Evaluation of turbulent time scale of linear revised k-ε models based on LES data

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ABSTRACT: It is well known that the standard k-ε turbulence model suffers from a serious drawback in overestimating turbulence kinetic energy, k, in the flow with impinging. In order to overcome this drawback, several revised k-ε models have been proposed. Recently, new linear revised k-ε models, namely S, Ω and S-Ω models, were proposed by Nagano & Hattori. The expressions for the turbulent time scale in these new revised k-ε models were modified based on a ‘mixed time-scale’ concept. This study investigated the spatial distribution of turbulent time-scale around a bluff body used in these three linear revised k-ε models, and the performance of these models was examined.

KEYWORDS: mixed time-scale, turbulent time-scale, linear revised k-ε models, large eddy simulation, cube, S-Ω model

1 INTRODUCTION
Large Eddy Simulation (LES) can produce better predictions of the entire flow pattern around a bluff body than the standard k-ε model does, however, it requires much more computational load and causes computational instability[1]. Thus the k-ε model is still favorable due to its cost-effectiveness. However, it is well known that the turbulence kinetic energy, k, is over-predicted at the stagnation point by using the standard k-ε model. As this problem causes erroneous predictions of the flowfields around this region, several revised k-ε models have been proposed in order to overcome this drawback of the standard k-ε model [2]. Recently, new linear revised k-ε models, namely S, Ω and S-Ω models, were proposed by Nagano & Hattori [3]. The expressions for the turbulent time scale in these new revised k-ε models were derived based on the ‘mixed time-scale’ concept. The present authors have examined the performance of these models when applied to flow over hilly terrains and flow around bluff bodies [4,5]. Through these studies, it was confirmed that these new revised k-ε models provided fairly accurate results without significant increase of computational load. However, there remain several points to be revised concerning the expression for turbulent time-scale.

This study investigated the spatial distribution of turbulent time-scale around a cube-shaped building geometry and the new linear revised k-ε models based on the ‘mixed time-scale’ concept were tested in two ways. Firstly, the correct distributions of flow pattern and Reynolds stresses around a cube were obtained by the LES computation. The turbulent time-scale was evaluated directly by the LES results using the linear eddy viscosity modeling, and its spatial distribution around a cube was compared with those given from the expressions adopted in S, Ω
and S-Ω models, which were also evaluated using the LES results, in order to investigate the relative performance of these models. Secondly, RANS computations obtained from these models were compared with the results of wind tunnel experiment and LES in order to examine the prediction accuracy.

2 OUTLINE OF COMPUTATION

2.1 Description of Flowfield

In this study, the accuracy of the new linear revised k-ε models was investigated when applied to simulate the flowfield around a cube-shaped building model placed within the surface boundary layer, for which wind tunnel measurements were reported by Murakami, Mochida and Hayashi [1].

2.2 Outline of computation

The standard Smagorinsky type sub-grid model was applied in LES computation, and the value of the Smagorinsky constant was set to 0.12. The numerical methods and the boundary conditions are given in the appendix. The revised k-ε models adopted in the computations were three different ‘mixed time-scale’ models [3] and the Durbin model[6]. Table 1 summarizes the expressions for time-scale employed in the models.

2.3 Method of the evaluation of the turbulent time scale

Table 2 shows the method for evaluating the turbulent time-scale by LES using the linear eddy viscosity modeling.

3 RESULTS AND DISCUSSION

3.1 Reattachment lengths given by LES

Table 3 compares the predicted reattachment lengths, \(X_R\) (on the roof) and \(X_F\) (behind the cube). The results of LES showed fairly good agreement with the experiment, although \(X_F\) was found slightly larger than the experimental value.

3.2 Distribution of eddy viscosity \(v_t\)

The distribution of eddy viscosity is illustrated in Figure 1. The values of eddy viscosity \(v_t\) were estimated by the results of LES using Eqn.(17) in Table 2. The negative eddy viscosity was observed in the area above the frontal corner of the cube. This negative value of eddy viscosity revealed that the inadequate ability of eddy viscosity model coped with in this region. It was because the Reynolds stress in the linear type revised k-ε models was evaluated from the mean velocity gradient \(S_{ij}\) (cf. Eqn.(15) in Table 2), and in the region near the separation around the front corner, where \(S_{ij} < 0\), the Reynolds stresses were much influenced by the flow property in the upstream region, where \(S_{ij} > 0\), through the effects of advection. Thus, the different sign between the Reynolds stresses given by LES data and that evaluated by the linear eddy viscosity model appeared in this region.

3.3 Distribution of turbulent time scale

Figure 2 shows the longitudinal profiles of the turbulent time-scales in front of the windward
face estimated by Eqns. (2)-(13) in Table 1 using the LES data. The values of the time-scales in the S model and the Durbin model showed better agreement with those evaluated directly using the LES data (Eqn. (18) in Table 2) except for the region near the wall, compared to those obtained by using the Ω and the S-Ω models.

Table 1 Model Equations (expressions for turbulent time-scale of revised k-ε)

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>S model</td>
<td>$T_s = 1/S$</td>
</tr>
<tr>
<td>S-Ω model</td>
<td>$T_\text{S-Ω} = 1/(S + \Omega)$</td>
</tr>
<tr>
<td>Ω model</td>
<td>$T_\Omega = 1/\sqrt{\Omega}$</td>
</tr>
<tr>
<td>Durbin</td>
<td>$T_\text{Durbin} = \min{1, 1/C_\mu \sqrt{\delta}}$</td>
</tr>
</tbody>
</table>

Here, $T$ is the turbulent time scale.

Table 2 The method for the evaluation of the turbulent time-scale using LES data

<table>
<thead>
<tr>
<th>Time Scale</th>
<th>Standard k-ε</th>
<th>S model</th>
<th>Ω model</th>
<th>S-Ω model</th>
<th>Durbin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1/H$</td>
<td>0.7H</td>
<td>1.2H</td>
<td>2.0H</td>
<td>2.0H</td>
<td>1.9H</td>
</tr>
<tr>
<td>$x_2/H$</td>
<td>0.5H</td>
<td>1.0H</td>
<td>2.0H</td>
<td>2.0H</td>
<td>1.4H</td>
</tr>
</tbody>
</table>

Table 3 Reattachment length

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$X_r$</th>
<th>$X_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. [1]</td>
<td>0.7H</td>
<td>1.2H</td>
</tr>
<tr>
<td>LES</td>
<td>0.7H</td>
<td>1.4H</td>
</tr>
</tbody>
</table>

In S, Ω, and S-Ω models, the $T$ (in Eqn.(1)) is replaced by the mixed time-scale $T_m$ given from Eqns.(4),(8) and (12).

In k-ε models, the eddy viscosity, $\nu_t$, is expressed as

$$\nu_t = C_\nu A T$$  \hspace{0.5cm} (1)
$$\tau = k/\epsilon$$  \hspace{0.5cm} (3)

$x$ and $u$: three components of spatial coordinates ($x = 1, 2, 3$: streamwise, lateral, vertical)

$u$: three components of instantaneous velocity vector

$<\cdot>$: ensemble-averaged value of $\cdot$

Figure 1 Vertical distribution of eddy viscosity $\nu_t$ evaluated by LES data

Figure 2 Longitudinal profiles of turbulent time-scales
3.4 Results of computations actually using k-ε models

The predicted reattachment lengths, $X_R$ and $X_F$, of all investigated k-ε models are compared in Table 3. Reverse flow was reproduced on the roof of the cube by all revised k-ε models, except the standard k-ε model. Among the revised k-ε models, the results of S model and Durbin model gave better agreement with experiment. The better prediction accuracy in these two models corresponded to the fact that the distribution of turbulent time-scale near the cube given by these models agreed well with the estimated value using the LES data (cf. Figure 2).

4 CONCLUSIONS

1) The spatial distribution of turbulent time scale was evaluated using LES data via the linear eddy viscosity modeling for the flowfield around a cube within the surface boundary layer.
2) Computations using four different revised k-ε models, which adjusted the turbulent time-scale according to the flow property, were conducted. The reattachment lengths on the roof predicted by the S model and the Durbin model showed better agreement in comparison to the other two revised k-ε models used in this study. This tendency corresponded to the results of the examination using the LES data, on the modeling of the turbulent time-scales around a cube employed in the linear revised k-ε models.

Appendix: The computational domain covered $15.7H(x_1) \times 9.7H(x_2) \times 6.0H(x_3)$. The domain was discretized into $198(x_1) \times 181(x_2) \times 99(x_3)$ grids. A second-order centered difference scheme was adopted for the spatial derivative. For the time advancement, the Adams-Bashforth scheme and Crank-Nicolson scheme were used for the convection terms and diffusion terms respectively. For the inflow boundary, the velocity fluctuations were generated using an artificial generation method proposed by Kataoka & Mizuno [8]. At the outflow boundary, a zero-gradient condition was used. The normal gradients of tangential velocity components and the normal velocity components were set to zero at the upper and side faces of computational domain. For the boundary condition at the solid walls, Werner and Wengle’s approach [9] was adopted, in which a linear or a $1/7$ power law distribution of the instantaneous velocity was assumed.

Note: The original mixed time scale models proposed by Nagano and Hattori were designed to be adopted in the low-Reynolds-number region near the solid surface. However, only the high-Reynolds-number part of the original model was used in this study.

REFERENCES

1 S. Murakami, A. Mochida, Y. Hayashi, Examining the k-ε model by means of a wind tunnel test and large-eddy simulation of the turbulence structure around a cube, J. Wind Eng. Ind. Aerodyn. 35 (1990): 87-100