Stabilized finite element method for thermal environmental flow in urban area

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ABSTRACT: This paper presents a numerical method for thermal environmental flow in urban area. The $k - \varepsilon$ model based on RANS is used for the basic equation of a flow field. The stabilized finite element method based on the SUPG/PSPG method is employed for the special discretization. The validity of this method is investigated through several numerical examples.

KEYWORDS: Finite Element Method, SUPG/PSPG Method, $k - \varepsilon$ Model, Isothermal Condition, Heat Balance at Ground Surface

1. INTRODUCTION

Currently, the average energy consumption density, in 23 wards, Tokyo, reaches to 40[W/m$^2$] and locally over 120[W/m$^2$]. The heat-island phenomenon becomes a serious social problem in urban area. The major cause of the heat-island phenomenon are based on the exhaust heat from automobile, dwelling house and building, and thermal storage effect by alteration of surface of ground and structures. It is very important to evaluate the thermal environment in urban area for the planning and designing of urban environment. Numerical simulation is one of the powerful tools to evaluate the thermal environment.

This paper presents a numerical simulation method for the evaluation of thermal environmental flow in urban area. The finite element method is employed for the special discretization method since the finite element method is easily applied to the complicated spatial domain. The incompressible Navier-Stokes equation based on the Bussinesq approximation is employed for the basic equation of flow field. The $k - \varepsilon$ model based on RANS is used for the turbulent model. The stabilized finite element method based on the SUPG (Streamline-Upwind / Petrov-Galerkin)
PSPG (Pressure Stabilizing / Petrov-Galerkin) method [1] is employed. For the temporal discretization, the implicit scheme based on the Crank-Nicolson method is employed. The efficiency and validity of this method is investigated by the comparison with the existing experimental and numerical results.

2. BASIC EQUATIONS AND FINITE ELEMENT METHOD

(1) Basic equation of non-isothermal condition

In this study, we deal with the incompressible viscous fluid in non-isothermal condition. The incompressible Navier-Stokes equation based on the Bussinesq approximation is employed for the basic equation of flow field as Eqs. (1) and (2). For the turbulent model, the $k - \varepsilon$ model based on RANS is used. Equation of the turbulence energy, equation of the energy dissipation rate and the equation of energy are expressed as Eqs. (3), (4) and (5), respectively.

Equation of motion;

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x_i} + \frac{2}{3} \frac{\partial k}{\partial x_i} - \frac{\partial}{\partial x_j} \left\{ \left( \nu + \nu_T \right) \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right\} + g\beta (\bar{\theta} - \theta_0) \delta_{ij} = 0 \quad (1)$$

Equation of continuity;

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (2)$$

Equation of turbulence energy;

$$\frac{\partial k}{\partial t} + \bar{u}_j \frac{\partial k}{\partial x_j} - \frac{\nu_T}{\sigma_k} \frac{\partial^2 k}{\partial x_j^2} - P_k + \varepsilon - G_k = 0 \quad (3)$$

Equation of energy dissipation rate;

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}_j \frac{\partial \varepsilon}{\partial x_j} - \frac{\nu_T}{\sigma_{\varepsilon}} \frac{\partial^2 \varepsilon}{\partial x_j^2} - \left( C_{\varepsilon 1} P_k - C_{\varepsilon 2} \varepsilon + C_{\varepsilon 3} G_k \right) \frac{\varepsilon}{k} = 0 \quad (4)$$

Equation of energy;

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{u}_j \frac{\partial \bar{\theta}}{\partial x_j} - \left( \alpha + \frac{\nu_T}{\sigma_{\theta}} \right) \frac{\partial^2 \bar{\theta}}{\partial x_j^2} = 0 \quad (5)$$

$$\nu_T = C_\mu \frac{k^2}{\varepsilon}, \quad P_k = \nu_T \bar{S}_{ij} \bar{S}_{ij}, \quad \bar{S}_{ij} = \sqrt{\frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)^2}, \quad G_k = g\beta \frac{\nu_T}{\sigma_{\theta}} \frac{\partial \bar{\theta}}{\partial x_i} \delta_{ij}$$

where, $\bar{u}_i$, $\bar{p}$ and $\bar{\theta}$ are the flow velocity, pressure and temperature to which the ensemble average operation is performed, respectively, $k$ is the turbulence energy, $\varepsilon$ is the energy dissipation rate, $\nu$ is the kinematic viscosity coefficient, $\nu_T$ is the
eddy kinematic viscosity coefficient, $\rho_0$ is the standard density. $g$ is the gravity acceleration, $\alpha$ is the thermal diffusivity coefficient, $\beta (=1/\theta_0)$ is the volume expansion coefficient, $\theta_0$ is the base temperature. And the constants values are assumed as follows.

$$C_\mu = 0.09, C_{\varepsilon 1} = 1.44, C_{\varepsilon 2} = 1.92, C_{\varepsilon 3} = 1.44$$
$$\sigma_k = 1.0, \sigma_\varepsilon = 1.3, \sigma_\theta = 0.5$$

(2) Stabilized finite element method

We employ the stabilized finite element method based on the SUPG / PSPG method[1] for the spatial discretization of Eqs. (1) and (2), and the SUPG method[1] for the Eqs. (3), (4) and (5). Using the P1 / P1 finite element (equal order linear interpolation for both velocity and pressure), and the finite element equations are obtained. For the temporal discretization for all time-marching basic equations, the Crank - Nicolson method is employed. The velocity of the advection term is approximated by the Adams-Bashforth method. To solve the discretized simultaneous equations, the element-by-element Bi-CGSTAB2 method is applied.

3. NUMERICAL EXAMPLE

In order to investigate the validity of this method, the present method is applied to the stratified flow analysis with a high temperature area on the ground surface. The computed results are compared with the existing experimental and numerical results[2].

(1) Numerical conditions

Fig.1 shows the analytical domain. The finite element division used is $53 \times 17 (x \times y)$. Minimum mesh length is assumed to be 0.1. For the boundary condition, the inlet velocity is assumed to the values obtained from the experimental data. The heat flow rate on the hot panel is assumed to 0.0013. For the numerical conditions, the Reynolds number ($Re$) is $2.9 \times 10^4$, the Prandtl number ($Pr$) is 0.71, the Grashof number ($Gr$) is $1.02 \times 10^9$, the time increment $\Delta t$ is $1.0 \times 10^3$, respectively.

![Fig. 1 Numerical Example](image-url)
Fig. 2 Temperature distribution

Fig. 3 Dimensionless temperature at LineA-D

(2) Numerical result

Fig.2 shows the computed temperature distribution. Fig.3 shows the comparison of the dimensionless temperature at Line A-D. The computed results obtained by the present method are good agreement with the existing experimental and numerical results[2].

4. CONCLUSION

This paper presents a numerical method for thermal environmental flow in urban area by the stabilized finite element method. The validity and efficiency of this method was investigated through the numerical examples. The consideration of the heat balance based on plants and ground surface are left in the future work.

REFERENCES
