Flutter and Its Application - Flutter mode and Ship Navigation

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1 INTRODUCTION

Galloping, Torsional flutter and coupled flutter, all of these are here classified as flutter, have been widely studied by numerous researchers. Their generation mechanisms have been clarified, through previous studies. These flutter phenomena are summarized from author’s viewpoints. Galloping is classified into unsteady and quasi-steady galloping, and into flow reattachment type galloping and flow non-reattachment type galloping. (Matsumoto and Laneville[2011], Matsumoto[2011]) In these, it is described that inner circulatory flow and separation bubble plays essentially important role for satisfactory of den Hartog Criteria[1956], \( \frac{dCF}{\alpha} < 0 \). Furthermore, inclined cable aerodynamics, those are rain-wind induced vibration and dry galloping, which have become greatly concerns to cable-stay bridge design0, are briefly explained from their generation point of view. On torsional flutter, pointing out important role of separation bubble, it is explained that the change of flutter of rectangular cylinder with different side ratio, B/D, from torsional flutter to coupled flutter occurs continuously with B/D, bounded approximately with B/D=10. (Matsumoto[1997]) Analogy of aerodynamic derivatives between H-shaped cylinders and rectangular cylinders is introduced (Matsumoto[2004]). Related to Tacoma Narrows Bridge failure, on sudden change of oscillation mode, from 5th symmetrical heaving mode to 1st asymmetrical torsional mode, another possible cause of increasing wind velocity is shown through wind tunnel test (Matsumoto[2004]). On coupled flutter, step-by-step (SBS) flutter analysis is introduced in contrast to conventional complex eigen value flutter analysis (Matsumoto[2010]). Classification of flutter branch and fundamental flutter mode is explained. Furthermore, on similar Selberg formula[1961] to estimate coupled flutter critical velocity for thin plate, is driven basing on simple assumption (Matsumoto[2010]). As application of knowledge of flutter, fundamental flutter mode, that is H-90 mode, definitely important role to produce propulsion force. A primary ship navigation test has been carried out and generation mechanism of propulsion force by flapping plate has been discussed. Navigation system by flapping plate might have significantly advantage of saving power because of intelligent utilization of flutter power generation system in flapping plate (Matsumoto and Ishizaki[2010]).

2 GALLOPING

2.1 Non-flow Reattachment Type Galloping

Galloping can be classified into “Quasi-steady Galloping” and “Unsteady Galloping” (Nakamura and Hirata[1996] and Matsumoto[2008]). “Quasi-steady Galloping” which is 1DOF heaving flutter, has been known to be catastrophic fluid-induced vibration, and negative slope associated with pitching angle of lift force coefficient, that is \( \frac{dCF}{\alpha} < 0 \), is definite condition for its excitation, as den Hartog Criteria[1956]. A numerous studies on quasi-steady galloping have been carried out to verify its generation mechanism from the point of flow fields
around bluff bodies. As summary, the characteristic flow fields related to appearance of negative lift slope, \( \frac{dC}{d\alpha} < 0 \), are classified into two different ones, that is inner-circulatory flow appearance on side surface of bluff body, which is originally proposed by Bearman[1972], and separation bubble formation on side surface of body. On former flow field, Nakamura[1981] lately pointed out “flow reattachment type pressure distribution flow”. For 2D rectangular cylinders with pitching angle of \( 0^\circ \), quasi-unsteady galloping caused by this flow field can be observed at the range of side-ratio, \( B/D \) (\( B \): along-wind length, \( D \): cross-wind length) between 0.75 and 2.8 in smooth flow (see Fig.1).

\[ \text{Matsumoto[2006]} \]

\[ \text{Nakamura[1996]} \]

Fig.1. \( \frac{dC}{d\alpha} \)-B/D Diagrams of Rectangular Cylinders

2.2 Flow Reattachment Type Galloping

On the later flow field, stalling phenomena of airfoil at stalling pitching angle has been well known. McAlister and Carr[1978] has shown clear visualized separated vortex and vortex patch during forced torsional motion of airfoil at stalling angle and Shimizu, Nishihara and Ishihara[2006], and Matsumiya, Nishihara and Shimizu[2010] studied on galloping on transmission line with snow, and indicated that flow reattachment and formation of separation bubble should excite galloping instability at critical pitching angle, as shown in Fig.2.
(a) Visualized Flow by CFD (Shimzu, Oka and Ishihara [2006])

(b) CL-α and CD-α Diagrams

(c) Flow Reattachment and Formation of Separation Bubble at α=20° and α=154° corresponding Galloping Appearance (Matsumiya, and Shimizu [2010])

Fig 2. Quasi-Steady Galloping of Transmission Line caused by Flow Reattachment and Formation of Separation Bubble (from Shimizu, Oka, Ishihara [2006] and Matsumiya, Nishihara and Shimizu [2010])
2.3 Unsteady Galloping

As far as “unsteady Galloping”, Nakamura and Hirata [1994] reported “Low Speed Galloping (LSG)” for bluffer rectangular cylinders with less than B/D = 0.75, which appeared at lower reduced velocity range than \( \text{Fr} = 1/\text{St} \). These bluffer cylinders do not show \( \frac{dC_L}{d\alpha} < 0 \), but \( \frac{dC_L}{d\alpha} > 0 \) as shown in Fig.1. Hirata [1993] precisely measured the unsteady pressure on side surface during LSG, and found the appearance of “reattachment type pressure” on side surface. Therefore, LSG must be unsteady galloping, which cannot be excited by den Hartog criteria. Appearance reduced Ranges of conventional quasi-steady galloping (Nakamura [1994] named it as High Speed Galloping (HSG) for contrast to LSG, and LSG of rectangular cylinders are illustrated as show in Fig.3 (by Nakamura [1994]).

![Fig 3. HSG and LSG of Rectangular Cylinders(Nakamura[1994])](image)

As another example of unsteady galloping, yawed circular cylinder with yawing angle of 45°, significant cross-flow response appears when Karman vortex (KV) shedding becomes weak as shown in Fig.4. On Generation of quas-isteady galloping in relation to mitigation/damaged Karman vortex (KV), further investigation might be needed.

![Model](image)
Furthermore, bluff rectangular cylinders, those side ratios are B/D=0.3, 0.4, 0.5, 0.6, 0.7, with fixed splitter plate in a wake show unsteady galloping. An example of $C_L$, $C_D$, $C_L'$, and Strouhal number vs. pitching angle, diagrams as shown in Fig.5 and its $H_{1*}$, which is related to aerodynamic damping, defined by Scanlan[1971], shows large positive magnitude. Basing on den Hartog Criteria, galloping is never excited, but violent galloping is observed in free vibration test for this case. (see Fig.5 and Fig.6) This galloping should be caused by “fluid-memory”, corresponding the undulation of separated flow in near wake generated by body motion. In another expression the lift force has significant phase lag from body motion. That means typical unsteady effect of lift force. Circular cylinder with a fixed splitter plate would show unsteady galloping.
Aerodynamic vibration of stay cable of cable stay bridge has frequently observed observed in dry state, that is without rain, or in wet state, that is in rainy state. Those are called as Rain Wind induced vibration (RWV) and dry galloping (DG). Recently author has summarized that three major factors play definitely important role for excitation of RWV and DG, independently and individually. Those are formation of water rivulet (WR) on upper cable surface originally pointed out by Hikami [1986], axial flow (AF) in a near wake originally pointed out by author [1989, 2008, 2010], and critical Reynolds number (Re_c) which has originally pointed out by Macdonald [2005]. All of these factors generate particular flow field around cable surface, that is “separation bubble” by local flow reattachment (Matsumoto [2011]). Taking into account of the appearance of “separation bubble” in “flow reattachment type galloping” in the process from non-flow reattachment state to flow reattachment state, on the other hand, the appearance of “inner circulatory flow” in “flow non-reattachment type galloping” in the process from flow separation state to flow reattachment state, DG at the range where Re number towards to Re_c, separated flow tends to closely moves to cable surface, “inner circulatory flow” should be generated. At Re_c separated flow reattaches on cable surface, then “separation bubble” is generated on cable surface (Bass [1985]). Thus, generation mechanism of inclined RIV and DG is summarized as indicated in Fig. 7 and Fig. 8.
3 TORSIONAL FLUTTER

It has widely known that airfoil shows torsional flutter near stalling angle and its flow field can be characterized by flow separation, flow reattachment and formation separation bubble. Fig.9 is visualized flow pattern during torsional motion of airfoil by Mc-Alister and Carr[1978]. It has been verified that unsteady change of size of separation bubble during torsional oscillation should be generation mechanism.
For rectangular cylinders with B/D from 5 to 20, unsteady pressure properties, those are amplitude of pressure coefficient, $C_p$, and phase, $\psi$, of negative pressure from the maximum angle of attack (for heaving motion: neutral position during downward motion and for torsional response nose up maximum) are shown in Fig. 6. It can be verified that flow separates from the leading edge of cylinder and flow reattaches on side surface for B/D=8, 10, 12.5, 15 and 20, and for B/D=5, flow tends to reattaches near at trailing edge. It should be noted that the maximum peak of $C_p$ appears at almost same position on side surface for all cylinders and phase, $\psi$, is almost identical in various side ratios. Furthermore, it should be noted that if relative angle of attack in heaving and torsional motions is identical, $C_p$ and $\psi$ are almost same. Basing on these unsteady characteristics, it is verified that flow field around rectangular cylinders with B/D=5 to 20 might be less affected by the after body-length. Heaving and torsional 2DOF differential equations are expressed as follows:

$$m \ddot{\eta} + c_{\eta} \dot{\eta} + k_{\eta} \eta = \rho b^3 \omega_f H_1 \dot{\eta} + \rho b^3 \omega_f \dot{H}_1 \eta + \rho b^3 \omega_f H_2 \dot{\phi} + \rho b^3 \omega_f \dot{H}_2 \phi$$

$$I \ddot{\phi} + c_{\phi} \dot{\phi} + k_{\phi} \phi = \rho b^3 \omega_f A_1 \dot{\phi} + \rho b^3 \omega_f \dot{A}_1 \phi$$

where, m and I: mass and mass inertia per unit length, $c_{\eta}$ and $c_{\phi}$ : heaving and torsional damping coefficient, $k_{\eta}$ and $k_{\phi}$ : heaving and torsional stiffness, $b$: half chord length, $\rho$: air density, $\omega_f$: flutter circular frequency, $H_1$ and $A_1$ (i=1-4): aerodynamic derivatives. These 8 aerodynamic derivatives can be obtained from unsteady pressure properties (Scanlan[1971]), those are $C_p$ and $\psi$, as follows:

$$H_1 = \frac{-V^2}{2b\omega^2 \eta_0} \int \tilde{C}_p \cos \psi dx = \frac{V^2}{2b\omega^2 \eta_0} \int \tilde{C}_p H_1 dx$$

$$H_4 = \frac{-V^2}{2b\omega^2 \eta_0} \int \tilde{C}_p \sin \psi dx = \frac{V^2}{2b\omega^2 \eta_0} \int \tilde{C}_p H_4 dx$$

$$A_1 = \frac{-V^2}{2b\omega^2 \eta_0} \int \tilde{C}_p x \cos \psi dx = \frac{V^2}{2b\omega^2 \eta_0} \int \tilde{C}_p A_1 dx$$
In particular, torsional instability can be measured from $A_2^*$. If $A_2^* > 0$, torsional flutter appears, on the contrary if $A_2^* < 0$, aerodynamically stable against torsional flutter. Positive or negative of $A_2^*$ can be determined by the position of diagram of $C_p H_2^*$, it means if it locates upstream side or downstream one from mid-chord point, $A_2^*$ shows negative or positive, respectively. (see Matsumoto[2005])

(a) by Forced Heaving Oscillation
Aerodynamic derivative of $A_2^*$, in consequence, changes continuously from negative to positive with increase of B/D as shown in Fig.10. The rectangular cylinder with B/D=10 shows almost $A_2^*=0$, and more bluffer cylinders with less than B/D=10 up to B/D=4 studied here, $A_2^*>0$, that means appearance of torsional 1DOF flutter. Rectangular cylinders with more than B/D=10 show coupled flutter instability.

On the aerodynamic derivatives between rectangular cylinders and H-shaped cylinders, there exists similarity as shown in Fig. 10. From this diagram, H-shaped cylinder possesses aerodynamically more bluff than rectangular cylinder, even though their side ratio,B/D, are identical.
Fig 11. Aerodynamic Derivative of $A_2^*$ of Rectangular Cylinders with Various Side Ratios (Matsumoto[2005])

(a) Analogy of Flutter Derivatives of H-Shaped and Rectangular Cylinders

(b) Comparison of Aerodynamic Derivatives, $A_1^*$, $A_2^*$, $H_1^*$ and $H_2^*$, of H-Shaped and Rectangular Cylinders
For bluff bodies which indicate torsional flutter, including rectangular cylinders, and H-shaped cylinders, aerodynamic interference between vortex-induced vibration (VIV) and torsional flutter becomes significant issues at particular frequency ratio in torsional frequency, $f_{ij0}$, and heaving one, $f_{i0}$. For example, original Tacoma Narrows Bridge had plate-girder with H-shape section with $B/D=5$. This bridge showed catastrophic torsional oscillation by 1st asymmetrical mode, after showing 5th heaving symmetrical mode under wind velocity less than 20m/s. Its frequency ratio, $f_{ij0}/f_{i0}$, corresponding these modes, is 2.5.

(a) Response of Original Tacoma Narrows Bridge in 1DOF Heaving, 1DOF Torsional and 2DOF Torsional Vibration Systems

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Fig.12 Analogy of Aerodynamic Characteristics between H-Shaped and Rectangular Cylinders (Matsumoto[2004])
The response characteristics investigated by wind tunnel test are shown in Fig.13. The Farquarhsen report [1949] pointed out sudden change of vibration mode, from 5th heaving to 1st torsional might be caused by broken of center diagonal stay cables by heaving VIV. However, the another explanation, basing on these wind tunnel test, might be possible. It might be caused by slight increase of wind velocity from 18m/s to 19m/s, which correspond to 16.3m/s to 17.2m/s as effective wind velocity crossing perpendicularly bridge girder, because of skewly crossing to the Tacoma Narrows Chanel by 25°. Furthermore, Looking at 35mm film taken by Farqursen group at the bridge site, during 5th mode torsional vibration of main span, vibration of side-span still maintained the frequency of 5th symmetrical heaving mode. Taking into account of velocity mitigation near land than the center part of channel. It should be natural in thinking that these different vibration modes should be caused by the difference of wind velocity. Also many cable strands of the main cable near at the cable band of center-diagonal stay cables had been severely damaged. Cable band does not move, in general, for heaving vibration, because of the role of center diagonal stay cable to prevent the torsional displacement of girder. The fact that many cable strands were severely damaged near cable band, cater stay cables must be survive after appearance of torsional vibration of girder.
4 COUPLED FLUTTER

4.1 SBS Flutter Analysis

2DOF Heaving and torsional differential equations are expressed by equation (2.1) and (2.2). Aerodynamic derivatives of thin plate can be expressed by Theodorsen function as follows:

\[ H_1^*(k) = -(2\pi / k)F(k) \]
\[ A_1^*(k) = (\pi / k)F(k) \]
\[ H_2^*(k) = -(2\pi / k)(-1/2 + F(k)/2 - G(k)/k) \]
\[ A_2^*(k) = (\pi / k)(-1/2 + F(k)/2 - G(k)/k) \]
\[ H_3^*(k) = -(2\pi / k)(F(k)/k + G(k)/2) \]
\[ A_3^*(k) = (\pi / k)(F(k)/k + G(k)/2) \]
\[ H_4^*(k) = -(2\pi / k)G(k) \]
\[ A_4^*(k) = (\pi / k)G(k) \]

where \[ C(k) = F(k) - iG(k) \]

(4.1)

Aerodynamic derivatives of thin plate can be expressed by Theodorsen function as follows:

2DOF coupled flutter can be characterized by four properties, those are frequency v.s. velocity (\(\omega-V\)) property, damping v.s. velocity (\(\delta-V\)) property, amplitude ratio v.s. velocity (\(\eta_0/\varphi_0-V\)) property and phase difference v.s. velocity (\(\psi-V\)) property. These characteristic values are, in general, analyzed by complex eigen value analysis.

On the other hand, author has developed Step-by-Step analysis to get these values. (Matsumoto[2010] )Branch switching in these values can be studied by use of SBS flutter analysis. Its procedure is briefly explained in Fig.10. Clarification of flutter branch is definitely important to understand of multi-mode flutter characteristics. Because coupled flutter should be coupled wind-induced vibration between heaving and torsional modes. Therefore, if flutter begins with HB or TB, then the other torsional modes or heaving modes, except of fundamental pairing mode, must mainly contribute to change flutter onset velocity, \(V_{cr}\), more or less, respectively. Moreover, through wind tunnel test, flutter onset in T0 mode might be milder than in H-90 mode. But on this flutter property, further studies should be needed.
Flutter properties analyzed by SBS and conventional Complex Eigen Value flutter analysis (CEV) for thin plate under certain structural dynamics are indicated in Fig. 14. Theodorsen function are used for aerodynamic derivative as indicated above.
Flutter values characterizing property, \( \omega, \delta, (\eta_0/\phi_0) \) and \( \psi \), are completely identical in the both cases of CEV and SBS, as shown in Fig.1, but flutter branch at higher velocity after \( V_\omega=10 \text{ m/s} \) are completely different in CEV analysis and SBS analysis. In SBS analysis, at a little bit higher velocity, 12.5 m/s, than \( V_\omega=10 \text{ m/s} \), flutter branch suddenly changes from heaving branch (HB) to torsional branch (TB) or from TB to HB. At higher velocity range, HB flutter appears in SBS analysis, on the other hand, TB flutter appears in CEV analysis. Looking at \( \psi - V \) diagram, \( \psi \) is close to frequency of torsional 1DOF. This apparently indicates that at higher velocity range than flutter onset, this \( \omega \) must correspond to TB not to HB. Therefore at near \( V_\omega \) branch switching, from HB to TB, should occurs to satisfy this \( \omega \)-property belonging to TB. Two flutter-values, \( \omega \) and \( \delta \), from four-ones have been spot-lightened, because of determination of reduced velocity \( V_r=V/b_\omega \), which characterizes unsteady lift and pitching moment, and flutter onset velocity, \( V_c \). However, the another two flutter values, \( (\eta_0/\phi_0) \) and \( \psi \), have been less paid attention in discussion of flutter instability. It should be noted that these two flutter-values characterize flutter branches and flutter modes. These are significantly important to precisely verify the flutter coupling mechanism in multi-modes, that is multi-mode flutter, mechanism and generation mechanism of propulsion force during heaving and torsional coupling motion.
4.2 Flutter Modes

In coupled flutter, 6 fundamental modes can be defined as follows:

1. H mode: This is pure 1DOF heaving mode without torsional displacement, in another expression, is pure torsional mode around rotational axis fixed at the infinite point from mid-chord point of plate.

2. T mode: This is pure rotational mode around rotational axis fixed at mid-chord point of plate, without heaving displacement at this point.

3. T0 mode: This is pure rotational mode around leading edge. At heaving motion (the lowest is maximum) at mid-chord point has no phase difference against torsional motion (windward nose-up positive) at this point.

4. T180 mode: This is pure rotational mode around trailing edge. In this mode, torsional maximum delays from heaving maximum by -180 degree.

5. H90 mode: In this mode, the heaving maximum delays from the torsional maximum by 90 degree. This mode appears from quasi-steady point if \( dC_L < 0 \).

6. H-90 mode: In this mode, the heaving maximum delays from the torsional maximum by -90 degree, it means the heaving maximum proceeds ahead to the torsional maximum by 90 degree. This mode appears from quasi-steady point if \( dC_L > 0 \).

These four modes, T0, T180, H90 and H-90, are illustrated in Fig.2.

![Figure 2: Fundamental flutter modes: T0, T180, H90, and H-90](image)

4.3 Flutter Branches

Flutter branches can be defined from Amplitude ratio, \( \frac{\eta_0}{\phi_0} \), as shown in Fig.16. Around flutter onset, amplitude ratio, \( \frac{\eta_0}{\phi_0} \), shows the minimum. At lower velocity than \( V_{cr} \), \( \frac{\eta_0}{\phi_0} \) for HB tends to infinite with decreasing velocity, it means heaving displacement dominates than torsional one. At \( V=0 \), torsional displacement becomes zero, \( \phi_0=0 \), it means coupling motion is pure heaving motion or rotational axis at infinite point in upstream- or downstream-ward. On the other hand, for \( \frac{\eta_0}{\phi_0} \) of TB decreases together with decreasing velocity approaching zero at \( V=0 \). At \( V=0 \), heaving displacement is zero \( \eta_0=0 \). This means that flutter properties can be defined as TB at lower velocity range than around flutter onset. Similarly at higher velocity than...
around flutter onset, HB and TB can be defined as HB and TB respectively basing upon flutter properties obtained by SBS flutter analysis.

At velocity range near \( V_{cr} \), flutter branch would be defined from phase difference, \( \psi \), as following procedure. Taking into account of flutter modes, H mode, T mode, T0 mode, T180 mode, H90 mode and H-90 mode, their torsional motion, \( \varphi(t) \), and heaving motion, \( \eta(t) \) are expressed respectively as follow:

**H mod e**: \( \eta(t) = \eta_0 \sin \omega t \)
\[ \varphi(t) = 0 \]  
(4.2)

**T mod e**: \( \eta(t) = \varphi_0 \sin \omega t \)
\[ \varphi(t) = 0 \]  
(4.3)

**T0 mod e**: \( \eta(t) = \varphi_0 \sin \omega t \)
\[ \eta(t) = \eta_0 \sin \omega t \]  
(4.4)

**T180 mod e**: \( \eta(t) = \varphi_0 \sin \omega t \)
\[ \eta(t) = -\eta_0 \sin \omega t \]  
(4.5)

**H90 mod e**: \( \eta(t) = \varphi_0 \sin \omega t \)
\[ \eta(t) = -\eta_0 \cos \omega t \]  
(4.6)

**H-90 mod e**: \( \eta(t) = \varphi_0 \sin \omega t \)
\[ \eta(t) = \eta_0 \cos \omega t \]  
(4.7)

When phase difference between torsion and heaving motion is \( \psi \), coupling of heaving and torsional motions can be expressed as follow depending on \( \psi \) which varies between \(-90^0\) and \(180^0\) in flutter property obtained by flutter analysis:

\( 180^0 > \psi > 90^0 \): \( \cos \psi < 0 \) and \( \sin \psi > 0 \) 
(4.8)

\( 90^0 > \psi > 0^0 \): \( \cos \psi > 0 \) and \( \sin \psi > 0 \) 
(4.9)

\( 0^0 > \psi > -90^0 \): \( \cos \psi > 0 \) and \( \sin \psi < 0 \) 
(4.10)

Then,

\[ \varphi(t) = \varphi_0 \sin \omega t \]  
(4.11)

\[ \eta(t) = \eta_0 \sin(\omega t - \psi) \]  
\[ = \eta_0 \sin\omega t \cos\psi - \eta_0 \cos\omega t \sin\psi \]  
(4.12)

For all terms on right hand of the upper equation, coupling motions can be resolved as follows:

\( 180^0 > \psi > 90^0 \): \( \varphi(t) = \varphi_0 \sin \omega t \)
\[ \eta(t) = -T180 \cos \psi - H90 \sin \psi \]  
(4.13)

\( 0^0 > \psi > -90^0 \): \( \varphi(t) = \varphi_0 \sin \omega t \)
\[ \eta(t) = -T0 \cos \psi - H - 90 \sin \psi \]  
(4.14)
\(90^\circ > \psi > 0^\circ : \varphi(t) = \varphi_0 \sin \omega t\)
\(\eta(t) = T0 \cos \psi + H90 \sin \psi\)  \hspace{1cm} (4.15)

In HB and TB consist in two fundamental flutter modes depending on their amplitude ratio, \(\eta_0 / \varphi_0\), and phase, \(\psi\), as indicated in Fig.3. In detail, for HB,
- at \(V=0\) : H mode only,
- at \(0<V<V_{cr}\) : T0 mode and H90 mode,
- at \(V_{cr}<V\) : T0 mode and H-90 mode,

and for TB,
- at \(V=0\) : T mode only,
- at \(0<V<V_{cr}\) : T0 mode and H-90 mode,
- \(V_{cr}<V\) : T0 mode and H-90 mode.

At near \(V_{cr}\), flutter branch, HB or TB, which consists of T0 mode and H-90 mode for amplitude ratio- velocity diagram, can be classified by their magnitude of coefficients, those are \(\sin \psi\) and \(\cos \psi\). If \(\psi=-45^\circ\), the contribution to flutter of TB and HB are identical. If \(\psi<-45^\circ\), HB plays more significantly for flutter onset than TB, and contrary if \(\psi>-45^\circ\), TB does than HB. Flutter branch when flutter occurs changes not only with different aerodynamic derivatives caused by change of geometrical shape of structures or with angle of attack of wind, but also with structural dynamics. Example of change of flutter branch is shown in Fig.17.

(a) Case1(\(B=27.8\text{m}, M=2.065\text{e04Kb/m}, I=1.90\text{e06Kgm2/m}, f_0=0.31\text{sec}^{-1}, f_{ij0}=0.62\text{sec}^{-1}, f_{ij0}/f_0=2.0, \delta_{ij}/\delta_{ii0}=0.02\)), where red plot for HB, blue plot for TB.
Fig 17 shows that flutter begins with TB(case1) or HB (case2), respectively. It should be noted that flutter branch at flutter onset changes by structural dynamics. On the other hand, Xu[2011] recently reported that flutter of full scale elastic model of the Sutong Bridge in China, the longest cable stayed bridge in the world, begins with To mode under the condition of horizontal wind, but it begins with H-90 mode under up-ward wind with the angle of attack of 3°. In this case, different aerodynamic derivatives change flutter branch at flutter onset.
Flutter branches and flutter modes of thin plate explained before are illustrated in velocity diagram and $\psi$-velocity diagram in Fig.18.

4.4 Selberg Formula

Selberg formula[1961] has been widely used for estimation of flutter critical velocity, in primary stage of design of long span bridge, as a desk work. $V_{cr}$ obtained from Selberg Formula is for thin plate section, so the exact $V_{cr}$ should be investigated by wind tunnel tests using scaled section-model or full scale elastic model, in indirect, measurement of aerodynamic derivatives or direct measurement of $V_{cr}$. However, how to be driven this useful Selberg Formula has been not clarified. Under following simple assumptions, significantly similar formula can be obtained (Matsumoto[2010]).

Assumptions:1. When torsional frequency, decreasing with wind velocity, is identical to heaving frequency, no-affected by wind velocity, that is, flutter appears.
2. $f_{ij}$ is characterized by only $A_3^*$, and $A_3^*$ is expressed by $A_3^*=(\frac{\omega}{k})(F(k)/k-G(k)/2))^{1/2}$
3. Using quasi-steady assumption,: $F(k)=1$ and $G(k)=0$, where $k$ is reduced velocity=$b\omega/V$.

Then $V_{cr}$ can be expressed similarly with Selberg Formula as follows:

$$V_{cr} = 3.71f_{ij}\rho(2b)^{1/2}(ml)^{1/2}/\left(1-\left(f_{ij}/f_{ij0}\right)^2\right)^{1/2}$$

(4.16)

Selberg Formula

$$V_{cr} = 3.81f_{ij}\rho(2b)^{1/2}(ml)^{1/2}/\left(1-\left(f_{ij}/f_{ij0}\right)^2\right)^{1/2}$$

(4.17)

obtained from upper assumptions for thin rectangular section

The $V_{cr}$-values calculated two formula under some structural dynamics and exact ones analyzed by CEV analysis and SBS analysis are compared in Fig. 15.
5 APPLICATION OF FLUTTER –GENERATION OF PROPULSION FORCE

5.1 Generation of Propulsion Force

According to former literatures, instantaneous propulsion force might be generated by coupling motion of heaving and pitching as term related to thin-airfoil theory, $F_{\text{pair-foil}}$, which consists in lift and drag force, $F_{\text{PL,CD}}$, and virtual mass effect, $F_{\text{pmass}}$, and jet disgorging term, $F_{\text{pjet}}$. Then, the propulsion force, $F_p$, can be expressed as follows:

Airfoil Theory term1:

$$ F_{\text{pair-foil}} = L(\alpha_{\text{re}}) \sin \alpha_{\text{re}}^* - D(\alpha_{\text{re}}) \cos \alpha_{\text{re}}^* $$

$$ = \left( \frac{1}{2} \right) \rho V_{\text{re}}^2 2b C_L(\alpha_{\text{re}}) \sin \alpha_{\text{re}} - \left( \frac{1}{2} \right) \rho V_{\text{re}}^2 2b C_D(\alpha_{\text{re}}) \cos \alpha_{\text{re}}^* $$

(5.1)

where, $L,D,C_L,C_D$: unsteady force and coefficients

$$ V_{\text{re}} = \left[ V^2 + \left( \frac{d \eta}{dt} \right)^2 \right]^{1/2} $$

(5.2)

$$ \alpha_{\text{re}}^* = \arctan \left( \frac{d \eta}{dt} / V \right) $$

(5.3)

$$ \alpha_{\text{re}} = \alpha_{\text{re}0} + \phi $$

(5.4)
Airfoil Theory term 2(Karman&Sears[1938]) : (Virtual Mass Effect):

\[ L_{\text{mass}} = \pi \rho \phi^2 \left( \frac{d^2}{dt^2} \eta + V \frac{d\phi}{dt} \right) \]  \text{normal to plate, at } \eta(t) = 0 \quad (5.5)

\[ F_{\text{prop}} = L_{\text{mass}} \sin \phi \text{ : propulsive component, at } t = 0 \quad (5.6) \]

Jet Disorging term:

\[ F_{\text{jet}} = \rho A_0 (V + v) \left( V + v_p \right) - v = \rho A_0 v_p (V + v) \quad (5.7) \]

where, \( v_p = v/2 \) \quad (5.8)

\( A_0 \) effective area at near trailing edge \text{ where jet passes into wake, } \nu_0: \text{disorging jet velocity through } A_0 \nu: jdisorging jet velocity in a wake

Propulsion force in term of airfoil theory, \( F_{\text{PLD}} \) and jet disorging term, \( F_{\text{jet}} \) show the maximum at having velocity maximum, that is pitching angle of zero in H-90 flutter mode, on the other hand \( F_{\text{mass}} \) shows the maximum at zero heaving velocity, that is maximum pitching displacement.

Propulsion force, , is classified into 6 cases depending combination of torsional and heaving displacement, as follows (Matsumoto and Ishizaki([2010])):

I. Upward heaving motion, \( \alpha_c^* < 0 \)
   1. Positive torsional angle, \( \varphi > 0 \)
      1.1 Absolute value of Torsional angle is larger than the one of relative angle of attack due to heaving velocity: \( |\varphi| > |\alpha_c^*| \) (case1)
      1.2 Absolute value of Torsional angle is smaller than the one of relative angle of attack due to heaving velocity: \( |\varphi| < |\alpha_c^*| \) (case2)
   1. Negative torsional angle (case3)

II. Downward heaving motion
   1. Positive torsional angle \( \varphi > 0 \)
   2. Negative torsional angle \( \varphi < 0 \)
      2.1 Absolute value of Torsional angle is larger than the one of relative angle of attack due to heaving velocity: \( |\varphi| > |\alpha_c^*| \) (case5)
      2.2 Absolute value of Torsional angle is smaller than the one of relative angle of attack due to heaving velocity: \( |\varphi| < |\alpha_c^*| \) (case6).

These 6 cases and Lift anf Drag induced by relative velocity are shown in Fig.16.
From these generation mechanism of Lift and Drag depending torsional and heaving displacement, the following conditions to generate the positive propulsion force, \( F_{PL&D} \), are definitely required:

For generation of lift force with the up-stream ward force component:

\[
(\eta_0 / b)\varphi_0 > V_r = V / b\omega
\]

For that the propulsive component of lift is larger than the one of drag:

\[
\gamma(\alpha_{re})\tan\alpha_{re} - 1 > 0
\]

Where,
- \( \eta_0 \): amplitude of heaving and torsional motion of flapping plate, respectively
- \( \varphi_0 \): circular frequency of flapping plate
- \( V \): propulsion velocity (or oncoming flow velocity)
- \( \gamma \): lift and drag ratio
- \( \alpha_{re} \): relative angle of attack \( (=\alpha_{re}^* + \varphi) \)
- \( \alpha_{re}^* \): angle of attack induced by heaving velocity \( (=\arctan(d\eta/dt)/V) \)

It is verified that positive propulsion force, \( F_{PL&D} \), can be always generated in H-90 flutter mode, if upper conditions are satisfied, as shown in Fig.17.
Previous studies on navigation mechanism of fish swimming (Tanaka and Nagai [1996]) and ship navigation by flapping plates (Terada, Yamamoto, Nagamatsu and Imaizumi [1998], Barannyk, Buckham and Oshkai [2010]) reported that the coupled motion on tail-fins of fish and flapping plate were controlled by H-90 flutter mode effectively generation of propulsion force. Tail-fin movement in fish swimming is shown in Fig. 22 (Tanaka & Nagai [1996]).

On propulsion force generated by jet disgorging, $F_{pjet}$, it has not been clarified at present as pointed by Barannyk [2010]. This jet disgorging might generate inverse Karman vortex in a wake, however, its detail mechanism and its effect on propulsion force should be studied more in a future.

5.2 Ship Navigation Test by Double Flapping Plates controlled Inverse Phase H-90 Modes

The propulsion force measurement has been conducted by use of ship model with 2.5Kgf in weight, 0.72m in length, 0.33m in width and 0.01m in submerged depth, as shown in Fig. 19. Two rigid, flexible and half-flexible two flapping plates (see Fig. 20) were installed at near trailing edge, and they were controlled in inverse phase in order to cancel the sway forces, mutually. Each flapping plate was controlled in H-90 mode, individually. The distance at each neutral position between two plates was 0.22 m. The size of flapping plates were, 0.04m(half chord length for rigid plate), 0.08m(half chord length for flexible plate), 0.06m(height for all plates). The thickness of plates were 1.5mm (for rigid and flexible plaste) and 2.0mm(for elastic plate). The pitching axis was fixed at up-stream-ward quarter-point for flexible plate-2 and at mid-chord point for the others. The frequency of flapping plate was mainly 4Hz and flow velocity in water channel for measurement of resistance force was carried out in approximately
0.24 m/s, 0.26 m/s, 0.30 m/s and 0.34 m/s. The propulsion forces were measured in still water. The amplitude of heaving and pitching motion was fixed as 0.02 m and 19.5°, respectively. Besides, navigation velocity of ship model was measured in still water. The maximum navigation speed for flexible flapping plate was observed as over 0.55 m/s in the case of f₀=4.35 Hz, and the one for rigid plate 0.45 m/s at f₀=4.35 Hz as well.

An example of time history of propulsion force for rigid plate with f₀=4 Hz at V=0 m/s is shown in Fig.10. The propulsion force varies with time with showing many local peaks, but mainly fluctuates with the frequency of flapping plate. The maximum propulsive force is roughly evaluated as 170gf. It should be noted that this force fluctuating property, that is one peak appearance in one cycle motion of two flapping plates. This one peak appearance can be explained by effect of enhanced jet disgorging between two flapping plate controlled in inverse phase motion. The enhanced jet flow appears once in one cycle motion of flapping plate.

Ship navigation speed diagrams with various flapping plates and frequency are indicated in Fig.13.
As shown in this figure, navigation speed gradually increases after start (t=0). The maximum speed is over 55 cm/s in the case of flexible flapping plates. For rigid plate, maximum speed is up to 45 cm/s as shown in Fig.27. Because of uncertainties on unsteady and non-linear force characteristics during coupling motion with large amplitude over stalling angle of attack and taking into account of appearance of quasi-steady behavior of H-90 flutter mode at high reduced velocity, it might be assumed to be quasi-steady force, $F_{p\text{quasi}}$, that is $F_{\text{airfoil}} \approx F_{p\text{quasi}}$, at a primary study.
5.3 Estimation of Propulsion Force induced Jet Disgorging, \( F_{\text{jet}} \) basing on CFD Analysis by Isogai[2006] for Dolphin Standing Swimming

Described before on uncertainties on evaluation of propulsion force generated by jet disgorging, \( F_{\text{jet}} \), might be approximately estimated from CFD results for standing swimming of dolphin in still water analyzed by Isogai[2006], under assumption of quasi-steady lift and drag forces. CFD result showed dolphin can generate upward propulsion force to cancel dolphin weight \( W = 138 \text{Kgf} \) by flapping motion of tail fin (\( b = 0.072 \text{m at center}, l = 0.432 \text{m}, \text{tail fin area} = 0.0377 \text{m}^2, \text{aspect ratio} = 4.96 \)) with frequency of 4.07Hz. Amplitude of torsional and heaving motions are 58.9° and 0.36m, respectively. The tail-fin motion is controlled in almost H-90 flutter mode, where phase is not -90° degree but -75.8°. In this case, maximum propulsion force of 2750 N can be generated at the instant of heaving maximum velocity, 9.25m/s. The flow field around tail fin at the moment of \( \frac{dn}{dt}_{\text{max}} \) is shown in Fig.28.

![Fig. 28 Flow Field around Tail Fin of Dolphin during Standing Swimming (CFD by Isogai[2010])](image)

It should be noted that jet disgorging velocity in a wake is \( v = 5.4 \text{m/s} \) in equation(5.7). Relative angle of attack, \( \alpha_{\text{re}} = \frac{dn}{dt}/V + \phi = 90 - 58.9° = 31.1° \), \( C_L(\alpha_{\text{re}}) = 1.0 \) and \( C_D(\alpha_{\text{re}}) = 0.8 \), of which values are for 3DOF delta wing measured by Okamoto[2008]. Then, basing on quasi steady lift
and drag forces, $F_{\text{Pquasi}}$, can be calculated as 1595.5N. Furthermore, if effective area, $A_0$, where jet is passing, near trailing edge of tail-fin can be expressed by $A_0 = 0.1555m^2$ ($= \eta_{\text{dx}} = 0.36m \times 0.432m$), besides jet disgorging up-ward velocity component is approximately 3.82m/s. Then from eq.(), maximum propulsion force at the moment of maximum heaving velocity, $dH/dt|_{\text{max}}$, generated by jet disgorging into wake, $F_{\text{Pjet}}$, is obtained as $F_{\text{Pjet}} = 1136N$. Therefore, the total maximum propulsion force, $F_{\text{Pmax}}$, at the moment of maximum heaving velocity is obtained as 2730.5 N by summation of $F_{\text{Pquasi}}|_{\text{max}}$ and $F_{\text{Pjet}}|_{\text{max}}$ at heaving velocity maximum. This value is similar to the maximum propulsion force obtained by CFD (by Isogai[2006]). Of course, there are many simplified assumption in this calculation, so these agreement might be eventual one. However, contribution ratios of $F_{\text{Pfoil}}|_{\text{max}} (= F_{\text{P_L&D}}|_{\text{max}})$ and $F_{\text{Pjet}}|_{\text{max}}$ to total maximum propulsion force, $F_{\text{Pmax}}$, might be roughly evaluated from this result. That means $F_{\text{Pjet}}|_{\text{max}}$ and $F_{\text{Pfoil}}|_{\text{max}}$ contribute to total maximum propulsion force, $F_{\text{Pmax}}$ by 42% and 58%, respectively.

5.4 Flutter Power Generation

The “concept of Flutter Power Generation(FPG)” has been proposed by Isogai [2003]. This is a practical application of coupled flutter. If forced torsional vibration to the plate or airfoil, then significantly intensive heaving vibration can be excited because of appearance of natural coupled flutter in flutter fundamental H-90 mode in this system. The point of this FPG is that enough small power for forced torsional motion can generate a big power by intensive heaving vibration of plate/airfoil. Isogai pointed that giving power for torsion is less 1% of obtaining power by heaving motion. At present, Abiru[2010] is demonstrating in the field (in small river), to show its efficiency.
Its system in wind tunnel is shown in Fig.29. The amplitude ratio between heaving response and forced torsional vibration for rectangular cylinder with B/D=20 measured FPG experiment under forced torsional vibration with p-p amplitude of 10° is compared with analytical results in Fig.30.

![Diagram](image)

(a) Rectangular Cylinder with B/D=20         (b) Rectangular Cylinder with B/D=5

Fig. 30 Comparison of Amplitude Ratio in FPG of Experimental results and Analytical ones for B/D=20 and 5, under Forced Torsional Vibration with p-p Amplitude of 10°. (Matsumoto[2006])

It should be noted that FPG system uses naturally appearing coupled flutter in H-90 flutter mode. In Ship navigation system with flapping plate has significant advantage in saving power for driving coupled motion to the flapping plate. The detail is described below.

5.5 Advantage of Flapping plate System for Ship Navigation

In order to generate the positive propulsion force by coupled torsional and heaving motion, equations (5.9) and (5.10) are definitely satisfied. Therefore naturally generated coupled flutter of plate-like sections cannot generate positive propulsion force, because in amplitude ratio between heaving and torsional motion, \( \frac{\eta_0}{\theta_0} \) vs. velocity diagram, equation (5.9) is not satisfied as shown in Fig.27. This \( \eta_0/\theta_0 \)-velocity diagram is same one in Fig.15 (or Fig18(a)). In another words, natural coupled flutter must generate negative propulsion force, which means generation downstream-ward force fluctuation during coupling motion. In particular, this peak of negative propulsion force appears twice, because of twice appearance of \( |\frac{d\eta}{dt}_{\text{max}}| \) in one cycle flutter motion, it means \( f_{Fp} \) (propulsion force frequency) is twice of \( f_{F} \) (flutter frequency). During heaving and torsional 2DOF coupled flutter with frequency of \( f_{F} \) windward vibration with 2\( f_{F} \) can be excited.

By the way, for ship navigation by the flapping plate, when ship starts from still state, initial power to generate the coupled motion, which satisfied equations (5.9) and (5.19), then ship moves with certain velocity, \( V \). At this moment in getting velocity \( V \), given torsional motion can generate heaving motion by the mechanism of Flutter Power Generation, therefore, to get continuously the positive propulsion force to navigate a ship, the lack heaving amplitude, \( \eta \), should be added to satisfied equations (5.9) and (5.10). Namely in ship navigation with flapping
plate, once ship starts, rather mount power would be saved aided by the mechanism of flutter power generation as illustrated in Fig.28.

A lot of further studies for needed for practical realization of ship navigation with flapping plate, however there must be significant advantage in ship navigation with flapping plate from the point of power saving.

6 CONCLUSIONS

In this paper, fundamental of flutter phenomena, including galloping, torsional flutter and coupled flutter, has been introduced and as application, it is explained that propulsion force can be intelligently produced by use of the fundamental flutter mode, H-90.

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7 REFERENCES

15. M. Matsumoto and et al., Inclined Cable Aerodynamics, Structural Design, Analysis & Testing, in Proc. of Structural Congress ’89, ASCE, 1989
17. M. Matsumoto and et al., Cable Vibration and Its Aerodynamics/Mechanical Control, Proc. of International Conference of Cable Stayed and Suspension Bridges, Deauville, IABSE, 1994
18. M. Matsumoto and et al., Torsional Flutter of Bluff Bodies, JWEIA, 1997, Vol.69, pp.871-881
22. M. Matsumoto and et al., Flutter Characteristics of H-Shaped Cylinders with Various Side-Ratios and Similarity to Rectangular Cylinder Ones, Proc. of the 5th International Colloquium on BBAA, Ottawa, 2004
23. M. Matsumoto and et al., The Flutter Instability Mechanism – Coupled flutter and Torsional Flutter, 2005
24. M. Matsumoto, Flutter Instability of Structures, Proc.of the 4th EAWCE (key note), Prague,, 2005
29. M. Matsumoto, Mechanism of wind & rain/wind induced cable vibrations –role of Karman vortex on inclined cable aerodynamics, Proceedings of Wind Induced Vibration of Cable Stay Bridges Workshop, St. Louis, Missouri, USA, 2006
34. M. Matsumoto and et al., Dry galloping characteristics and its mechanism of inclined/yawed cables, J of Wind Eng. and Industrial Aerodyn. 2010, 98, pp317-327
36. M. Matsumoto & H. Ishizaki, Study on Propelling Forces by taking into account of Flutter Modes, Proc. of Advanced in Interaction and Multi-scale Mechanics(AIMM’10), Jeju, Korea, 2010
42. M. Okamoto & Y. Jinba, (2008), Experimental Study of Aerodynamic Characteristics of Wing Platform at Low Reynolds Number , pp42-50,Bullitein of Akita Liberal Engineering High School, No.44,
44. A. Selberg, Oscillation and aerodynamic Stability of Suspension Bridges, ACTA, Polytechnica Scannavica, Civil Engineering, Civil Engineering and Construction Series , 1961