An analytical model of linked twin tall buildings and modal property analysis

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Abstract

In order to study effects of link on modal properties of linked tall buildings, an analytical model of linked buildings that can consider varied locations and numbers of links is introduced. Then effects of link are investigated by parametric analysis about stiffness, location and number of links, which provide a reference for preliminary design of linked tall buildings and a foundation for subsequent dynamical response analysis.

1 Introduction

Due to increasingly esthetical requirements, limited land resources in urban area, and some special function such as observation deck, sky swimming pool, there are more and more tall buildings in close proximity connected by sky-bridge or sky-bridges, i.e., linked buildings (Xie and Irwin, 1998, Huang, et al., 2001, Lim and Bienkiewicz, 2009). Two main types of coupling effect, i.e., aerodynamic and structural couplings (Lim and Bienkiewicz, 2009), may arise in the wind-resistant design of these linked buildings. The former one is caused by the cross-correlation of wind loadings between buildings. The latter one represents the effect of connection parts, which can synchronize wind-induced structural vibrations. In this way, the structural coupling introduced by links may lead to a more efficient wind-resistant design. Therefore, it is very important to incorporate effects of link on modal properties of linked tall buildings and their effects on the wind-induced vibration responses.

Lim and Bienkiewicz (2011) introduced a simplified six-degree-of-freedom model of twin linked buildings with a skybridge, by which structural frequencies and mode shapes at skybridge level can be derived analytically. This model, however, only considers the case of one link and makes some assumption about mode shape, which may not be applied for all cases. Therefore, an analytical that can consider varied cases of link or links is introduced in this paper. The accuracy of the analytical model is verified by comparing with the model of finite element method (FEM). The effect of links on modal properties is examined by parametric analysis about stiffness, location and number of links using the analytical model, which can give a reference for preliminary design of linked tall buildings and provide a foundation for subsequent dynamical response analysis.

2 Modelling of structural coupling effect of link

In order to study the effect of link on modal properties of linked buildings as shown in Fig.1, which is an example of two linked twin tall buildings (tower 1 and 2) connected by n links, the sub-matrix of mass and stiffness of each sub-part, i.e., two towers and links, should be formed at first. Then, the total mass and stiffness can be assembled to obtain structural frequencies and mode shapes by eigenvalue analysis of the characteristic equation. Assuming that links are fixed-fixed beams without the stiffness of out-plane, the stiffness matrix of link $K_L$ for the deformation of two ends of link $(x_{L1}, x_{L2}, y_{L1}, y_{L2},$...
\[ \begin{bmatrix}
  k_a & -k_a & 0 & 0 & 0 & 0 \\
  -k_a & k_a & 0 & 0 & 0 & 0 \\
  0 & 0 & 12k_b & -12k_b & 6k_bl & 6k_bl \\
  0 & 0 & -12k_b & 12k_b & -6k_bl & 6k_bl \\
  0 & 0 & 6k_bl & -6k_bl & (4+\beta)k_bl^2 & (2-\beta)k_bl^2 \\
  0 & 0 & 6k_bl & -6k_bl & (2-\beta)k_bl^2 & (4+\beta)k_bl^2 
\end{bmatrix} \]

\[ KL = \begin{bmatrix}
  k_a & -k_a & 0 & 0 & 0 & 0 \\
  -k_a & k_a & 0 & 0 & 0 & 0 \\
  0 & 0 & 12k_b & -12k_b & 6k_bl & 6k_bl \\
  0 & 0 & -12k_b & 12k_b & -6k_bl & 6k_bl \\
  0 & 0 & 6k_bl & -6k_bl & (4+\beta)k_bl^2 & (2-\beta)k_bl^2 \\
  0 & 0 & 6k_bl & -6k_bl & (2-\beta)k_bl^2 & (4+\beta)k_bl^2 
\end{bmatrix} \]

where \( x_{Lt}, y_{Lt} \) and \( \theta_{Lt} \) \((t=1 \text{ or } 2)\) are deformations of end of links in the two translational directions and torsional directions along axis \( z \); \( l \) is the length of link; axial stiffness \( k_a = EA/l \); bending stiffness \( k_b = \frac{EI}{l^3} \), where \( I = \frac{1}{1+\beta} \) and \( \beta = \frac{12\mu EI}{GAl^2} \); \( \mu \) is a correction factor, for rectangular beam, \( \mu = 1.2 \).

It should be noted that the above stiffness matrix \( KL \) is formed at points connecting main towers and link. In order to assemble it to the total stiffness matrix, it should be transferred to the mass center of the related floor. According to the relationship shown in Fig. 2, the stiffness matrix of link for the deformation of mass center of tower can be derived as

\[ K'_L = HK_LHT \]

where \( H \) is the transformation matrix,

\[ H = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  -r \sin \alpha & 0 & r \cos \alpha & 0 & 1 & 0 \\
  0 & -r \sin \alpha & 0 & -r \cos \alpha & 0 & 1 
\end{bmatrix} \]

Once we have obtained all sub-matrices, total structural matrices can be obtained. Lumping the mass of link to the connected floors of two towers equally, the total mass matrix \( M \) and stiffness
matrix $K$ can be expressed as

$$M = \text{diag}\left([M_1, M_2, M_1, M_2, I_1, I_2]\right) + \frac{1}{2} E \times \text{diag}\left([M_L, M_L, M_L, M_L, r^2 M_L, r^2 M_L]\right) \times E^T$$

(4)

$$K = \text{diag}\left(K_{1x}, K_{2x}, K_{1y}, K_{2y}, K_{1\theta}, K_{2\theta}\right) + E(K_L \otimes U_n)E^T$$

(5)

where $M_1, M_2, I_1, I_2$ and $M_L$ are mass matrix and moment of inertia of tower 1 and 2, and mass matrix of links, respectively; $K_{rs}$ ($r=1$ or 2, $s=x, y$ or $\theta$) is the stiffness matrix of tower $r$ in direction $s$; $U_n$ is $n \times n$ identity matrix; $\otimes$ is Kronecker tensor product; $E=\text{diag}(E_1, E_2, E_1, E_2, E_1, E_2)$, and $E_i$ is a location matrix to show relative location relationship between links and each tower. Taking tower 1 for example, $E_i=[e_1, e_2, \ldots, e_{k}, \ldots e_n]$, each $e_k$ is a vector as

$$e_k=\{0 \quad \cdots \quad 1 \quad \cdots \quad 0\}^T$$

(6)

here, since one link usually is connected with one story of Tower 1, elements of $e_k$ are zero except the $i$th element with value of one, which mean that the $i$th floor of tower 1 is connected with the $k$th link. In this way, the matrix $E_1$ and $E_2$ can be formed. Now we have obtain the total mass and stiffness matrix, the characteristic equation can be formed and then structural model properties such as frequencies and mode shapes can be calculated directly by eigenvalue analysis.

### 3 Verification of the analytical model

An example of 40-story twin linked buildings with links at the top three floors is chose to illustrate the accuracy of the above analysis model. The mass matrix of each tower is obtained by condensing the mass to the mass center of each floor. And the stiffness matrix of each tower can be calculated by flexibility method based on FEM model. Then the modal analysis can be conducted by both the analytical model and the FEM model. Tab.1 compares natural periods of the two methods. It can be seen that the two results coincide well, in other words, the accuracy of the analytical model is verified. Therefore, we can replace the time-consuming FEM model with the analytical model for subsequent analysis, which can improve efficiency of analysis.

<table>
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<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<tr>
<td>Analytical model</td>
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<td>4.182</td>
<td>3.368</td>
<td>1.779</td>
<td>1.656</td>
<td>1.553</td>
<td>1.553</td>
<td>1.417</td>
<td>1.234</td>
<td>0.743</td>
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<td>4.184</td>
<td>3.334</td>
<td>1.779</td>
<td>1.675</td>
<td>1.553</td>
<td>1.553</td>
<td>1.417</td>
<td>1.229</td>
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<td>-0.02</td>
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<td>-0.03</td>
<td>0.01</td>
<td>0.42</td>
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</tr>
</tbody>
</table>

### 4 Parametric analysis

In order to study effects of properties of link such as stiffness, location and number of links on the modal properties of linked tall buildings, a parametric analysis is conducted. The relationship between frequencies of the first eight modes and the bending stiffness is shown in Fig.3. It can be seen that the stiffness has no effect on the first two periods or frequencies as the first two mode shapes of two towers are symmetrically translational. For the higher mode, the natural periods generally decrease with the increase of stiffness due to the complexity of mode shapes, which means the link can stiffen.
these modes. However, once these modes of two towers become symmetrical, these links have no effect on the period again as the right hand of mode 6 shown in Fig.3. As for mode shapes (results are not shown due to the limit of space), the results show that with the increase of stiffness, the coupling effect within mode between translational motion in direction y and torsional motion will increase. Moreover, with the increase of stiffness of link, lower mode shapes that have relatively simple shape originally will present complex shapes, some even show point of contraflexure, which means the common assumption in high frequency base balance (HFBB) technique, i.e., linear translational mode shape and uniform torsional mode shape may be not appropriate for linked buildings. It should be noted that once the stiffness reaches a large value, the stiffen effects for the linked buildings will hardly increase with the stiffness. Due to the limit of space, the results of sensitivity analysis of other parameters are not shown here.

Figure 3 Relationship between period and bending stiffness

5 Conclusions
An analytical model of linked buildings that can consider different location and number of links is derived to study effects of link on modal properties of linked tall buildings. Then effects of link are investigated by parametric analysis about stiffness, location and number of links, which provide a reference for preliminary design of linked tall buildings and a foundation for subsequent dynamical response analysis.

References

