Coupled vibration between wind-induced internal pressures and large span roof for a two-compartment building with openings

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ABSTRACT: The equations governing wind-induced internal pressure responses for a two-compartment building are derived based on some reasonable assumptions when the roof responds in dynamic and quasi-static manners, respectively. The gain functions, power spectra, and RMS values of internal pressure coefficients in both compartments are numerically analyzed. The results show that the dynamic model must be used to calculate the responses of internal pressure when the roof is flexible, while the simplified quasi-static model can be used for rigid roof. With increased roof natural frequency, all the three resonance frequencies of the “internal pressures - roof” coupled system increase; the peak internal pressure response also increases at the first two frequencies, but decreases at the third resonance frequency; the RMS internal pressure coefficients also increase, but their difference decreases.

KEYWORDS: building with internal partitioning and openings, wind-induced internal pressure, roof flexibility, governing equations, gain functions, coupled system.

1 INTRODUCTION

The dominant openings might be created due to either impact of flying debris or failure of windows and doors in severe wind storms. In the laminar uniform wind, an initial internal pressure overshoot which is higher than the mean external pressure at the opening will be produced when a sudden opening is created, then the response of internal pressure will exhibit damped oscillation, and it finally will keep constant with the external pressure. However, the overshoot phenomenon existed in smooth flow is lost amidst fluctuating turbulent wind in real life [1]. The steady-state internal pressure response for a building with a dominant opening can also be described with the Helmholtz acoustic resonator model. The resonance phenomenon will be occurred if there is sufficient energy at the Helmholtz frequency for the external pressure at the opening. So it will have a profound effect on building safety.

The effect of structure flexibility on internal pressure has been studied a lot for the single-cell building with a dominant opening. Vickery [2] treated the building deflections as quasi-static and suggested an effective volume method to consider the effect of building flexibility. Sharma and Richards [3] derived an equation governing the internal pressure for a building with a dominant opening and flexible roof, but the effect of external pressure on the roof movement failed to be considered. Yu et al [4] further considered the joint action of external and internal pressures on roof, and presented a linear internal pressure model. Sharma [5] derived an equation governing the response of internal pressure in any flexible building. Furthermore, a non-linear internal pressure governing equation was presented when the roof structure responds in a quasi-static manner, and the roof is under the joint action of external and internal pressures. Guha [6] derived an equation governing the internal pressure for a building with a dominant opening and flexible roof, however only considered the effect of internal pressure on roof. The studies of wind-induced internal pressures for a two-compartment building are much less compared with the single-cell building. Sharma [7] presented linearized equations governing the internal pressures for
a rigid and non-leaky two-compartment building. Yu et al [8] developed non-linear governing equations by considering the effect of background leakage. However, there are no theoretical and experimental studies on the effect of roof flexibility.

In this paper, the equations governing wind-induced internal pressure responses for a two-compartment building when the roof responds in dynamic and quasi-static manners is derived. Then, the gain functions, roof displacement, power spectra and RMS values of internal pressures are numerically analyzed.

2 GOVERNING EQUATIONS FOR A FLEXIBLE BUILDING

2.1 Basic assumptions

The object is a two-compartment building with flexible roof. There is an opening in the windward wall and an opening in the partition wall, and there are no background leakages. Besides, some reasonable assumptions are made as follows:

(i) The wall is rigid, while the roof is flexible.

(ii) The partition wall only plays a role of wind shield, but it doesn’t provide support to the roof.

(iii) The roof behaves linearly.

(iv) The roof will produce a tiny rotation angle in theory for the fluctuating internal pressure in the compartment without an external opening is a little higher than that in the other one. However, it can be ignored for the roof is large and the wall is rigid.

(v) Internal pressures are completely relevant in their own compartments. This is the precondition for the theory of the internal pressure response and has already been validated by experiments [9].

(vi) The adiabatic law is applied for air in both compartments. It means the state equation governing the variation of the internal air pressure and density becomes:

\[ \frac{P_i}{\rho_i} = C \]  

where \( \gamma \) is the specific heat ratio for adiabatic air and is generally equal to 1.4, \( C \) is a constant, and \( P_i, \rho_i \) are the internal air pressure and density, respectively. After differentiating both sides of equation (1) and simplifying it, then:

\[ \frac{dP_i}{\rho_i} = \frac{\gamma P_a}{\rho_i} \frac{d\rho_i}{\rho_i} \]  

where \( P_a \) is the atmospheric pressure, which is the steady-state internal pressure.

2.2 Governing equations

The simplified model of internal pressures for a two-compartment building with flexible roof and openings is shown in Fig. 1. \( P_w, P_{i1} \) and \( P_{i2} \) are the transient wind pressures of windward wall, compartment 1 and compartment 2, respectively. \( L_{o1} \) and \( L_{o2} \) are the effective lengths of “air slugs” in the opening 1 and 2. \( A_{o1} \) and \( A_{o2} \) are the areas of the opening 1 and 2. \( \forall_{o1} \) and \( \forall_{o2} \) are the internal volumes of compartment 1 and 2, \( \forall = \forall_{o1} + \forall_{o2} \). \( c_1 \) and \( c_2 \) are the flow coefficients at the opening 1 and 2. \( m_r, k_r \) and \( \zeta_r \) are the mass, stiffness and damping ratio of the roof, respectively. \( A_{r1} \) and \( A_{r2} \)
are the roof areas of compartment 1 and 2, \(A_r = A_{r1} + A_{r2}\). \(x_1\) and \(x_2\) are the displacements of “air slugs” in the opening 1 and 2, \(x_r\) is the displacement of flexible roof.

According to the unsteady form of the Bernoulli equation [2], the relationship between the airflow rate and the transient pressure drop for opening 1 is given by:

\[
\rho_0 L c_1 A_{oi} \dot{x}_1 + \frac{1}{2} \rho_0 c_1 A_{oi} C_{l1} \dot{x}_1 = c_1 A_{oi} q_0 (C_{pi} - C_{pi0}) \quad (3)
\]

Similarly, the relationship between the airflow rate and the transient pressure drop for opening 2 is given by:

\[
\rho_0 L c_2 A_{oi} \dot{x}_2 + \frac{1}{2} \rho_0 c_2 A_{oi} C_{l2} \dot{x}_2 = c_2 A_{oi} q_0 (C_{pi} - C_{pi2}) \quad (4)
\]

where \(C_{pi} = P_i / q_0\) and \(C_{pi2} = P_{i2} / q_0\) are the transient internal pressure coefficients of compartment 1 and compartment 2, \(C_{pw} = P_w / q_0\) is the transient external pressure coefficient on the windward wall. \(\rho_0\) is the air density, \(q_0 = \frac{1}{2} \rho_0 U_0^2\) is the reference dynamic pressure of the approaching flow.

The changes in volumes of both compartments are:

\[
\Delta \forall_1 = \forall_1 - \forall_{01} = A_{r1} x_r - c_1 A_{oi} x_1 + c_2 A_{oi} x_2 \quad (5)
\]

\[
\Delta \forall_2 = \forall_2 - \forall_{02} = A_{r2} x_r - c_2 A_{oi} x_2 \quad (6)
\]

Then, according to the assumption (vi):

\[
-q_0 \frac{\Delta C_{pi}}{\Delta \forall} \approx -q_0 \frac{dC_{pi}}{d\forall} \frac{P_{i}}{P_{i0}} \frac{\dot{\forall}}{\forall_0} \quad (7)
\]

Substitution of equations (5) and (6) into equation (7), and let \(\nu = \frac{\forall}{\forall_0} = 1 + \frac{x}{H}, \dot{\nu} = \frac{\dot{x}}{H}, \ddot{\nu} = \frac{\ddot{x}}{H}\), there are:
\[ C_{p1} = \frac{\gamma P_a}{q_0 \gamma_0} \left[ c_1 A_{o1} x_1 - c_2 A_{o2} x_2 - A_i H(\nu - 1) \right] \]  
\[ C_{p2} = \frac{\gamma P_a}{q_0 \gamma_0} \left[ c_2 A_{o2} x_2 - A_2 H(\nu - 1) \right] \]

where \( H \) is the ridge height of the building.

Simultaneously solving equations (8) and (9) results in:

\[ x_1 = \frac{q_0 C_{p1} \gamma_0 + q_0 C_{p2} \gamma_0}{\gamma P_a c_1 A_{o1}} + \frac{A_1 H(\nu - 1)}{c_1 A_{o1}} \tag{10} \]

\[ x_2 = \frac{q_0 C_{p2} \gamma_0}{\gamma P_a c_2 A_{o2}} + \frac{A_2 H(\nu - 1)}{c_2 A_{o2}} \tag{11} \]

Derivation of both sides of equations (10) and (11) leads to:

\[ \dot{x}_1 = \frac{q_0 \dot{C}_{p1} \gamma_0 + q_0 \dot{C}_{p2} \gamma_0}{\gamma P_a c_1 A_{o1}} + \frac{A_1 H \dot{\nu}}{c_1 A_{o1}} \tag{12} \]

\[ \dot{x}_2 = \frac{q_0 \dot{C}_{p2} \gamma_0}{\gamma P_a c_2 A_{o2}} + \frac{A_2 H \dot{\nu}}{c_2 A_{o2}} \tag{13} \]

\[ \ddot{x}_1 = \frac{q_0 \ddot{C}_{p1} \gamma_0 + q_0 \ddot{C}_{p2} \gamma_0}{\gamma P_a c_1 A_{o1}} + \frac{A_1 H \ddot{\nu}}{c_1 A_{o1}} \tag{14} \]

\[ \ddot{x}_2 = \frac{q_0 \ddot{C}_{p2} \gamma_0}{\gamma P_a c_2 A_{o2}} + \frac{A_2 H \ddot{\nu}}{c_2 A_{o2}} \tag{15} \]

Substitution of equations (12~15) into equations (3~4) yields:

\[ \frac{\rho_a L_{e1} \gamma_0}{\gamma P_a c_1 A_{o1}} \ddot{C}_{p1} + \frac{\rho_a L_{e2} \gamma_0}{\gamma P_a c_2 A_{o2}} \ddot{C}_{p2} + \frac{\rho_a L_{e3} \gamma_0}{c_3 A_{o3}} \ddot{\nu} + \frac{\rho_a C_{e1}}{2(c_1 A_{o1})^2} \left[ q_0 \gamma_0 A_{o1} \ddot{C}_{p1} + q_0 \gamma_0 A_{o2} \ddot{C}_{p2} \right] + \frac{\rho_a C_{e2}}{2(c_2 A_{o2})^2} \left[ q_0 \gamma_0 A_{o2} \ddot{C}_{p2} \right] + \frac{\rho_a C_{e3}}{2(c_3 A_{o3})^2} \ddot{\nu} + \frac{\rho_a}{\gamma P_a} \left( q_0 \gamma_0 \dot{C}_{p1} + q_0 \gamma_0 \dot{C}_{p2} + \dot{\nu} \right) + C_{p1} = C_{pw} \tag{16} \]

\[ \frac{\rho_a L_{e2} \gamma_0}{\gamma P_a c_2 A_{o2}} \ddot{C}_{p2} + \frac{\rho_a L_{e2} \gamma_0}{c_2 A_{o2}} \ddot{\nu} + \frac{\rho_a C_{e2} \gamma_0}{2(c_2 A_{o2})^2} \left[ q_0 \gamma_0 \ddot{C}_{p2} \right] + \frac{\rho_a}{\gamma P_a} \left( q_0 \gamma_0 \dot{C}_{p2} + \dot{\nu} \right) + C_{p2} - C_{p1} = 0 \tag{17} \]

Equations (16~17) are the general equations governing the internal pressure responses for a two-compartment building with flexible roof.
When the roof of the building is rigid, i.e. $\nu = 1$, $\dot{\nu} = 0$ and $\ddot{\nu} = 0$, the equations (16~17) can be simplified as follows:

$$\frac{\rho L_1}{\gamma P_a c_1 A_{o1}} C_{p1} + \frac{\rho L_2}{\gamma P_a c_1 A_{o2}} C_{p2} + \frac{\rho C_{f1} q_0 V_{o1}}{2(\gamma P_a c_1 A_{o1})^2} \left( \ddot{C}_{p1} + \frac{\dot{V}_{o1}}{V_{o1}} \dot{C}_{p1} \right) - \frac{\rho C_{f2} q_0 V_{o2}}{2(\gamma P_a c_2 A_{o2})^2} \left( \ddot{C}_{p2} + \frac{\dot{V}_{o2}}{V_{o2}} \dot{C}_{p2} \right) + C_{p1} = C_{PW}$$  (18)

$$\frac{\rho L_2}{\gamma P_a c_2 A_{o2}} C_{p2} + \frac{\rho C_{f2} q_0 V_{o2}}{2(\gamma P_a c_2 A_{o2})^2} \left( \ddot{C}_{p2} + \frac{\dot{V}_{o2}}{V_{o2}} \dot{C}_{p2} \right) - C_{p1} = 0$$  (19)

If the damping coefficients in equations (18) and (19) are linearized, they will become the internal pressure governing equations presented by Sharma [7]. Thus, the Sharma’s governing equations [7] are a special case of the equations (16) and (17) derived in this paper.

2.2.1 Responses of internal pressure when the roof responds in a dynamic manner

In reality, many large span industrial building, their component structures, in particular the roof, exhibit greater flexibility than the wall. So they will behave in a dynamics manner in strong wind. As for a two-compartment building shown in Fig. 1, the response of the roof to the internal pressures will be governed by:

$$m_2 \ddot{x}_r + 2m_2 \zeta_2 \omega \dot{x}_r + m_2 \omega^2 x_r = A_1 q_0 C_{p1} + A_2 q_0 C_{p2}$$  (20)

The non-dimensional form of the equation (20) is:

$$\ddot{\nu} = \frac{q_0 (A_1 C_{p1} + A_2 C_{p2})}{m_H} - 2\zeta_2 \omega \nu - \omega^2 (\nu - 1)$$  (21)

Equations (21) and (16~17) are the dynamic model of internal pressures for a two-compartment building with flexible roof. The equation (21) can be solved simultaneously with equations (16~17) to obtain the coupling responses of internal pressures and roof for specified parameter conditions.

2.2.2 Responses of internal pressure when the roof responds in a quasi-static manner

The dynamics model (Equations (21) and (16~17)) is a second-order differential equations, and it is hard to be solved for its complexity.

When the natural frequency of the roof is larger than the first expected Helmholtz frequency, the internal pressure responses are dominated by average and background components, while the resonance component can be ignored for its little contribution. Thus, the roof deflection is proportional to the applied load at all times [2], i.e. a quasi-static manner [3].

$$\nu = 1 + \frac{q_0 (C_{p1} + C_{p2})}{k_b}$$  (22)

where $k_b = (2\pi f_0)^2 \nu \rho m / A_r^2$ is the building bulk modulus defined as the ratio of increase in pressure to volumetric strain.

Substitution of the time derivation of $\nu$ into equations (16~17), and let $b = \gamma P_a / k_b$, yields:
\[
\frac{\rho_c L_{c1}(\varphi_{o1} + b\varphi_{o0})}{\gamma P_c A_{o1}} \dot{C}_{p1} + \frac{\rho_c L_{c1}(\varphi_{o2} + b\varphi_{o0})}{\gamma P_c A_{o1}} \dot{C}_{p2} + \frac{\rho_c L_{c1} q_0}{2(\gamma P_c A_{o1})^2} \left( \varphi_{o1} + b\varphi_{o0} \right) \dot{C}_{p1} + \left( \varphi_{o2} + b\varphi_{o0} \right) \dot{C}_{p2} \right] + C_{p1} = C_{pw}
\]

\[
\frac{\rho_c L_{c2} \varphi_{o2}(1+b)}{\gamma P_c A_{o2}} \left( \dot{C}_{p2} + \frac{b}{1+b} \dot{C}_{p1} \right) + \frac{\rho_c L_{c2} q_0 \varphi_{o2}^2 (1+b)^2}{2(\gamma P_c A_{o2})^2} \left( \dot{C}_{p2} + \frac{b}{1+b} \dot{C}_{p1} \right) - C_{p1} + C_{p2} = 0
\]

Equations (23–24) are the quasi-static model of internal pressures for a two-compartment building with flexible roof. It is actually an approximate model of the dynamic model when the natural frequency of roof is considerably larger than the lower Helmholtz frequency.

3 NUMERICAL EXAMPLE
Take a two-compartment building as an example to analyze the dynamic and quasi-static responses of internal pressure. The two-compartment building is similar as the TTU test building, whose basic parameter values are as follows:

\[
\begin{align*}
\varphi_{o1} &= 200m^3, & \varphi_{o2} &= 300m^3, & A_{o1} &= 2m^2, & A_{o2} &= 2m^2, & L_{c1} &= \sqrt{\pi A_{o1}}/4, & L_{c2} &= \sqrt{\pi A_{o2}}/4, \\
C_{1} &= 0.6, & c_2 &= 0.6, & C_{L1} &= 1.2, & C_{L2} &= 1.2, & U_0 &= 30m/s, & \gamma &= 1.4, & P_a &= 101300Pa, & \\
\rho_a &= 1.22kg/m^3, & A_1 &= 50m^2, & A_2 &= 75m^2, & H &= 4m, & m_1 &= 3500kg, & \zeta &= 0.1, & \omega &= 2\pi f_r, & \\
f_r &= 3Hz, 5Hz, 7Hz, 10Hz.
\end{align*}
\]

The time history of wind speed can be simulated by the superposition of the harmonic [10] based on Davenport spectrum, and then the time history of external pressure coefficient at the windward opening can be further obtained according to the quasi-steady theory. The responses of internal pressure can be calculated by inputting the external pressure coefficient obtained in last step into the governing equations derived in the present paper. The simulated wind speed and its verification are shown in Fig. 2.

![Figure 2. (a) Simulated time-history wind speed and (b) its verification](image)

The gain functions of internal pressures calculated from the dynamic and quasi-static models for different roof flexibilities are shown in Fig. 3, in which Figs 3a,b,c,d are the results of compartment 1 and Figs 3e,f,g,h are the results of compartment 2. The two-compartment building
with flexible roof and openings is similar to a coupled dynamic system which contains three “mass-spring-damping” subsystems. It can be seen from Fig.3 that three resonance peaks are evident from dynamic analysis while only two resonance peaks are obtained from quasi-static analysis. Furthermore, the first two resonance frequencies obtained from dynamic analysis is the same as that obtained from quasi-static analysis. Besides, the internal pressure response in compartment 2 is higher than that in compartment 1 calculated from both models. However, the third resonance peak gradually disappears when the roof natural frequency gradually becomes larger, and the gain function obtained from the dynamic model is gradually consistent with that from quasi-static model.
The gain functions of internal pressures in both compartments, the roof displacement over external pressure at the windward opening calculated from dynamic model for different roof natural frequencies are shown in Fig.4. It is shown from Figs 4a,b that three resonance frequencies of the “internal pressure - roof” coupled system increase with increased roof natural frequency. The first two resonance peaks of internal pressure are also increase with increased roof natural frequency, while the third resonance peak decreases and finally disappears. It is shown from Fig 4c that the restoring force of the roof increases with increased roof natural frequency at the first two resonance frequencies, and the restoring forces at the first resonance frequency are much larger than these at the last two resonance frequencies.

The power spectra of internal pressure coefficients in both compartments obtained from the dynamic model for different roof flexibilities are shown in Fig.5. It is shown that the resonance energy of internal pressure in compartment 2 at the first resonance frequency is higher than that in compartment 1 for all kinds of roof flexibilities. The resonance energies of internal pressure in both compartments at the third resonance frequency decrease with increased roof natural frequency. This is in agreement with the analytical results of Fig.3.

The RMS internal pressure coefficients in both compartments and RMS roof displacement for different roof flexibilities are list in Table 1. It is shown that the RMS internal pressure coefficient in compartment 2 is large than that in compartment 1. The RMS internal pressure coefficients obtained from both models increase with increased roof natural frequency, but their difference decreases. It is because with increased roof natural frequency, i.e. the roof flexibility gradually decreases, the additional damping also decreases. Moreover, the RMS value of the roof displacement decreases with increased roof natural frequency.
Figure 5. Power spectra of internal pressure coefficients in both compartments

Table 1. RMS internal pressure coefficients in both compartments and RMS roof displacement for different roof flexibilities

<table>
<thead>
<tr>
<th>Roof frequency (Hz)</th>
<th>$\tilde{C}_{pm}$ (Dynamic analysis)</th>
<th>$\tilde{C}_{pm}$ (Quasi-static analysis)</th>
<th>$\bar{x}_r$ (Dynamic analysis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Compartment 1</td>
<td>Compartment 2</td>
<td>Compartment 1</td>
</tr>
<tr>
<td>3 Hz</td>
<td>0.236</td>
<td>0.250</td>
<td>0.233</td>
</tr>
<tr>
<td>5 Hz</td>
<td>0.243</td>
<td>0.262</td>
<td>0.242</td>
</tr>
<tr>
<td>7 Hz</td>
<td>0.247</td>
<td>0.269</td>
<td>0.246</td>
</tr>
<tr>
<td>10 Hz</td>
<td>0.251</td>
<td>0.274</td>
<td>0.249</td>
</tr>
</tbody>
</table>

4 CONCLUSIONS

As for a two-compartment building with flexible roof and openings, the equations governing wind-induced internal pressure responses are derived based on some reasonable assumptions when the roof responds in dynamic and quasi-static manners. The unsteady form of the Bernoulli equation, the law of mass conservation, and adiabatic equation are used for the derivation. A two-compartment building similar to the TTU test building is taken as an example to analyze the characteristics of gain functions, power spectra and RMS values of the internal pressure coefficients and roof displacement. Some main conclusions are drawn:

1) The governing equations derived in the present paper can be used to analyze the internal pressure responses for a two-compartment building with flexible roof and openings. The dynamic model must be used to calculate the responses of internal pressure when the roof is flexible, otherwise, the simplified quasi-static model can be instead.
(2) The first two resonance frequencies of the “internal pressure - roof” coupled system obtained from the dynamic model is consistent with that obtained from the quasi-static model. The internal pressure response in compartment 2 is larger than that in compartment 1. When the roof natural frequency gradually becomes larger, the gain functions obtained from the dynamic model are gradually in accord with these from the quasi-static model.

(3) With increased roof natural frequency, the three resonance frequencies increase, and the first two resonance peaks of internal pressure are also increase, while the third resonance peak decreases and finally disappears. Besides, the restoring forces of the roof at the first two resonance frequencies also increase with increased roof natural frequency.

(4) The RMS internal pressure coefficient in compartment 2 is large than that in compartment 1. The RMS internal pressure coefficients obtained from both the dynamic and quasi-static models increase with increased roof natural frequency, but their difference decreases. Moreover, the RMS value of the roof displacement also decreases with increased roof natural frequency.

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