frequency shifts on the natural frequency of vibration. Dynamic interaction between vortex shedding and cylinder body can also lead to significant oscillation in in-line direction at a double-frequencies of the vortex shedding, due to oscillating drag component exerted on the cylinder. In-line amplitude is typically one-order smaller compared to the values of transverse counterpart, and hence, the in-line response has surprisingly little effect on the transverse response in a regime where the mass ratios are greater than 6. However, Jauvtis and Williamson [1] showed that, as the mass ratios are reduced to below 6, stream-wise response makes dramatic changes in the fluid-structure interaction, yielding massive vibration with three times diameters of peak-to-peak amplitudes. The response branch characterized by this massive oscillation was called by Jauvtis and Williamson [1] as a ‘super-upper’ branch, and the associated periodic vortex mode, in which a triplet of vortices are formed in each half cycle, was defined as a ‘2T’ mode. More recently, a number of research studies have provided important insights into the 2-dof responses [2, 3].

The previous studies of 2-dof VIV response of cylinder with varying natural frequency ratios have been limited to relatively larger Reynolds numbers [4-6]. In the laminar flow regime, the combined in-line and transverse VIV may show quite different response characteristics; this

expectation is derived from the numerical work by Lucor and Triantafyllou [7], since they showed that the two distinct response peaks, which originally appeared in the experiment as the frequency ratio close to 2.0, do not appear in the 2-D numerical simulations, although they explained that the coarse resolution in the frequency ratios does not allow to verify the presence of this double peak. It is, therefore, more desirable to explore the response characteristics of VIV for varying natural frequency ratios in the laminar flow regime.

In this work, we present computational results for VIV response of single cylinder, which is free to vibrate in 2-dof with varying in-line to transverse natural frequencies, corresponding ratio values varies between 1.0 and 2.0. The Reynolds number, based on the diameter of cylinder, is fixed to be 150; therefore, laminar 2-D flow assumption is expected to be valid in this investigation. The main objective of the present work is to systematically study the effects of natural frequencies on 2-dof VIV responses of single cylinder at laminar flow regime. The VIV response is characterized in terms of vortex shedding frequencies, motion trajectories, amplitudes and aerodynamic forces exerting on cylinder. Vortex shedding modes are also scrutinized to reveal the effects of natural frequencies on the unsteady wake patterns behind the cylinder.

2 GOVERNING EQUATIONS AND NUMERICAL METHODS

2.1 Governing equations

The fluid flow is governed by the two-dimensional, incompressible, Navier-Stokes equations, which is expressed in terms of the primary variables in Cartesian coordinate system as follows:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

where u_i (or u_j) is the velocity component in the x_i (or x_j) coordinate direction and p is the pressure divided by the fluid density, ρ . The dimensionless parameter of Reynolds number, Re , is based on the cylinder diameter, D , the free-stream velocity, U_∞ , and the kinematic viscosity of the fluid, ν ; i.e., $Re = U_\infty D / \nu$.

As the flexible body is under the action of flow-induced forces, it may exhibit oscillatory motions in both in-line and cross-flow directions. The motion of the body can be modeled via a mass-spring system, and is governed by the motion equations, respectively, in in-line and transverse directions as follows:

$$\ddot{X} + 4\pi\xi_x f_{nx} \dot{X} + 4\pi^2 f_{nx}^2 X = \frac{C_D}{2M_{rx}} \quad (3)$$

$$\ddot{Y} + 4\pi\xi_y f_{ny} \dot{Y} + 4\pi^2 f_{ny}^2 Y = \frac{C_L}{2M_{ry}} \quad (4)$$

Here, $\ddot{X}$, $\dot{X}$ and X denote the in-line acceleration, velocity and displacement of the cylinder, respectively; while $\ddot{Y}$, $\dot{Y}$ and Y represent the same quantities corresponding to the transverse motion. f_{nx} and f_{ny} represent the natural frequencies of the cylinder respectively along

in-line and cross-flow directions; $\xi_x = \xi_y = \xi$ are the structural damping ratios; $M_{rx} = M_{ry} = M_r = m/\rho D^2$ are the reduced masses of the body, where m is the mass of the cylinder per unit length. C_D and C_L are the instantaneous drag and lift coefficients, respectively, and calculated as $C_D = 2F_D/\rho U_\infty^2 D$, $C_L = 2F_L/\rho U_\infty^2 D$, where F_D , F_L are the forces acting upon the cylinder in the in-line and cross-flow directions. The fluid forces are computed by performing an integration, involving both the pressure and viscous stresses, around the surface of the cylinder. The reduced velocity of the cylinder is based on the natural frequency in transverse direction; i.e., $U_r = U_\infty/f_{ny}D$.

2.2 Numerical formulations

To account for the cylinder motions, an Arbitrary Lagrangian-Eulerian (ALE) formulation of the incompressible Navier-Stokes equations is employed for the solution of fluid flows with moving boundaries [8, 9]. The ALE formulation is easily implemented by modifying convective velocity in equation (1) into $c_i = u_i - w_i$, where w_i is the i th component of the grid velocity vector.

A stabilized second-order characteristic-based-split method, as explained in Bao et al. [10], is employed for the discretization of governing equations in ALE form.

Dynamic equations of cylinder motions are solved using an explicit time integration method that was presented in Placzek et al. [11]. They reported that the numerical damping in this algorithm was dramatically reduced by combining a centered upwind and downwind scheme for the prediction of the displacement. The integration step is described as follows:

(i) The cylinder acceleration is explicitly predicted from the previous time step,

$$\ddot{X}^{n+1} = \frac{C_D^n}{2M_{rx}} - 4\pi\xi_x f_{nx} \dot{X}^n - 4\pi^2 f_{nx}^2 X^n \quad (5)$$

$$\ddot{Y}^{n+1} = \frac{C_L^n}{2M_{ry}} - 4\pi\xi_y f_{ny} \dot{Y}^n - 4\pi^2 f_{ny}^2 Y^n \quad (6)$$

(ii) The cylinder velocities and displacements at $n+1$ time step are evaluated using linear approximation:

$$\dot{X}^{n+1} = \dot{X}^n + \Delta t \ddot{X}^{n+1} \quad (7)$$

$$X^{n+1} = X^n + \Delta t[(1-\theta)\dot{X}^n + \theta\dot{X}^{n+1}] \quad (8)$$

$$\dot{Y}^{n+1} = \dot{Y}^n + \Delta t \ddot{Y}^{n+1} \quad (9)$$

$$Y^{n+1} = Y^n + \Delta t[(1-\theta)\dot{Y}^n + \theta\dot{Y}^{n+1}] \quad (10)$$

where, θ is the blending factor, Placzek et al. [11] suggested that $\theta=0.5$ can produce the smallest numerical damping; hence, in the present simulation, we also take the same value.

quencies (f_{nx} , f_{ny}) with the reduced velocity are also shown in this figure. In the f_{nx}/f_{ny} range 1.0-1.75, the St_y curves coincide with each other, indicating that the in-line motions have insignificant effect on the frequency synchronization features ($U_r=3.0-7.0$). At $f_{nx}/f_{ny}=2.0$, the U_r range associated with the resonant response is equal to that for the lower f_{nx}/f_{ny} , however at $U_r=4.0$, the corresponding value of St_y is closer to the f_{ny} curve of the cylinder. A similar curve trend in the St_x is observed in Figure 2(b); the St_x curves for the range $f_{nx}/f_{ny}=1.0-1.75$ also nearly collapse into a single variation curve, whereas a slight difference for $f_{nx}/f_{ny}=2.0$ is noticed at the lower reduced velocity range. An important frequency feature associated to the dual resonant response can be identified in Figure 2, that is, at $f_{nx}/f_{ny}=1.0$, the St_x curve is located upper side of the f_{nx} , indicative of the non-occurrence of dual resonant response. At $f_{nx}/f_{ny}=1.25-1.75$, an intersection of the St_x curve and corresponding f_{nx} curve is evidenced in the plot, showing that frequency synchronization never occurs at this range. A striking frequency behavior is observed at $f_{nx}/f_{ny}=2.0$, where the St_x goes up along the f_{nx} curve in the U_r range from 4.0-7.0, giving a powerful evidence for the occurrence of frequency synchronization in the in-line direction. From the above observation, it may be inferred that, in contrast to the higher Reynolds number flow, the condition for the occurrence of dual resonant response in the laminar flow regime is strictly restricted, and the associated f_{nx}/f_{ny} is about the value of 2 in a very narrow range.

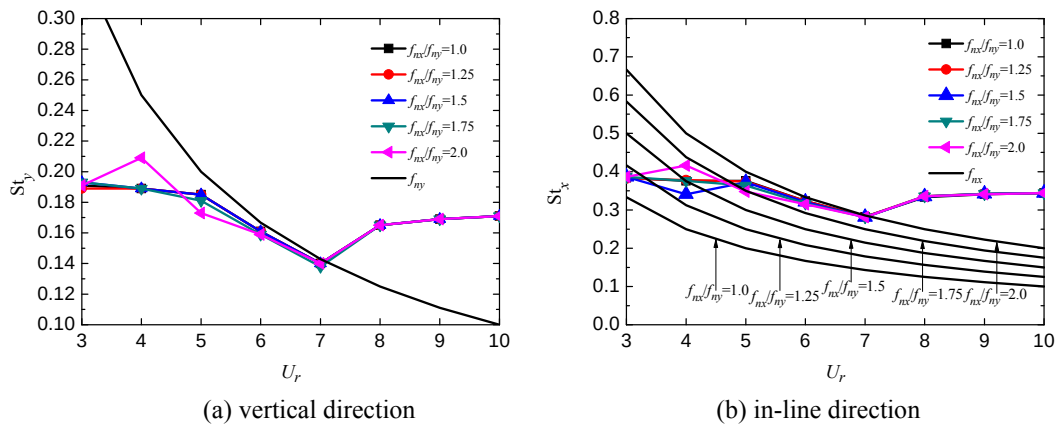


Figure 2. Variation of Strouhal numbers of two-degree-of-freedom as a function of reduced velocity at different natural frequency ratios

4.2 Cylinder orbital trajectories

In general, the combined in-line and cross-flow vibrations of a single cylinder typically show a Figure-eight motion, due to the fact that the dominant frequency in the drag oscillation is twice that for the lift. The same response in terms of cylinder orbit is also noticed in the current simulations. We select the two cases of $f_{nx}/f_{ny}=1.0, 2.0$ to display the cylinder orbital trajectories versus the reduced velocity in Figure 3, in which (a) represents the result associated to the ‘single-resonant’ only in the transverse direction, while (b) corresponds to the dual-resonant response in two degrees of freedom. It can be observed that, except the case of $U_r=4.0$ at $f_{nx}/f_{ny}=1.0$, which shows a fairly messy unrepeatable orbit, most of the cylinder motion appears to be a regular periodic Figure-eight trajectories. However, it is not the case in higher Reynolds number flows. Dahl et al. [6] reported that, as the nominal natural-frequency ratio is near 1, many of the orbital shapes appear irregular, due to the fact that in-line motion does not settle into a regular trajectory. In Figure 3, the path direction in the orbital motion is also marked by a letter ‘C’ or ‘CC’, which represent the direction of the cylinder motion at the top position of the figure-eight path;

i.e., ‘C’ means clockwise , while ‘CC’ denotes counterclockwise direction. It is found that at $f_{nx}/f_{ny}=1.0$, the path direction appears to be either clockwise or counterclockwise trajectories; however, at $f_{nx}/f_{ny}=2.0$, where dual-resonant response could occur, only counterclockwise trajectory accompanies the cylinder motion. This observation is similar with that reported in Dahl et al. [5]. They also argued that the path direction and the orbital shape are important indicators of the appearance of higher harmonic forcing, and show that the counterclockwise path direction is highly repeatable.

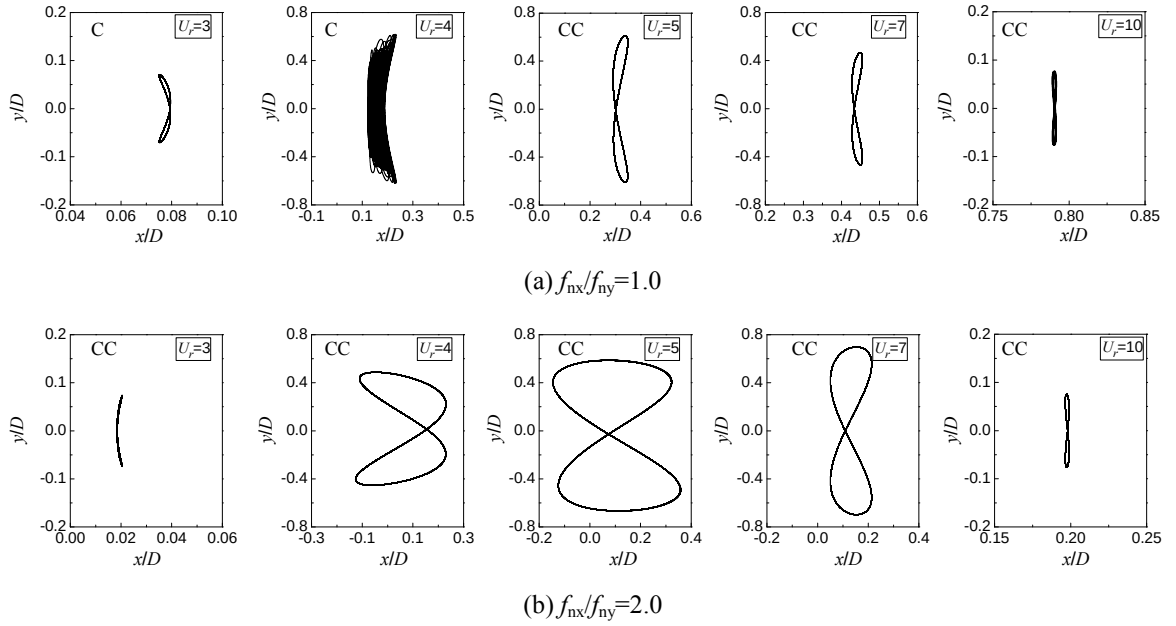


Figure 3. Comparison of Lissajous figures of the trajectory for a single cylinder at different natural frequency ratios; CC (Counterclockwise): trajectory moves upstream at the top of the figure-eight; C (Clockwise): trajectory moves downstream at the top of the figure-eight.

4.3 Cylinder response amplitudes

When the mass ratios are less than 6.0, a dramatic change occurs in the fluid-structure interactions and hence, the freedom to oscillate in line greatly affects the transverse vibrations [1]. In this subsection, the effects of the combined 2-dof vibrations on the amplitudes of the cylinder motions are studied at different natural frequency ratios. Figure 4 shows the variation of the non-dimensional maximum vibration amplitudes with U_r for both transverse and inline directions; i.e., Y_{max}/D , X_{max}/D . As observed in Figure 4(a), in the f_{nx}/f_{ny} range from 1.0 to 1.5, the resonant response curves collapse together and exhibit similar variation trend against the reduced velocity. The maximum amplitudes reach a peak value at a reduced velocity, to which the resonant response begins, with the values around $Y_{max}/D=0.615$. At $f_{nx}/f_{ny}=1.75$, the response curve shifts slightly, with an increased peak value of $Y_{max}/D=0.722$. A dramatic change is observed as the frequency ratio increases up to 2.0, where the peak amplitude shifts to occur at a larger reduced velocity of $U_r=6.0$, with the value rapidly increased to as large as $Y_{max}/D=0.86$. It is worth noting that, in contrast to the earlier experiments, two distinct response peaks could appear when the frequency ratio is near the value of 2.0 [4,12], however, in the present simulations, we does not observed the presence of the double-peak response. The same numerical results are also reported

recently by Lucor and Triantafyllou [7]; they also ascribed this to the lower level of the resolution of the simulation parameters.

Corresponding to the response curve of the transverse amplitude, a distinct response in the in-line amplitude of the cylinder motion is excited at $f_{nx}/f_{ny}=2.0$, on which the cylinder undergoes dual-resonant vibrations. The peak value of the non-dimensional amplitude, X_{max}/D , shifts to a larger reduced velocity as that in the transverse direction, and it reaches a value as high as $X_{max}/D=0.256$; this value is at the same level as reported in the experiment of Dahl et al. [6].

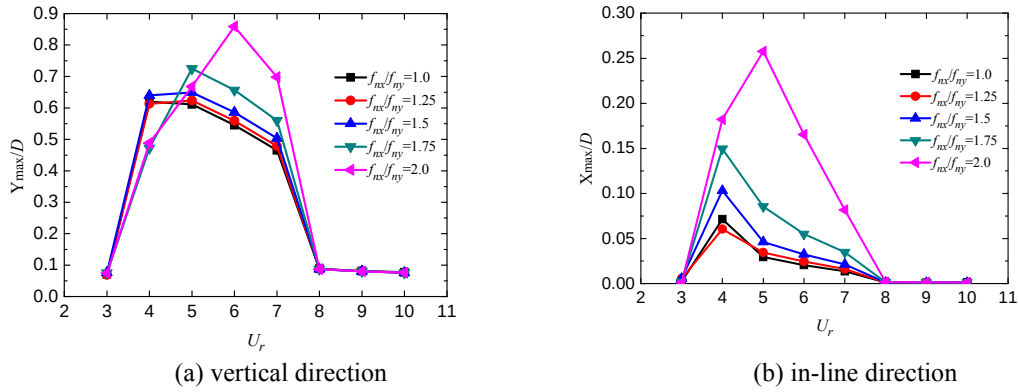


Figure 4. The non-dimensionalized maximum vibration amplitudes in two-degree-of-freedom as a function of reduced velocity at different natural frequency ratios.

4.4 Aerodynamic forces

In the following, we further investigate the statistical characteristics of the forces exerted on the cylinder. Figure 5 shows the variation of the drag and lift coefficients against the reduced velocity, including mean drag coefficient ($\bar{C}_D$) and lift (C'_L) coefficients. As seen in Figure 5(a), the force curves of mean drag are nearly coincided in the range of $f_{nx}/f_{ny}=1.0$ -1.75, while a marked increase is evidenced as the natural frequency increases to $f_{nx}/f_{ny}=2.0$. The lift oscillation appears to be less sensitive to the variation of f_{nx}/f_{ny} ; the peak response is reduced from $C'_L=1.419$ to 0.992 at the same reduced velocity as the f_{nx}/f_{ny} increases to 2.0, in Figure 5(b).

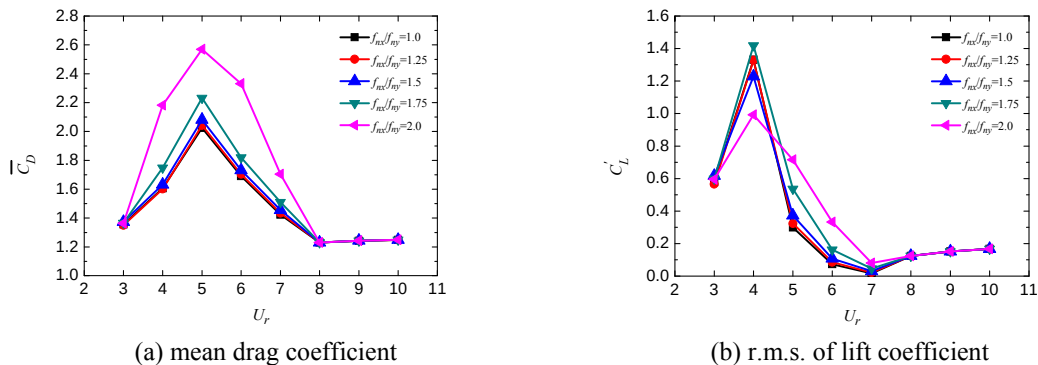


Figure 5. Variations of drag and lift force coefficients with the reduced velocities at different natural frequency ratios.

4.5 Vortex shedding modes

Figure 6 provides the instantaneous wake structure in terms of fully developed vorticity fields for selected natural frequency ratios of $f_{nx}/f_{ny}=1.0$ and 2.0 . As seen in Figure 6(a), a classical 2S mode (two single vortices of opposite sign are shed per cycle) becomes the predominant vortex shedding pattern for the $f_{nx}/f_{ny}=1.0$. Even so, there exist some differences in the wake structures associated with the different reduced velocities. For example, an irregular unstable vortex street is observed at $U_r=4.0$, which is consistent with the unrepeatable cylinder motions as shown in Figure 3(a). In addition, the vertical or horizontal spacing between the successive vortices may be different for the different reduced velocities. For example, at $U_r=5.0$, a two-row vortex structure is formed in the wake, in which the vertical spacing is relatively large. When compared to the results from the case of transverse-only motions [13], the wake pattern for the 2-dof VIV at $f_{nx}/f_{ny}=1.0$ is basically consistent with the former situations. It is not surprising, because under this condition, the limited oscillation in the in-line direction has only very restricted impact on the wake.

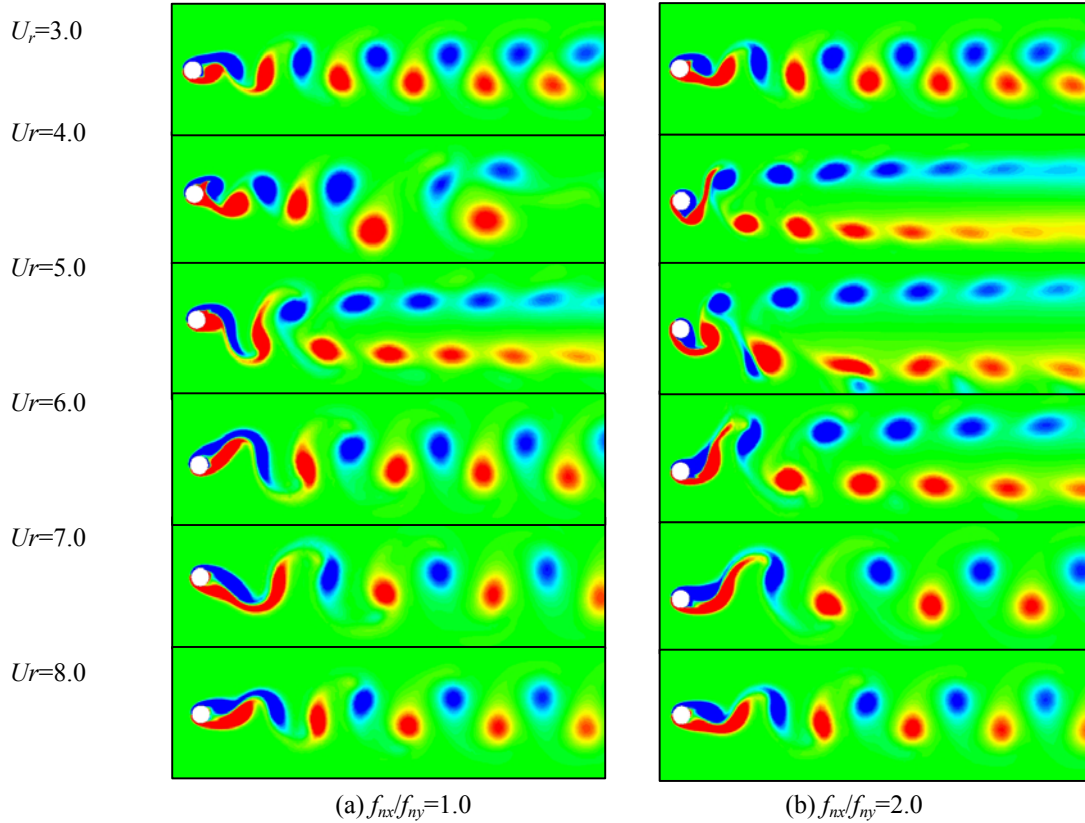


Figure 6. Variation of instantaneous vorticity fields for the fully developed, unsteady flow past a freely vibrating cylinder as a function of reduced velocity at different natural frequency ratios.

A dramatic change in the wake structure is found at $f_{nx}/f_{ny}=2.0$, as in Figure 6(b). From the in-line amplitude curve in Figure 4(b), we have observed that the $X_{\max}/D$ increases with increases in reduced velocity, and reaches its peak at $U_r=5.0$, after then, it drops down with the further increase of the reduced velocity. Correspondingly, the wake pattern varies with the variation in the amplitude of the cylinder motions. As the reduced velocity changes from 3.0 to 4.0, the wake re-

tains the 2S mode; however, it becomes significantly wider and leads a two-row configuration behind the cylinder. As the reduced velocity further increases to reach $U_r=5.0$, the in-line amplitude reaches its peak value; correspondingly, the wake pattern transforms from 2S mode to P+S mode (a single vortex and a pair of opposite signed vortices are released per cycle of shedding). After the in-line motion reaches the peak, with the decrease in the $X_{\max}/D$, the wake is appeared as two-row vortex streets firstly, and then it changes into a classical 2S mode in a single vortex street.

5 CONCLUSION

In the present study of 2-dof vortex-induced vibration: the cylinder is immersed in steady cross-flow of velocity U_∞ and is free to vibrate both in the streamline and transverse directions, and the associated reduced masses of the structures are equal to each other and take very low value: $M_r=2.0$. To encourage high amplitude oscillations, the structural damping coefficients in both directions are set to zero; i.e., $\xi=0.0$. The reduced velocity, U_r (based on the natural frequency in transverse direction), is systematically increased within the range $U_r=3.0-12.0$ in increments of one. For each reduced velocity, five frequency ratios are considered; i.e., $f_{nx}/f_{ny}=1.0, 1.25, 1.5, 1.75$ and 2.0 .

Based on the comprehensive analysis of the results, we could draw following findings:

It is found that interaction of the 2-dof dominant force frequencies is significant in the dual-resonant response regime, where the peak frequencies of the drag and lift signals can appear at the same value. In the laminar flow regime, the condition for the occurrence of dual-resonant response is strictly limited around $f_{nx}/f_{ny}=2.0$.

The cylinder orbital trajectory shows that at $f_{nx}/f_{ny}=1.0$, the path direction can appear to be either clockwise or counterclockwise; however, at $f_{nx}/f_{ny}=1.0$, where dual-resonant response could occur, the cylinder orbit is only in counterclockwise patterns. The combined 2-dof vibrations significantly amplify the transverse response of the cylinder, and the peak response delays to appear at a higher reduced velocity. Double-peak response, which occurs at higher Reynolds number flow, is not observed in the present simulation; this difference may be ascribed to the Reynolds number effect. The oscillating drag component is most sensitive to the variation of the natural frequency ratios, and maximized within the dual-resonant regime.

A 2S wake pattern is accompanied to the VIV of single cylinder in the transverse-only resonant response state, however, the vortex distance in both in-line and transverse directions varies with the reduced velocity. As the cylinder experiences dual-resonant response, the wake may convert from 2S to P+S pattern, which would associate to the state where the in-line response reaches its maximum amplitude.

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