

Turbulence simulation in wavelet domain based on Log-Poisson model: univariate and multivariate wind processes

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ABSTRACT: The wind in atmospheric boundary layer is characterized as turbulent flow at a high Reynolds number. The complexity in turbulence is mainly from its nonlinear and multi-scale structure. This paper simulates the wind process utilizing the wavelet domain based on the Log-Poisson model where both multi-scale structure and the intermittency could be properly represented. Following simulating a univariate process, a methodology for simulating multivariate processes is also established, where a correction technique based on the coherence of the wavelet coefficients at each scale is utilized. The resulting simulation follows the characteristic features of the sample record and offers a more realistic simulation than the spectral representation based methods.

KEYWORDS: Turbulence; non-Gaussian; wavelet; Log-Poisson model; multivariate processes

1 INTRODUCTION

The wind process in atmospheric boundary layer is characterized as turbulent flow at a high Reynolds number. A physical origin of the complexity in turbulence is due to the nonlinearity controlled by the Navier-Stokes equations. As a result, turbulent flows are naturally described based on the statistics theory. Typically, turbulent fluctuations are assumed to be Gaussian. Conventional wind process simulation, such as spectral representation, is based on this assumption. Another physical origin of the complexity in turbulence is due to the multi-scale property resulting from nonlinear interactions, where the turbulence are better described utilizing the statistics of velocity difference at two locations compared with that of velocity fluctuations at a single location. As a strong discontinuous nature of turbulence, internal intermittency seems only changing the velocity statistics slightly (a broad banded probability density distribution (PDF) compared to Gaussian distribution), while it has significant effects on the statistics of the velocity difference (a considerably larger tail PDF compared with Gaussian distribution).

In order to take into account the effects of intermittency, several phenomenological models have been introduced in the literature, such as Log-normal and other multifractal models. Obviously, the conventional wind process simulation cannot take intermittency into account. On the other hand, these random cascade models are convenient to be incorporated in the wavelet based simulation^[1,2]. Specifically, the recently developed Log-Poisson model^[3], which is claimed to be able to reveal the physical mechanism of the intermittency phenomenon, is utilized in this study. As a result, the intermittency will be preserved in the simulated turbulence utilizing wavelet domain. The boundary condition effects on the energy containing (EC) range and the non-local relationship between the scales in the multiplicative cascade process of the inertial subrange are included in the simulation. In addition, the wavelet domain also facilitates to include non-stationary feature in the simulation often observed in atmospheric turbulence. Based on this developed framework, univariate and multivariate wind process simulation schemes are proposed and their efficacy is demonstrated by examples.

2 ANALYSIS OF WIND PROCESS

Conventional strategies of simulating wind processes are usually restricted to the second-order statistics with implied Gaussian assumption. These strategies can be generalized by Karhunen-Loeve (K-L) expansion and numerically realized with the Galerkin Scheme^[4]. The generalized Gaussian process simulation scheme is given as^[5]

$$V(\theta, t) \approx \sum_{k=1}^N \lambda_k \xi_k(\theta) \phi_k(t) \approx \sum_{k=1}^N \lambda_k \xi_k(\theta) \left[\sum_{m=1}^N d_m^{(k)} \psi_m(t) \right] \quad (1)$$

where θ is the original random variable; $V(\theta, t)$ is the target random process with respect to time t ; $\{\lambda_k\}$ are the eigenvalues of K-L expansion; $\{\xi_k(\theta)\}$ is the identical independent distribution (i.i.d) standard Gaussian sequence; $\{\phi_k(t)\}$ are the eigenfunctions corresponding to $\{\lambda_k\}$; $\{d_m^{(k)}\}$ are the coefficients corresponding to the prescribed truncated orthogonal basis $\{\psi_m(t), m=1, 2, \dots, N\}$. Suppose Fourier basis is selected, the simulation is equivalent to the spectral representation. On the other hand, the simulation can be extended to the nonstationary Gaussian process if wavelet basis is selected^[5].

A typical turbulent flow field is not necessarily Gaussian as demonstrated by some experimental data^[e.g.,6]. Specifically, the PDFs of velocity differences at the inertial scale exhibit a significant longer tail than Gaussian distribution, which is related to the intermittency in turbulence. Conventional schemes based on the second-order statistics are insufficient to describe such a non-Gaussian process. On the other hand, since typical dynamical information in turbulence is contained in the scaling quantities, based on which the wavelet coefficients are generated, wavelet expansion utilizing higher-order statistics is an appropriate tool to analyze and simulate such a multi-scale process with non-Gaussian PDF.

2.1 Scaling properties of turbulence

There are two significant scaling quantities describing the multi-scale structure of turbulence: the energy dissipation rate ε and the velocity difference δv . In this study, the characteristics of δv are utilized. For a turbulence flow, within the inertial range, the p^{th} order moment of δv_l at scale l has a power-law dependence on l ^[3],

$$\langle \delta v_l^p \rangle \sim l^{\tau_p} \quad (2)$$

where $\langle \rangle$ denotes the expectation of the random variable; τ_p represents the exponent at the p^{th} order moment. Within the EC range, δv is related to the boundary condition and may exhibit nonstationary features as it reflects the external mechanism.

2.2 Statistical property of the Log-Poisson random cascade model

The universal scaling law can be realized via a Log-Poisson process, which is a random multiplicative cascade process^[1]. Within the inertial range, for the case of $p=1$, the velocity difference δv_{l_1} at scale l_1 and the velocity difference δv_{l_2} at scale l_2 has the following relation^[1,3]

$$\delta v_{l_1} = W_{l_1, l_2} \delta v_{l_2} = \left[\left(\frac{l_2}{l_1} \right)^{\gamma-1} \beta^{n_{h,l_2}} \right]^{\frac{1}{3}} \delta v_{l_2} \quad (3)$$

where W_{l_1, l_2} is the multiplicative factor between δv_{l_1} and δv_{l_2} ; γ and β are the scale-related parameters and need to be estimated based on the real turbulence flow, however, refer-

ence [3] suggests that $\gamma = 2/3, \beta = 2/3$; n_{l_1, l_2} is the independent Poisson random variable with mean λ_{l_1, l_2} , satisfying the following relation^[T,3]

$$\lambda_{l_1, l_2} = -\frac{\gamma}{\beta-1} \ln \frac{l_2}{l_1} \quad (4)$$

It should be noted that the above relation from l_2 directly to l_1 is identical to l_2 via l_3 to l_1 .

3 SIMULATION OF WIND PROCESS

The wavelet technique is utilized here to simulate the multi-scale structure of the turbulence flow. Suppose only δv_l in the EC range depends on the external mechanism, it is reasonable to attribute the nonstationary part in the turbulence flow to δv_l within this range. Therefore, it is necessary to determine the critical scale l_c , which pinpoints the EC range and the inertial range. Usually, l_c is assumed corresponding to the frequency f_c which makes $f\hat{S}_{vv}(f)$ maximum, where $\hat{S}_{vv}(f)$ is the power spectrum of the wind velocity $V(t)$ with zero mean value. The frequency f_c is a function of mean wind velocity an integral scale as Karman spectrum is utilized.

Haar wavelet function is chosen here since it represents a clear physical meaning of velocity difference. Correspondingly, the relation between the wavelet scale order j and the dominant frequency f_j of j th Haar wavelet function can be determined^[Appendix A]. Particularly, the relation between j_c and f_c is shown as

$$j_c = n + \text{int} \left\{ \log_2 \left(\frac{f_c}{0.742} \right) \right\} \quad (5)$$

where n is the highest wavelet scale order used in the simulation.

3.1 Simulation of univariate wind process

Use $m_v(b_k, a_j)$ to denote the wavelet coefficient at the a_j scale and b_k time position, associated with the normalized form $\zeta(b_k, a_j)$. With Haar wavelet transform, $m_v(b_k, a_j)$ is closely related to δv at l_j scale^[Appendix B].

3.1.1 Simulation scheme

The simulation scheme of univariate wind process in wavelet domain is presented in Fig. 1, which is described as follows:

- (1) Determine j_c using Eq. (5);
- (2) Within the EC range, $\{\delta v_{l_{j,j}}\} (j=1, \dots, j_c; i=1, \dots, 2^{n-1})$ are produced from *i.i.d* standard Gaussian sequence and transformed to $\{\zeta(b_k, a_j)\} (j=1, \dots, j_c; k=1, \dots, 2^{j-1})$;
- (3) Within the inertial range, $\{\delta v_{l_{j,j}}\} (j=j_c+1, \dots, n; i=1, \dots, 2^{n-1})$ are produced independently from $\{\delta v_{l_{j-1,j}}\}$ or $\{\delta v_{l_{j_c,j}}\}$ base on

$$\delta v_{l_{j,j}} = W_{j,j-1} \delta v_{l_{j-1,j}} \quad \text{or} \quad \delta v_{l_{j,j}} = W_{j,j_c} \delta v_{l_{j_c,j}} \quad (6)$$

where

$$W_{j,j-1} = \left(\frac{l_{j-1}}{l_j} \right)^\gamma \beta^{n_{j,j-1}}, \quad W_{j,j_c} = \left(\frac{l_{j_c}}{l_j} \right)^\gamma \beta^{n_{j,j_c}} \quad (7)$$

and then transform the generated $\{\delta v_{l_{j,j}}\}$ to $\{\zeta(b_k, a_j)\} (j=j_c+1, \dots, n; k=1, \dots, 2^{j-1})$;

(4) $m_v(b_k, a_j)$ is adjusted from $\zeta(b_k, a_j)$ according to the wavelet coefficients correlation at the same scale, which shares the following relation with conventional spectral density for stationary process^[Appendix C]

$$m_v(b_k, a_j) = \zeta(b_k, a_j) \times \sqrt{\left\langle |m_v(b_k, a_j)|^2 \right\rangle} = \zeta(b_k, a_j) \times \sqrt{2^{a_j} \int_{-\infty}^{\infty} |\tilde{\psi}_{a_j, b}(f)|^2 S_{VV}(f) df} \quad (8)$$

where $\tilde{\psi}_{a_j, b}(f)$ is the Fourier spectrum of wavelet function $\psi_{a_j, b}(t)$ at scale a_j ; $S_{VV}(f)$ is the conventional spectral density with mean velocity. For a non-stationary process, the above equation is only approximately satisfied, while the exact energy distribution of wavelet coefficients should refer to the technique as Eq.(1). In essence, the eddies could be treated as scale-dependent stationary within the inertial range as they depend mainly on the internal mechanism of the turbulence rather than the external condition;

(5) Synthesize the wind process v from the generated $m_v(b_k, a_j)$.

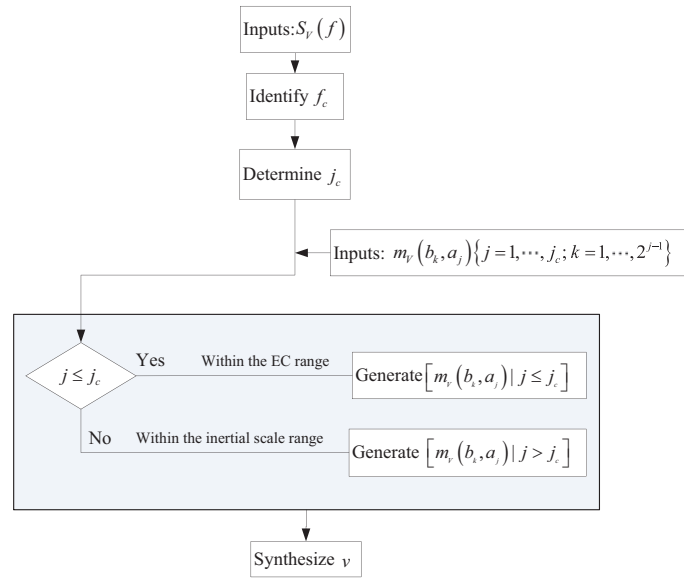


Figure 1 Simulation of univariate wind process.

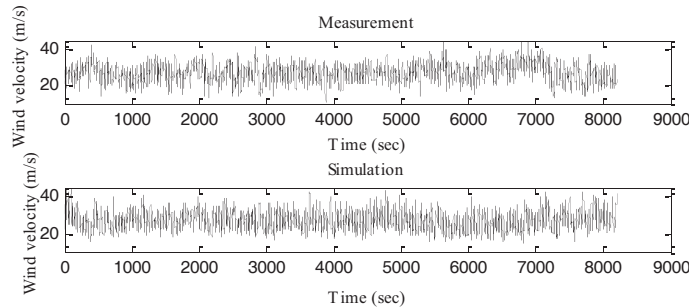


Figure 2 The measurement and simulation result of a univariate wind process.

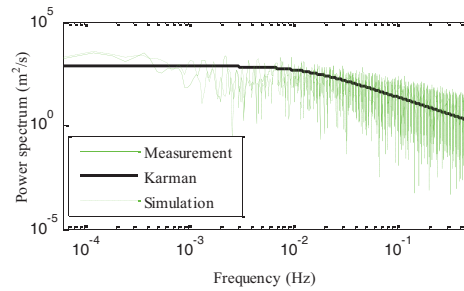


Figure 3 Power spectrum

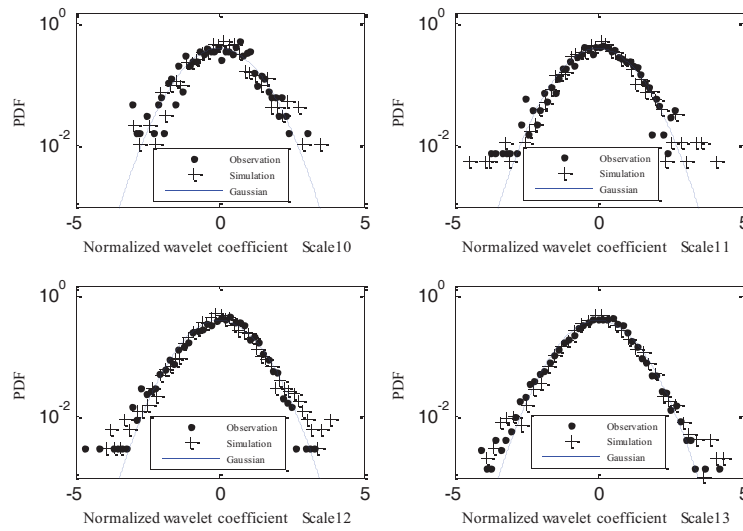


Figure 4 PDF of wavelet coefficients at some typical inertial scales.

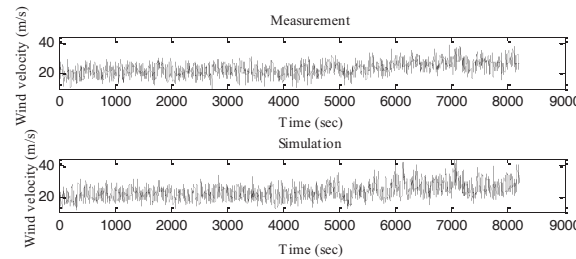


Figure 5 The measurement and simulation result of univariate nonstationary wind process.

3.1.2 Case study

Hurricane Katrina is utilized in this study to demonstrate the effectiveness of the proposed simulation framework. The measured data and the simulation result are shown in Fig. 2. ℓ_u is estimated as 247m and $j_c = 7$. The power spectra based on the measured and simulated data are presented in Fig. 3 together with Karman spectrum. PDFs of the wavelet coefficients at some typical inertial scales are given in Fig. 4. There is 4.59% error between the variances of the simulated process and of the measurement. A nonstationary case is shown as Fig. 5.

3.1.3 Discussion

It is interesting to notice that reference [1] has used a similar approach to simulate a univariate stationary wind process. This study offers a reasonable strategy for simulating nonstationary

wind. More importantly, there is a major distinction in the treatment of the Log-Poisson model between this study and reference [1]:

(1) Within the inertial range, it is known that the universal scaling law is obtained from the ergodicity assumption which implies stationarity, however, the algorithm for generating the wavelet coefficients in reference [1] distorts the ergodicity in three aspects. First, the odd wavelet coefficients are generated by involving an algebraic average of the wavelet coefficients at larger scale, while the even wavelet coefficients are not, which are given here^[1],

$$\tilde{\zeta}(b_{k=2i+1}, a_j) = s \times (2/3)^{m/3} 2^{-1/9} \frac{\sqrt{2} [\tilde{\zeta}(b_{k=i}, a_{j-1}) + \tilde{\zeta}(b_{k=i+1}, a_{j-1})]}{2} \quad (9)$$

$$\tilde{\zeta}(b_{k=2i}, a_j) = s \times (2/3)^{m/3} 2^{-1/9} \tilde{\zeta}(b_{k=i}, a_{j-1}) \quad (10)$$

where s is a binomial random variable taking the value +1 or -1; m is equivalent to $n_{j,j-1}$ in this study. The ergodicity assumption indicates that $\zeta(b_k, a_j)$ should have identical property at each scale while reference [1] obtains the odd and even terms using different schemes; second, both $\tilde{\zeta}(b_{k=2i}, a_j)$ and $\tilde{\zeta}(b_{k=2i+1}, a_j)$ deviate from the actual random variables $\zeta(b_k, a_j)$ since another random variable s is involved in the expression. The case of $\tilde{\zeta}(b_{k=2i+1}, a_j)$ is worse due to the average operation previously discussed; third, there should be 2^{j_n-1} basic *i.i.d* Gaussian random variables to generate the wavelet coefficients at each scale instead of 2^{j_c-1} .

(2) Reference [1] simply regards the wavelet coefficient $\tilde{\zeta}(b_k, a_j)$ equivalent to the velocity difference δv_{l_j} and supposes they have the same statistical property. Actually, inferred by the wavelet decomposition algorithm, $\tilde{\zeta}(b_k, a_j)$ should be determined based on the contribution from all the $\delta v_{l_{j,d}}$ s. The weighting of the contribution from each $\delta v_{l_{j,d}}$ depends on the selected wavelet function. For example, for Haar wavelet basis, which is utilized in this study, the relation between the velocity difference and the wavelet coefficients is presented in Appendix A.

(3) In reference [1], it is stated "when the large fluctuations occurred at a certain scale j , the coefficient fluctuations become large around the same time at the other j : the local self-similarity was realized with the algorithm in this study." Actually, this local strong fluctuation is a false appearance caused by the manipulation in the algorithm: all the samples of $\tilde{\zeta}(b_k, a_j)$ in the inertial range are generated from the same series of samples $\tilde{\zeta}(b_k, a_{j_c})$. In this regard, the difference between the samples of $\tilde{\zeta}(b_k, a_{j_c})$ will be passed down which causes similar corresponding difference between $\tilde{\zeta}(b_k, a_j)$ at all smaller scales. Actually, the universal scaling law holds true only in the statistical meaning rather than the sample meaning, therefore, the samples of $\tilde{\zeta}(b_k, a_j)$ should be generated independently from the samples of $\tilde{\zeta}(b_k, a_{j_c})$, instead of reusing at each time.

3.2 Simulation of multivariate wind processes

The available information in simulating multivariate wind processes is the cross power spectrum $S_{XY}(f)$ and the trends represented by $\bar{m}_X(b_k, a_j)$ and $\bar{m}_Y(b_k, a_j)$ ($j=1, \dots, j_c$; $k=1, \dots, 2^{j-1}$) at the target locations X and Y. The main object is to generate $\bar{m}_X(b_k, a_j)$ and $\bar{m}_Y(b_k, a_j)$ ($j=j_c+1, \dots, n$; $k=1, \dots, 2^{j-1}$) from the wavelet coefficient cross spectral density at the same scale between X and Y. The relation is shown as follows for stationary processes^[Appendix C]

$$\tilde{C}_{m_X m_Y}(f, a_j) = \int_{-\infty}^{\infty} C_{m_X m_Y}(\tau, a_j) e^{-i\tau\omega} d\tau = 2^a \left| \psi_{a_j, b}(f) \right|^2 S_{XY}(f) e^{i\tau f} \quad (11)$$

where $\tilde{C}_{m_X m_Y}(\omega, a_j)$ is the wavelet coefficient cross spectral density (CPD) at the same scale. Similar to the univariate case, for non-stationary processes, the above equations are approximately satisfied.

3.2.1 Simulation scheme

The simulation scheme of multivariate wind processes in wavelet domain is presented in Fig. 6, which is described as follows:

- (1) Generate respectively the initial wavelet coefficients $m_X^0(b_k, a_j)$ and $m_Y^0(b_k, a_j)$ using the univariate simulation strategy described in section 3.1;
- (2) Within the EC range, produce the final wavelet coefficients $m_X(b_k, a_j)$ and $m_Y(b_k, a_j)$ using conventional multivariate Gaussian simulation strategy at each scale;
- (3) Within the inertial range, estimate the first four moments $d_{X,i,j}^T$ and $d_{Y,i,j}^T$ ($j = j_c + 1, \dots, n$; $i = 1, \dots, 4$) at each scale from the produced $m_X^0(b_k, a_j)$ and $m_Y^0(b_k, a_j)$. Calculate the target $\tilde{C}_{m_X m_Y}^T(f)$ from $S_{XY}(f)$ and set the initial coherence value $\gamma_{m_X m_Y}^D = \gamma_{m_X m_Y}^T$, the iteration number $it=1$, where

$$\gamma_{m_X m_Y}^2(f) = \frac{|\tilde{C}_{m_X m_Y}^T(f)|^2}{\tilde{C}_{m_X m_X}^T(f) \tilde{C}_{m_Y m_Y}^T(f)} \quad (12)$$

- (4) A set of Gaussian correlated processes $m_X^G(b_k, a_j)$ and $m_Y^G(b_k, a_j)$ ($j = j_c + 1, \dots, n$) are generated using a standard multivariate Gaussian simulation algorithm^[8];
- (5) Transform respectively the obtained m_X^G and m_Y^G through forward Modified Hermite Transformation and spectral correction^[9,10] to the non-Gaussian m_X^{NG} and m_Y^{NG} which matches their target $\tilde{C}_{m_X m_X}^T(f)$, $\tilde{C}_{m_Y m_Y}^T(f)$ and target moments $d_{X,i,j}^T$, $d_{Y,i,j}^T$;
- (6) Measure the coherence $\gamma_{m_X m_Y}^{NG}$ from the generated m_X^{NG} and m_Y^{NG} and compare it with the target coherence $\gamma_{m_X m_Y}^T$ to determine the error as

$$err = \|\gamma_{m_X m_Y}^{NG} - \gamma_{m_X m_Y}^T\| \quad (13)$$

- (7) If the error is below the acceptance level, the iteration ends. If not, update $\tilde{C}_{m_X m_Y}^D$ by

$$\tilde{C}_{m_X m_Y}^D(it+1) = \tilde{C}_{m_X m_Y}^D(it) + \tilde{C}_{m_X m_Y}^D(it)(\tilde{C}_{m_X m_Y}^T - \tilde{C}_{m_X m_Y}^{NG}) \quad (14)$$

until the error is accepted;

- (8) Generate the wind processes v_X and v_Y from $m_X(b_k, a_j)$ ($j = 1, \dots, j_c$), m_X^{NG} and $m_Y(b_k, a_j)$ ($j = 1, \dots, j_c$), m_Y^{NG} respectively.

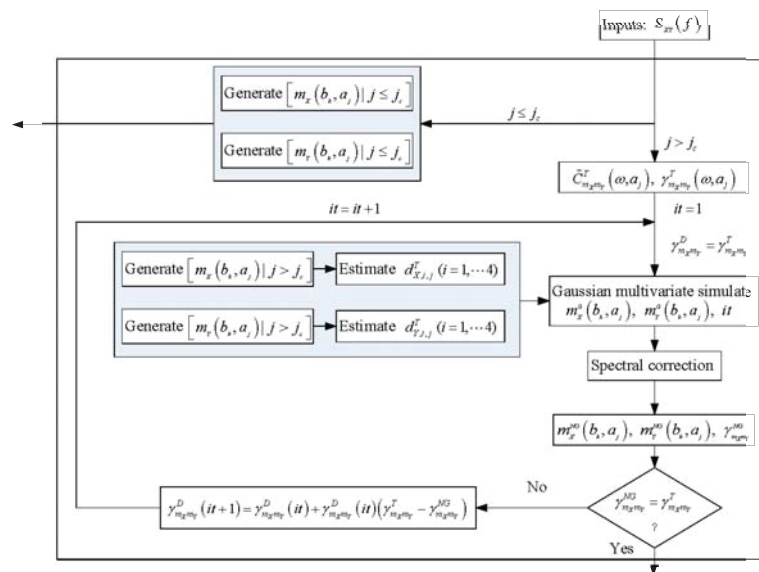


Figure 6 Simulation of multivariate wind processes.

3.2.2 Case study

Two measurements drawn from different passages of Hurricane Katrina are employed to simulate the multivariate wind processes. The generated time histories are show in Fig. 7. The cross correlation coefficients of the measurement and simulation exhibit good agreement (Fig. 8).

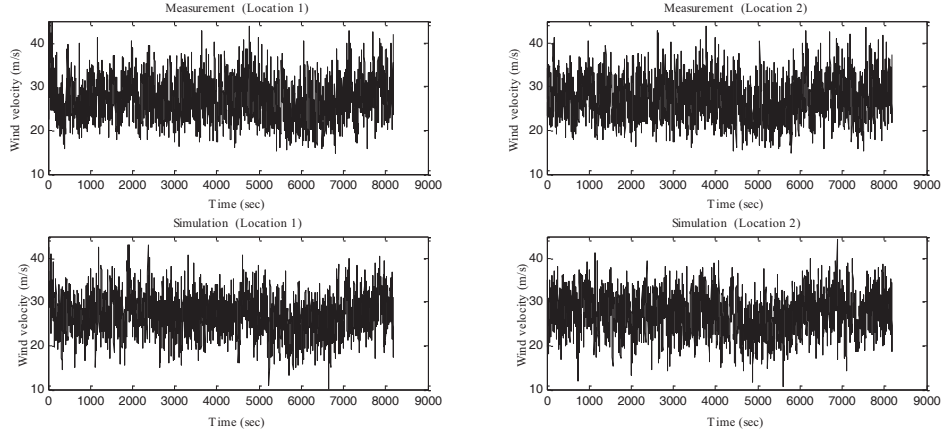


Figure 7 The measurements and simulation results of multivariate wind processes.

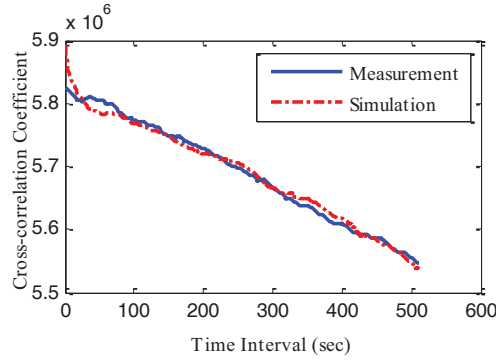


Figure 8 Coefficients of cross-correlation function.

4 CONCLUSION

A methodology of simulating univariate and multivariate turbulence flows in wavelet domain is proposed. This method, utilizing the Log-Poisson random cascade model, emphasizes the internal intermittency effect of the turbulence fluctuations in this simulation. The simulation results agree with both the power spectrum and the statistical information presented by the scaling law, thus present a better representation of the natural wind process than the conventional methods which only considers the second-order moments. Furthermore, in the simulation of the multivariate processes, the wavelet coefficients are iteratively updated to match the wavelet coefficients coherence at the same scale, thereby guaranteeing the consistence of the cross correlation function between the measurement and the simulation results.

5 ACKNOWLEDGEMENTS

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6 APPENDIX

A. When time interval is 1sec, the Fourier spectrum of Haar wavelet function is shown as

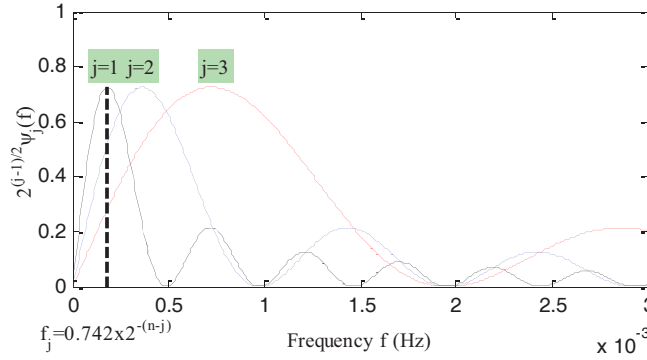


Figure 9 Absolute Fourier spectrum of Haar wavelet

B. The Discrete Haar wavelet reconstruction algorithm is shown as follows:

$$m_V(b_k, a_j) \sim \frac{1}{2^{n-j}} \sum_{i=2^{n-j} \times (k-1) + 1}^{2^{n-j} \times k} \delta v_{l_{j,i}}, \quad (k=1, \dots, 2^{j-1}) \quad (15)$$

In order to explain clearly the relation between velocity difference δv_{l_j} and wavelet coefficient $m_V(b_k, a_j)$, the above algorithm is expressed in unnormalized form. Specifically, suppose a wind process $v(t)$ including 8 discrete points, there are

$$\begin{aligned} m_V(b_1, a_3) &\sim \delta v_{l_{3,1}} = v_1 - v_2, \quad m_V(b_2, a_3) \sim \delta v_{l_{3,2}} = v_3 - v_4, \dots, m_V(b_4, a_3) \sim \delta v_{l_{3,4}} = v_7 - v_8 \\ m_V(b_1, a_2) &\sim \frac{1}{2} \sum_{i=1}^2 \delta v_{l_{2,i}} = \frac{1}{2} [(v_1 - v_3) + (v_2 - v_4)], \quad m_V(b_2, a_2) \sim \frac{1}{2} \sum_{i=3}^4 \delta v_{l_{2,i}} = \frac{1}{2} [(v_5 - v_7) + (v_6 - v_8)] \\ m_V(b_1, a_1) &\sim \frac{1}{4} \sum_{i=1}^4 \delta v_{l_{1,i}} = \frac{1}{4} [(v_1 - v_5) + (v_2 - v_6) + (v_3 - v_7) + (v_4 - v_8)] \end{aligned} \quad (16)$$

When applied in the reconstruction like the simulation in this study, $\{\delta v_{l_{j,i}}\}$ represent a *i.i.d* sequence. Therefore, $m(b_k, a_j)$ is another random variable different from δv_{l_j} , furthermore, indicated by Central Limit Theorem, $m(b_k, a_j)$ approaches Gaussian distribution closer than δv_{l_j} .

C. The relation between the wavelet coefficients cross-correlation at the same scale and the conventional cross-correlation is shown as^[11]

$$\begin{aligned} C_{m_X m_Y}(b_1, b_2, a_1, a_2) &= \left\langle \int_{-\infty}^{\infty} m(b_1, a_1) m^*(b_2, a_2) \right\rangle \\ &= \left\langle \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(t_1 - b_1, a_1) \psi^*(t_2 - b_2, a_2) x(t_1) y^*(t_2) dt_1 dt_2 \right\rangle \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(t_1 - b_1, a_1) \psi^*(t_2 - b_2, a_2) \langle x(t_1) y^*(t_2) \rangle dt_1 dt_2 \\ &= \sqrt{2^{a_1 + a_2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(2^{a_1} t_1 - b_1) e^{-i t_1 f} \psi^*(2^{a_2} t_2 - b_2) e^{i t_2 f} R_{XY}(t_1, t_2) e^{i(t_2 - t_1)f} dt_1 dt_2 \end{aligned} \quad (17)$$

If X and Y are both stationary, the above equation can be written further as

$$\begin{aligned} C_{m_X m_Y}(b_1, b_2, a_1, a_2) &= \sqrt{2^{a_1+a_2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(2^{a_1} t_1 - b_1) e^{-i t_1 f} \psi^*(2^{a_2} t_2 - b_2) e^{i t_2 f} \int_{-\infty}^{\infty} S_{XY}(f) df dt_1 dt_2 \\ &= \sqrt{2^{a_1+a_2}} \int_{-\infty}^{\infty} \tilde{\psi}_{a_1, b_1}(f) \tilde{\psi}_{a_2, b_2}^*(f) S_{XY}(f) df \end{aligned} \quad (18)$$

Then it can derive the following expressions

$$C_{m_X m_Y}(\tau, a_j) = 2^{a_j} \int_{-\infty}^{\infty} \left| \tilde{\psi}_{a_j, b}(f) \right|^2 S_{XY}(f) df \quad (19)$$

$$\tilde{C}_{m_X m_Y}(f, a_j) = 2^{a_j} \left| \tilde{\psi}_{a_j, b}(f) \right|^2 S_{XY}(f) \quad (20)$$

$$C_{m_Y m_Y}(\tau, a_j) = 2^{a_j} \int_{-\infty}^{\infty} \left| \tilde{\psi}_{a_j, b}(f) \right|^2 S_{VV}(f) df \quad (21)$$

$$\tilde{C}_{m_Y m_Y}(f, a_j) = 2^{a_j} \left| \tilde{\psi}_{a_j, b}(f) \right|^2 S_{VV}(f) \quad (22)$$

Obviously, a stationary process implies a scale-dependent stationary process.

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