Modal Analysis for Wind-induced Response of Spatial Structures based on the POD Characteristics of Wind Pressure

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ABSTRACT: Proper Orthogonal Decomposition (POD) is an approach used to derive the most efficient coordinate system for observing individual phenomena. In this paper, considering the fast convergence of the contributions of both the structural vibration modes to the whole structural response and the POD eigenvectors to the wind pressure distribution on the whole surface, order-reduced modal analysis strategy was presented. Based on the POD characteristics of the measured wind pressure filed on the surface of a spherical rigid model in wind tunnel, comparative analysis between the presented order-reduced model analysis strategy and the conventional approach were carried out for a corresponding single-layer reticulated shell as an example, which shows that the improved modal analysis strategy can effectively reduce the order of the structural modal equation group for wind-resistant analysis of spatial structures.

KEYWORDS: modal analysis, POD, wind-induced response, large-span spatial structures.

1 INTRODUCTION

Proper Orthogonal Decomposition (POD) is an approach used to derive the most efficient coordinate system for observing individual phenomena\textsuperscript{[1-3]}. In this paper, considering the time-consuming problem in time-history analysis, order-reduced model analysis strategies were presented based on the fast convergence of the contributions of both the structural vibration modes to the whole structural response and the POD eigenvectors to the wind pressure distribution on the whole surface. With this method, several so-called important modes can be selected out by considering the correlation of structural vibration modes and the proper orthogonal POD modes of the measured wind pressure to reduce the order in conventional modal analysis system. Finally, with the data from the wind tunnel test for a spherical shell model, the presented model analysis strategy and the conventional approach was comparatively investigated, which shows that the improved modal analysis strategy can effectively reduce the order of the structural modal equation group for wind-resistant analysis of large-span spatial structures.

2 TRADITIONAL DYNAMIC AND MODAL ANALYSIS METHODS

Based on the theory of the finite element (FEM) method, the vibration equations for large-span spatial structures subjected to wind loading can be expressed as follows:

\[
[M][\dot{U}] + [C][\dot{U}] + [K][U] = \{F_w(t)\}
\]

where, \([M],[C]\) and \([K]\) are the mass, damping and stiffness matrix, and \([U],[\dot{U}]\) and \([\ddot{U}]\) are the displacement, velocity and acceleration vectors, respectively; \(\{F_w(t)\}\) is the wind load vector.

Suppose the displacement vector can be separated as

\[
[U(t)] = \sum_{j=1}^{N} b_j(t)[\phi_j]
\]
Then, Eq.(1) can be rewritten as a series of single-degree-of-freedom dynamic equations as
\[ m_j \ddot{b}_j + c_j \dot{b}_j + k_j b_j = f_j(t) \] (3)
where, \( m_j \), \( c_j \), \( k_j \) and \( f_j \) are the generalized mass, damping, stiffness and external force, respectively, corresponding to the \( j \)-th modal coordinate. They can be calculated by
\[ \begin{align*}
    m_j &= \{\phi_j\}^T [M] \{\phi_j\}; \\
    c_j &= \{\phi_j\}^T [C] \{\phi_j\}; \\
    k_j &= \{\phi_j\}^T [K] \{\phi_j\}; \\
    f_j &= \{\phi_j\}^T [F_w(t)]
\end{align*} \] (4)

3 PROPER ORTHOGONAL DECOMPOSITION

Based on wind tunnel test, information on the second statistical moments of wind pressure among all the measurement points can be obtained in the form of a covariance matrix, \([C_p]\), as:
\[ [C_p] = [S_p]^T [R_p] [S_p] \] (5)
where, \( S_{pj} = \begin{cases} \sigma_{pi} & i = j \\ 0 & i \neq j \end{cases} \), \( R_{pj} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \), \( \sigma_{pi} \) is the standard deviation of wind pressure process at point \( i \); \( \rho_{pj} \) is the correlation coefficient of wind pressure between point \( i \) and \( j \).

With a proper orthogonal decomposition of the correlation coefficients matrix, we can obtain
\[ [R_p][\Phi] = [\Lambda][\Phi] \] (6)
where, \([\Lambda] = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)\), and \( \lambda_i \) is the square root of the \( i \)-th eigenvalue, \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N \); \([\Phi] = [\phi_1, \phi_2, \ldots, \phi_N]\), and \( \phi_i \) is the corresponding orthonormal eigenvector, \( i = 1, 2, \ldots, N \).

By using the first \( M \) orthonormal POD eigenvectors as a coordinate system and utilizing their orthogonality, expanding wind pressure data to each point on the whole surface can be carried out within an acceptable error\(^[6]\). The expanded wind pressure field can be expressed as
\[ p_i^e(t) = \sum_{m=1}^{M} a_{mi}(t) \phi_{mi}^e, \quad a_{mi}(t) = \sum_{k=1}^{N} p_k(t) \phi_{nk} \Delta s_k / \sum_{k=1}^{N} \phi_{nk}^2 \Delta s_k \] (7)
where, \( \{\phi_{mi}\} \) and \( \{\phi_{mi}^e\} \) is the \( m \)-th original and expanded orthonormal eigenvector, respectively, \( m = 1, 2, \ldots, M \), \( i = 1, 2, \ldots, N \), and \( n \) is the node number of the analysis model; \( a_{mi}(t) \) is the \( m \)-th principal coordinate; \( p_k(t) \) is the measured wind pressure data at point \( k \), \( k = 1, 2, \ldots, N \), \( N \) is the number of measuring points; \( \Delta s_k \) is the representative area of point \( k \).

4 ORDER-REDUCED MODAL ANALYSIS METHODS

4.1 Utilizing the orthogonality of the POD eigenvectors to expand the wind load vector

According to Eq.(7), the external wind load vector, \( \{F_w(t)\} \) in Eq.(1), can be expanded as
\[ F_{w1}(t) = p_i^e(t) A_i = A M \sum_{m=1}^{M} a_{mi}(t) \phi_{mi}^e \] (8)

Approximately, if the difference among the representative area at each measuring point can be ignored, the generalized external force in Eq.(4) can be simplified as follows:
\[ f_j = \{\phi_j\}^T A \sum_{m=1}^{M} a_{jm}(t) \phi_m = A \sum_{m=1}^{M} \alpha_{jm} a_{mi}(t), \quad m = 1, 2, \ldots, M, \quad j = 1, 2, \ldots, N \] (9)
where, \( \alpha_{jm} \) are the modal participation factors for \( a_{mi}(t) = \{\phi_j\}^T \phi_m \).

Therefore, considering the convergence of the POD eigenvectors to the wind pressure distribution on the whole surface, the number of the eigenvectors, \( M \) in Eq.(8), can be reduced.
4.2 Utilizing the orthogonality of the POD eigenvectors to expand the displacement vector

Considering the orthonormality of the vibration modes and the POD eigenvectors, it is also possible to use both of them to describe the final displacement response of the structures. As in Eq.(2), the POD eigenvectors of the measured wind field can be used as

\[ \{U(t)\} = \sum_{j}^{N} b_j(t)\{\varphi_j\} = \sum_{m}^{M} d_m(t)\{\phi_m\} \]

(10)

Thus, Eq.(1) can be rewritten as a series of SDOF equations as in Eq.(3) with \( d_m \) instead of \( b_j \), and \( m_m, c_m, k_m \) and \( f_m \) are the generalized mass, damping, stiffness and external force, respectively, corresponding to \( a_m(t) \), as given in Eq.(4) with \( \{\phi_m\} \) instead of \( \{\varphi_j\} \).

From Eq.(10), \( d_j(t) \) can be obtained based on the orthonormality of the POD eigenvectors as

\[ d_m(t) = \{\phi_m\}^T \sum_{j}^{N} b_j(t)\{\varphi_j\} = \sum_{j}^{N} a_m b_j(t) \]

(11)

5 NUMERICAL EXAMPLES

5.1 Analysis model and wind tunnel tests

A Kiewitt-type single-layer reticulated spherical shell is used as the analysis model in this paper, as shown in Figure 1. \( L=120m, f=40m \). The sections of all elements in the analysis model are assumed as \( \phi 200 \times 8mm \) tube. All the joints on the bottom were assumed fixed. Wind tunnel tests on a scaled model according to the length scale, 1:400, has been conducted with simultaneous measurement of the surface pressure with a sampling frequency of 1000Hz in the boundary layer wind tunnel of Wind Engineering Research Center, Tokyo Polytechnic University [5-6].

5.2 Analysis results for the order-reduced modal analysis methods

From Eq.(8)–(10) it can be seen that, if there are good convergences of the contributions of both the vibration modes to the whole response and the POD eigenvectors to the whole wind pressure field, the coefficients, \( a_{mj} \), should also show a very fast convergence as the orders of the vibration mode or the eigenvector increase. Figure 2 gives the PSDs of the first ten principal coordinates, \( a_n(t) \), and Figure 3 gives the normalized coefficients, \( a_{mj} \), according to different vibration mode considering the POD modes from No.1 to 100 and No.1 to 10 comparatively, which shows the above idea for the reticulated shell. Meanwhile, several so-called important vibration modes can be easily selected out from Figure 3, e.g., the No.2, 4, 6, 8, 10, 13, 34, 51, 55 and 88 modes in this case. Figure 4 gives the typical displacement responses obtained from the presented model analysis strategy (by the dashed line) and the conventional approach (by the continuous line).
Figure 3. Normalized coefficients according to the vibration mode with different number of the POD modes

(a) From No.1 to No.100

(b) From No.1 to No.10

Figure 4. Typical responses obtained from the presented and the conventional model analysis approach

In Figure 4, \( C_{Xu272} \) is the displacement coefficient in the \( X \) direction at joint 272, and \( CZ_{u1} \) is the displacement coefficient in the \( Z \) direction at joint 1. Although only the selected vibration modes and the first ten POD modes have been used in the presented model analysis, the differences between the obtained results are very limited, which can establish the efficiency of the presented model analysis strategy based on the POD characteristics of the wind pressure.

6 CONCLUSIONS

Considering the fast convergence of the contributions of both the structural vibration modes to the whole structural response and the POD eigenvectors to the wind pressure distribution on the whole surface, the presented order-reduced modal analysis strategy in this paper can effectively reduce the order of the structural modal equation grouping in modal analysis for wind-induced response of the spatial structures. Meanwhile, the so-called important vibration modes can be easily selected out. Furthermore, a large-span single-layer reticulated shell was used as an example in this paper to demonstrate the idea and the efficiency based on the wind tunnel tests.

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