Two dimensional numerical simulations of vortex-induced vibration responses of H-shaped bridge deck

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ABSTRACT: Two dimensional numerical simulations of vortex-induced vibration response of H-shaped bridge deck with 2DOF was conducted with the techniques of dynamic mesh and UDF in Fluent software. The energy of the vertical vibration mode and torsional vibration mode were also calculated to investigate to the energy transition mechanism from one vibration mode to another vibration mode. Numerical results show that the drag coefficient and Strouhal number of the stationary H-shaped bridge deck agree well with the results reported by Larsen. The lock-in of the vertical vibration mode of the H-shaped bridge deck was found when the reduced wind velocity is in the range from 1.23 to 2.43. The flutter critical reduced wind velocity of the H-shaped bridge deck is about 5.26, which corresponding to flutter critical wind velocity of 12.5m/s for the original H-shaped bridge deck. When the reduced wind velocity is 5.26 and 6.31 separately, the energy of the H-shaped section with 2DOF is dominated by the vertical vibration mode firstly, then the energy of it in the torsional vibration mode increase dramatically and the flutter instability occurred.

KEYWORDS: numerical simulation; vortex-induced vibration; H-shaped bridge deck; energy of vibration mode; dynamic mesh; UDF

1 INTRODUCTION
Vortex-induced vibrations (VIV) are motions induced by the vortex shedding behind the bluff bodies. VIVs can occur in many engineering situations, such as large span bridge decks, offshore structures, and other engineering structures. The famous first Tacoma Narrows Bridge main deck exhibited the vortex-induced vibrations after opening to the traffic, which was finally destroyed by wind (Billah K.Y., et al., 1991). With the development of computational performance of computers, the numerical methods on VIVs of structures with sample geometry shapes, such as circular section, square section and rectangular section are used by many researchers (Murakami S., et al., 1997; Owen J.S., et al., 2007; Placzek A., et al., 2009; Sarwar M.W., Ishihara T., 2010, Liu Z.W., 2011). The aerodynamics and structural responses of four generic cross-section shapes developed from the well-known plate girder section of the first Tacoma Narrows bridge were investigated by Larsen with discrete vortex methods (Larsen A., 1998). The aerodynamic performance of the main deck of the first Tacoma Narrows bridge was investigated by many researchers (Matsumoto M., et al., 2008).

In present paper, a new gird generation method was developed to simulate vortex-induced vibrations of H-shaped bridge deck of Tacoma Narrows Bridge with two-degree-of-freedom (2DOF). The energy of the vertical vibration mode and torsional vibration mode were also calculated to investigate to the energy transition mechanism from one vibration mode to another vibration mode.
2 NUMERICAL METHODS

2.1 Governing equations

The fluid flow computation is in the framework of the finite volume method (FVM). The Reynolds-averaged Navier-Stokes (RANS) equations can be written as

\[
\frac{\partial u_i}{\partial x_i} = 0
\]

\[
\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(2\mu S_{ij} - \rho u_i u_j)
\]

(1) (2)

where \( u, p \) present the time-average value of velocity and pressure; \( \mu \) is the molecular viscosity, and \( S_{ij} \) is the mean stress tensor. The small-scale fluctuations of velocity relating to the turbulence are reduced as \( u'_{ij}, u'_{ij} \), which is referred to as the Reynolds stresses.

The shear stress transport (SST) \( k - \omega \) turbulence model was used in the present work. It models the Reynolds stresses with two transport equations for the turbulent kinetic energy \( k \) and the specific dissipation rate \( \omega \). The SST \( k - \omega \) model shows good performance in predicting the adverse pressure gradient flows. The unsteady segregate solution was adopted in the calculations. The SIMPEC algorithm was used to solve the pressure-velocity coupled equations. The implicit first-order scheme was used for the unsteady terms. The PRESTO algorithm was used for pressure terms. The third-order upwind difference scheme QUICK was applied to the convection terms in the momentum equations and the \( k \) and the \( \omega \) transport equations.

2.2 Modeling of fluid-structure interaction

The elastically mounted cylinder with 2DOF under the vortex-induced forces can be idealized as shown in Figure 1. The differential equations governing the vortex-induced vibration of the 2DOF system with damping are given as follows,

\[
m\ddot{y} + c_y\dot{y} + k_y y = F_y(t)
\]

\[
I_m\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha \alpha = M_\alpha(t)
\]

(3) (4)

where \( m \) is the mass of the structure, \( c_y \) is the vertical damping, \( k_y \) is the vertical stiffness of the structure, \( y \) is the vertical displacement, respectively. \( I_m \) is the inertial moment of mass, \( c_\alpha \) is the torsional damping, \( k_\alpha \) is the torsional stiffness of the structure, \( \alpha \) is the rotational displacement. \( F_y(t) \) is the lift force in \( y \) direction, and \( M_\alpha(t) \) is the torque around the \( z \) axis, which can be obtained by conducting fluid analysis around the structure section.

To analyze the structural response of the two-degree-of-freedom system, the equations of structural motion were integrated using Newmark-\( \beta \) method. The computational flow chart of numerical simulation of VIV of H-shaped bridge deck with 2DOF used in present paper is given in Figure 2.

2.3 Geometry and mesh generation

The original Tacoma Narrows bridge deck was chosen to investigate the wind-induced vibration responses with numerical methods given in present paper. The geometry and structural data of the original Tacoma Narrows bridge deck were taken from the paper by Larsen (Larsen A., 1998). The main parameters of the main deck are given in Table 1.
Figure 1 VIV model, computational region and boundary condition of the H-shaped bridge deck

Grid and boundary conditions

Numerical simulation of flow around the stationary H-shaped bridge deck

Is the lift and torque coefficients is stable?

Yes

Define the dynamic boundaries of the H-shaped bridge deck use the UDFs in Fluent

Solve the equations (1), (2) using SST k-ω model in Fluent

No

Solve the equation (3), (4) separately by Newmark-β method and writing response results to the out.dat file.

Renew the mesh by DEFINE-CG-MOTION(VIV,dt,vel,omega,time,ditime)

Is the response of H-shaped bridge deck stable?

No

Yes

End

Fig. 2 Flow chart of numerical simulation of VIV of H-shaped bridge deck with 2DOF
Table 1 Main parameters of H-shaped deck of 1st Tacoma Narrows Bridge (Larsen A., 1998)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Original value</th>
<th>Scale</th>
<th>Model value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The width of the deck, $B$</td>
<td>m</td>
<td>11.887</td>
<td>1/50</td>
<td>0.23774</td>
</tr>
<tr>
<td>The height of the deck, $H$</td>
<td>m</td>
<td>2.40</td>
<td>1/50</td>
<td>0.048</td>
</tr>
<tr>
<td>The inertial mass per meter of the deck, $m_i$</td>
<td>kg / m</td>
<td>$4.25 \times 10^3$</td>
<td>1/50$^2$</td>
<td>1.70</td>
</tr>
<tr>
<td>The mass per meter of the deck, $m$</td>
<td>kg·m$^2$ / m</td>
<td>$177.73 \times 10^3$</td>
<td>1/50$^4$</td>
<td>0.02844</td>
</tr>
<tr>
<td>The first asymmetric vertical vibration frequency, $f_h$</td>
<td>Hz</td>
<td>0.13</td>
<td>10/1</td>
<td>1.3</td>
</tr>
<tr>
<td>The first asymmetric torsional vibration frequency, $f_\alpha$</td>
<td>Hz</td>
<td>0.20</td>
<td>10/1</td>
<td>2.0</td>
</tr>
<tr>
<td>Ratio of wind velocity, $\lambda$</td>
<td>/</td>
<td>/</td>
<td>1/5</td>
<td>/</td>
</tr>
<tr>
<td>The damping ratio, $\xi$</td>
<td>%</td>
<td>0.005</td>
<td>1.0</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The computational region and boundary conditions of the H-shaped bridge deck are shown in Figure 1. As shown in Figure 1, the whole computational domain was divided into three different blocks, namely, rigid region, buffer region and stationary region. The displacements and velocities of the rigid region are same as that of the H-shaped bridge deck. The region between the rigid region and the stationary region was modeled as buffer region in which the mesh can be renewed at each time step. The turbulence intensity was of the order of 0.5% at the velocity inlet boundary. And the inlet velocity was in the range from 0.38m/s to 3.0m/s, which corresponding to the reduced wind velocity $V / f_\alpha B$ from 1.23 to 9.71. Symmetry condition was used for the top and bottom sides of the computational domain. The outflow condition was used for the right side of the outlet boundary of the computational domain. The no-slip wall condition was used for the H-shaped bridge deck.

A block-structured grid was used with coarser structured mesh in the stationary region, and the unstructured mesh was used for the buffer region between the stationary region and the rigid region of the computational domain, whereas fine structured mesh was used for the rigid region near the H-shaped bridge deck. The height of the first layer of the mesh near the H-shaped bridge deck is $\delta = 0.0017B$. An overview of the grid of the region is shown in Figure 3(a), and the local view of the mesh around the H-shaped bridge deck is shown in Figure 3(b).
3 NUMERICAL RESULTS

3.1 Numerical simulation of flow around the stationary H-shaped bridge deck

To verify the accuracy of the mesh, the numerical simulations of flow around the stationary H-shaped bridge deck were conducted firstly. The inlet velocity was 10 m/s, and symmetry condition was used for the top and bottom sides of the computation domain. The pressure outlet condition was used for the right side of the outlet boundary of the computational domain. The no-slip wall condition was used for the H-shaped bridge deck. The aerostatic coefficients and FFT of the lift coefficients of the H-shaped section are given in Figure 4.

From the Figure 4, it can be seen that the drag coefficient of it is 0.238 with the reference width of B, which is slightly less than the result of \( C_d = 0.28 \) reported by Larsen (Larsen A., 1998). The Strouhal number is about \( St = fH/V = 0.1219 \), which is slightly larger than the result of \( St = 0.11 \) given by Larsen (Larsen A., 1998).

3.2 Numerical simulation of wind-induced vibration of the oscillating H-shaped bridge deck

The numerical simulations of wind-induced vibration of the H-shaped section with 2DOF were conducted with the method given in present paper under the wind velocity of from 0.38 to 3.0 m/s. Fig. 5 gives the time history of wind-induced vibration responses of the H-shaped bridge deck of the first Tacoma Narrows bridge for the wind velocity of 0.57 m/s, 0.75 m/s, 2.50 m/s, and 3.0 m/s.

(a) \( V_m = 0.57 \text{ m/s} \)

(b) \( V_m = 0.75 \text{ m/s} \)
The wind-induced vibration responses of the H-shaped bridge deck with 2DOF are given in Figure 6. From the Figure 6(a), it can be seen that the VIV of the heaving vibration mode of the H-shaped bridge deck was found, and the lock-in reduced wind velocity located from 1.23 to 2.43, which corresponding to the wind velocity from 1.90 to 3.75 m/s for the prototype bridge. The maximum RMS value of the non-dimensional displacement is about 0.04 when the wind velocity for the numerical model is 0.57 m/s. At this wind velocity, the maximum amplitude is about 0.06H (H is the height of the H-shaped section). As shown in Figure 6(b), the torsional vibration angle of the H-shaped bridge deck increasing suddenly when the wind velocity is about 2.5 m/s, which means that the flutter critical wind velocity is about 2.5 m/s for the numerical model. The flutter critical wind velocity is about 12.5 m/s for the original Tacoma Narrows bridge deck, which is less than the wind velocity of 18.7 m/s (Matsumoto M., et al., 2008).

3.3 Energy transformation of VIV of the H-shaped bridge deck

The energy transformation from one vibration mode to another one was calculated to investigate the mechanism of VIV of the H-shaped bridge deck with 2DOF. The energy of the vertical vibration mode and rotation vibration mode of the VIV of the H-shaped bridge deck are defined as follows,
\[ E_1 = \frac{(my^2 + ky^2)}{2.0} \]  
\[ E_2 = \frac{(I_m\alpha^2 + k_a\alpha^2)}{2.0} \]

where \( m \) is the mass structure, \( k \) is the spring stiffness of the bridge deck in vertical direction, \( y \) and \( \dot{y} \) are the vertical vibration displacement and velocity separately, \( E_1 \) is the energy of the structure system of the vertical vibration mode. \( I_m \) is the inertial moment of the mass of the bridge deck, \( k_a \) is the spring stiffness in torsional direction of the bridge deck, \( \alpha \) and \( \dot{\alpha} \) are the rotation angle and angular velocity separately, \( E_2 \) is the energy of the structure system of the torsional vibration mode.

The total energy of the VIV of the H-shaped bridge deck is given as follows,

\[ E = E_1 + E_2 \]

where \( E \) is the total energy of the H-shaped bridge deck with 2DOF.

The time history of energy of the vertical vibration mode and rotation vibration mode of the H-shaped bridge deck for wind velocity of 0.51m/s to 3.0m/s are given in the Figure 7.

As shown in Fig.7, the energy of the H-shaped bridge deck with 2DOF is mainly dominated by the vertical vibration mode when the wind velocity is in the range from 0.38 to 0.63m/s. While the energy of it is mainly dominated by the torsional vibration mode when the wind velocity is in the range from 0.75 to 1.0m/s. The energy of the H-shaped bridge deck with 2DOF is mainly dominated by the vertical vibration mode when the wind velocity is in the range from 1.2 to 2.0m/s. When the wind velocity is 2.5m/s and 3.0m/s separately, the energy of the H-shaped bridge deck with 2DOF is dominated by the vertical vibration mode firstly, then the energy of it in the torsional vibration mode increase sharply and the flutter instability occurred.
4 CONCLUSIONS

The gird generation method and UDF (User defined function) program developed in present paper was used to simulate the wind-induced vibration of the H-shaped bridge deck with 2DOF. The wind-induced vibration responses of the H-shaped bridge deck with 2DOF were calculated to investigate the VIV responses and flutter instability of it. Numerical results show that the drag coefficient and Strouhal number of the stationary H-shaped bridge deck agree well with the results reported by Larsen. The lock-in of the vertical vibration mode of the H-shaped bridge deck was found when the reduced wind velocity is in the range from 1.23 to 2.43. The flutter critical reduced wind velocity of the H-shaped bridge deck is about 5.26, which corresponding to flutter critical wind velocity of 12.5m/s for the original H-shaped bridge deck. When the reduced wind velocity is 5.26 and 6.31 separately, the energy of the H-shaped section with 2DOF is dominated by the vertical vibration mode firstly, then the energy of it in the torsional vibration mode increase sharply and the flutter instability occurred.

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