Nonlinear aerodynamic forces on square section: numerical study

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ABSTRACT: The paper presents a numerical simulation of nonlinear aerodynamic forces on square section by using computational fluid dynamic (CFD) method. The square section is subjected to a forced asymptotic transverse oscillation of varying amplitude. The aerodynamic forces on the square section are computed by multiple-domain mesh technique together with unstructured dynamic meshes. The instantaneous frequencies and amplitudes of lift force are identified from its time history by continuous wavelet transform (CWT) in terms of CWT ridges. The relationship between the non-dimensional amplitude and phase angle of frequency response component of lift force and the amplitude of forced oscillation is well demonstrated. Simulated results are compared with previous data at several reduced velocities. It shows that the proposed method can identify the general nonlinear features of aerodynamic forces in association with varying excitation amplitude.

KEYWORDS: square section; nonlinear aerodynamic force; large amplitude oscillation; numerical simulation; continuous wavelet transform; lock-in region

1 INTRODUCTION

For predicting wind-induced responses of long-span cable-supported bridges, the limitation of conventional linear aerodynamic methods has been recognized and nonlinear aerodynamic forces on the bridge attract more and more attentions [1]. Chen and Kareem [2, 3] separated the aerodynamic forces into the low- and high-frequency components in accordance with the effective angle of incidence. Diana et al. [4] proposed a rheological model in the time domain to reproduce nonlinear aerodynamic forces due to large changes in the angle of attack caused by turbulence and deck motion. They further carried out wind tunnel experiments to verify the rheological model [5]. Wu and Kareem [6] utilized artificial neural network framework with embedded cellular automata scheme to capture the hysteretic nonlinear behavior of aerodynamic systems.

This paper aims at investigating the relationship between the amplitude of aerodynamic forces on and the amplitude of forced oscillation of a bluff body, in which the square cylinder is selected. The nonlinear aerodynamic forces on the square cylinder forced in harmonic oscillation have been investigated through experiments [7, 8] and computational fluid dynamic (CFD) methods [9, 10]. The continuous wavelet transform (CWT) based on the complex Morlet wavelet [11] has been successfully applied to analyse the transient vibration behaviour of structures [12] and to detect structural damage [13]. However, the combination of these two problems to find the relationship between the amplitude of aerodynamic forces on and the amplitude of forced oscillation of a square section in this paper is original. The square section is subject to a forced asymptotic transverse oscillation of varying amplitude. The aerodynamic forces on the square sec-
tion are computed by the CFD method and are further analysed by using CWT. The captured features of the aerodynamic forces are compared with previous experiment and CFD results.

2 NUMERICAL SIMULATION OF NONLINEAR AERODYNAMIC FORCES

2.1 Dynamic mesh algorithm

To determine nonlinear aerodynamic forces on an oscillating square section of large amplitude with uniform flow inlets, a dynamic mesh algorithm called the domain decomposition algorithm [14] is used. Two-dimensional (2D) simulation is conducted and the computational domain is shown in Figure 1. The domain called the rigid boundary layer mesh region is connected with the square section and has the same movement as the square section during its oscillation. Very fine structured quadrangular meshes are used within this region. The domain called the dynamic mesh region is modified through the interface between this region and the rigid boundary layer mesh region while the square section is subjected to a forced oscillation. The initial triangular meshes are used in the dynamic mesh region and will be adjusted according to the early work of Batina [15] during the square section oscillation. The domain called the static mesh region is meshed by relatively coarse and fixed structured rectangular meshes.

2.2 Governing equations

The grid velocity \( V_g \) of the moving mesh within the rigid boundary layer region can be determined by the boundary condition of the oscillating square section. The grid velocity \( V_g \) of the moving mesh in the dynamic mesh region can be determined by the deformed and re-meshed grids. The grid velocity \( V_g \) of the mesh within the static region is zero.

After \( V_g \) is determined, the Reynolds-averaged Navier-Stokes (N-S) equations for 2D incompressible unsteady fluid, which will be solved by the finite volume method, are modified and given as:

\[
\frac{\partial}{\partial t} \int_{\Omega} \rho (V - V_g) \cdot n d\Sigma = 0
\]

\[
\frac{\partial}{\partial t} \int_{\Omega} \rho u (V - V_g) \cdot n d\Sigma + \int_{\Omega} \left( p + \frac{2}{3} \rho k \right) i \cdot n d\Sigma = \int_{\Omega} \mu_\text{ef} \nabla v \cdot n d\Sigma
\]

\[
\frac{\partial}{\partial t} \int_{\Omega} \rho v (V - V_g) \cdot n d\Sigma + \int_{\Omega} \left( p + \frac{2}{3} \rho k \right) j \cdot n d\Sigma = \int_{\Omega} \mu_\text{ef} \nabla v \cdot n d\Sigma
\]

where \( \Omega \) represents the control volume of quadrangular/triangular meshes whose boundary is moving; \( \Sigma \) is the mesh boundary (one-dimensional) surface, whose outward unit normal vector is \( n \); \( V = u \hat{i} + v \hat{j} \) is the velocity vector of the fluid; \( p \) is the fluid pressure; \( k \) is the turbulent kinetic energy; \( \mu_\text{ef} = \mu + \mu_t \) is the total fluid viscosity; and \( \mu \) is the dynamic viscosity. The turbulent viscosity \( \mu_t \) is obtained through a two-equation renormalization group (RNG) \( k-\epsilon \) turbulence model [16]. Standard values are used for the model constants: \( C_{\mu} = 0.0845 \), \( C_{1_\mu} = 1.42 \) and \( C_{2_\mu} = 1.68 \). The grid velocity of the moving mesh is also introduced into the transport equations of \( k \) and \( \epsilon \). Advancement in time is accomplished by the first-order implicit Euler scheme. The pressure-implicit SIMPLE algorithm is used to solve the pressure-velocity coupling. The diffusive terms are solved by a second-order central difference scheme and the convection terms are computed by means of the second-order upwind scheme. The velocity field of the steady-state solution that satisfies the continuity condition is used as the initial condition of flow velocity field.
2.3 Forced asymptotic oscillation

Vortex-excited oscillation is an important instability phenomenon. The shedding vortex behind a bluff body can produce a lateral periodic force. Consequently, the vortex-excited oscillation is usually considered as a forced oscillation. Large and limited amplitude oscillation may occur once reduced velocity goes into the so-called lock-in region. Therefore, the square section is defined as a rigid body and is subjected to a forced asymptotic transverse oscillation of varying amplitude in this study. The oscillating displacement of the square section is expressed as

\[ A(t) = D_A(t) \sin(2 \pi ft) \]

where \( A_0 \) represents the initial amplitude of the harmonic oscillation; \( \lambda \) is a constant of very small value; and \( f \) is the frequency of forced oscillation.

2.4 Definition of aerodynamic lift coefficient

In present study, only unsteady lift force \( F_L(t) \) (positive if upward) is investigated. The corresponding lift coefficient is defined as:

\[ C_L(t) = \frac{F_L(t)}{0.5 \rho U^2 H} \]

where \( H \) is the width of square section; and \( U \) is the mean speed of incoming flow.

3 IDENTIFICATION OF NONLINEAR AERODYNAMIC LIFT FORCE USING CWT

In CFD simulation, the lift coefficient \( C_L(t) \) of the square section subjected to a forced asymptotic oscillation is basically asymptotic as well. If the force coefficient varies with time at a dominant frequency, it can be expressed by

\[ C_L(t) = D_L(t) \cos(\phi_L(t)) \]

where \( D_L(t) \) and \( \phi_L(t) \) are limited to slowly time-varying functions, \( D_L(t) \geq 0 \) and \( \phi_L(t) \in [0, 2\pi] \) [17]. \( D_L(t) \) is defined as the instantaneous amplitude of nonlinear aerodynamic lift. The corresponding instantaneous frequency of the nonlinear aerodynamic lift is introduced as

\[ \omega_L(t) = \omega_L(t) \]

where the sign ' denotes the derivative with respect to time \( t \). The CWT method with the complex Morlet wavelet is used to identify instantaneous features of nonlinear aerodynamic lift force, and the complex Morlet wavelet is given as

\[ \psi(t) = \frac{1}{\sqrt{2\pi}} e^{-0.5t^2} \cdot e^{i\omega_c t} \]

where \( \omega_c = 2\pi f_c \geq 5.0 \) and \( f_c \) is the centre frequency of the mother wavelet.

By applying the CWT with the complex Morlet wavelet to Equation (4), the first term of the wavelet transform can be approximated as [18]

\[ T_{cz}(a, b) \approx 0.5D_L(b)e^{i\omega_c(t)} \psi^*(a\phi_L(b)) \]

where \( a \) is the scale parameter; \( b \) is the shift parameter in time; and \( \psi^*(\cdot) \) stands for complex conjugate of Fourier transform of \( \psi(\cdot) \). For a mono-component signal, the modulus of the wavelet transform is
transform $T_{CL}(a,b)$ is essentially maximum in the neighborhood of a curve $a_r(b)$, called the ridge of the wavelet transform, which satisfies the following condition [18].

$$a_r(b) = \frac{\partial R}{\partial b}$$ (8)

The snake penalization method proposed by Carmona et al. [18] is selected in this study to search the ridge, by which the instantaneous frequency of nonlinear aerodynamic lift can be determined through Equation (8). On the other hand, the instantaneous amplitude of nonlinear aerodynamic lift force can be found from Equation (7) in terms of the modulus. The accuracy of the above-mentioned method for the determination of instantaneous features of nonlinear aerodynamic lift force is dependent upon the time and frequency resolutions, which are merely restricted by the mother wavelet resolution. For the complex Morlet wavelet, the time and frequency resolutions at a given frequency $f_i$ in the CWT can be expressed as [19]

$$\Delta t_i = \frac{t_i}{\sqrt{2} f_i} \quad \Delta f_j = \frac{f_j}{2\pi \sqrt{2} f_i}$$ (9)

Therefore, the centre frequency $f_c$ of the mother wavelet is a critical parameter in defining the resolution capability. In this study, the wavelet entropy method proposed by Lin and Qu [20] is applied to determine the centre frequency $f_c$. The subsequent investigation reveals that the nonlinear aerodynamic lift coefficient may vary with time at several frequencies although the forced oscillation is at a single frequency. Therefore, the lift coefficient shall be expressed as [19]

$$C_{L_i}(t) = \sum_{j=1}^{m} D_{L_i}(t)\cos[\varphi_{L_i}(t)]$$ (10)

The amplitude $D_{L_i}(t)$ as well as the derivatives $\varphi'_{L_i}(t)$ also slowly vary with time. The wavelet transform, being a linear transform, of the lift coefficient by using the Morlet wavelet yields

$$T_{CL}(a,b) \approx 0.5 \sum_{i=1}^{n} D_{L_i}(b)e^{-i\varphi_c(b)(b)}\psi^*[a\varphi'_{L_i}(b)]$$ (11)

After the ridges are found using the snake penalization method, the instantaneous frequency and amplitude for each of the concerned frequency components in the lift coefficient $C_{L_i}(t)$ can be determined. Phase angle $\varphi_{L_i}(b)$ can be also determined from the complex function $T_{CL}(a,b)$.

4 NUMERICAL RESULTS AND DISCUSSIONS

4.1 Numerical model of oscillating square section

A square section of width $H=0.15m$ is selected as an example. The whole computational domain is sketched in Figure 1 and the corresponding parameters defining three regions as described in Section 2.1 are also designated. The height of body-fitted meshes is about 1.5mm or 2.0mm. The initial computation meshes are shown in Figure 2, in which the rigid boundary layer region has 14,712 quadrangular meshes, the dynamic mesh region has 13,398 triangular meshes, and the static mesh region has 7,898 quadrangular meshes. The initial angle of attack at the equilibrium position is set as zero. The inflow is set as uniform flow with a mean speed $U$ and a turbulence intensity of 0.5%. The reduced velocity $V_r=U/fH$ varies from 5.0 to 12.0, in which the frequency $f$ of forced oscillation is 4.0Hz. The Reynolds number $Re=\rho UH/\mu$ also varies from $3.1\times10^4$ to $7.4\times10^4$. The initial amplitude $A_0$ of harmonic oscillation is 5mm. Parameters $t_1$, $t_2$, $t_3$ and $t_4$ in Equation (2) are set as $14/f$, $59/f$, $72/f$ and $119/f$, respectively. The iteration time step is $3\times10^{-4}s$. 
4.2 Characteristics of flow around the fixed square section

To demonstrate the effectiveness of RNG \(k-\varepsilon\) turbulence model, the unsteady flow field around the fixed square section at zero incidence of wind is simulated. In this simulation, the computational meshes are as same as the meshes shown in Figure 2. The mean speed of incoming flow is \(U=4.2\text{m/s}\) and the Reynolds number is \(Re=4.3\times10^4\). The iteration time step is \(3\times10^{-4}\text{s}\). The computed time-averaged velocity \(<u>\) along the centre line is shown in Figure 3 and compared with previous experiment and CFD results [21-24]. The computed results are in good agreement with experimental results and better than the previous computational results, but the size of reverse
flow region is smaller than experiment results. The latter may be induced by large turbulent viscosity in the RNG k-ε turbulence model. The mean surface pressure $C_p$ around the square section is shown in Figure 4 and compared with previous experiment and CFD results [25-28, 10]. The corners of body are indicated in figure. The present results demonstrate a good agreement with experimental results. The Strouhal number of the fixed square section is $St=0.129$ by analyzing the present lift force time history. This is close to the experimental result $St=0.12$ [29] and the CFD results $St=0.14$ [9] and $St=0.128$ [10].

4.3 Frequency response component of lift force on the oscillating square section

As an example, the computed lift coefficient $C_L(t)$ is shown in Figure 5 at $V_r=6, 7$. The centre frequency of the complex Morlet wavelet is determined by using the wavelet entropy method [20]. The centre frequencies are set as $f_c=4.0$Hz and $f_c=4.5$Hz for $V_r=6, 7$, respectively. The corresponding scalograms obtained by CWT are shown in Figure 6. A unique ridge corresponding to the frequency response component of lift force can be found at $V_r=7$ since oscillation is in lock-in region. Two ridges can be observed at $V_r=6$ when the forced amplitude is small. One ridge is related to vortex-shedding frequency and the other is corresponding to frequency response component of lift force. As the forced amplitude gradually increases, oscillation will gradually go into lock-in region. Therefore, two ridges gradually merge into one at $V_r=6$ in Figure 6. This observation is in good agreement with the experiment result [7]. The ridge can be identified by snake penalization method [18].

The frequency response component of the lift coefficient can be expressed as

$$C_L(t) = D_{CL}(t) \sin[2\pi ft + \beta_{CL}(t)]$$

where the instantaneous amplitude $D_{CL}(t)$ equals a certain value $D_{L}(b)$ defined in Equation (11), such as $D_{L1}(b)$; phase angle $\beta_{CL}(t)$ can be determined from $\varphi_{L}(b)$ defined in Equation (11), such
as $\varphi_{LA}(b)$. Therefore, the instantaneous amplitude $D_{CL}(t)$ and phase angle $\beta_{CL}(t)$ of frequency response component can be determined from the corresponding wavelet ridge.

The instantaneous amplitude $D_{CL}(t)$ and the corresponding phase angle $\beta_{CL}(t)$ at different reduced velocities are shown in Figure 7 when the square section is subjected to a forced asymptotic transverse oscillation of varying amplitude defined in Equation (2). It is clear that the relation between instantaneous amplitude $D_{CL}(t)$ and excitation amplitude $D_{AF}(t)$ is nonlinear. In general, the instantaneous amplitude $D_{CL}(t)$ for reduced velocity $V_r$ in lock-in region is larger than that for reduced velocity $V_r$ outside lock-in region. An exceptional case is observed at $V_r=6$. When excitation amplitude is small, the instantaneous amplitude $D_{CL}(t)$ at $V_r=6$ is relatively small. However, the instantaneous amplitude $D_{CL}(t)$ rapidly increases when excitation amplitude reaches above 16mm. One can observe from Figure 6 that the oscillation in this case is gradually going into lock-in region. This reveals that the range of lock-in region increases with the

![Image of Figure 7](image1)

(a) Instantaneous amplitude $D_{CL}(t)$
(b) Instantaneous phase angle $\beta_{CL}(t)$

Figure 7 Simulated instantaneous amplitude and corresponding phase angle of frequency response component of lift coefficient at different reduced velocities

![Image of Figure 8](image2)

(a) Forced amplitude $D_{AF}=15mm$
(b) Forced amplitude $D_{AF}=20mm$

Figure 8 Comparison of amplitude of frequency response component between present and previous results

![Image of Figure 9](image3)

(a) Excitation amplitude $D_{AF}=15mm$
(b) Excitation amplitude $D_{AF}=20mm$

Figure 9 Comparison of phase angle of frequency response component between present and previous results
increasing excitation amplitude. This observation is in good agreement with the experimental result of Otsuki et al. [7]. The phase angle $\beta_{CL}(t)$ at different reduced velocities is relatively steady as excitation amplitude increases. The phase angle $\beta_{CL}(t)$ is negative or positive for wind speed $U$ below or above the resonance speed. It is clear that positive phase angle $\beta_{CL}(t)$ is required for vortex-induced oscillation which only occurs once the reduced velocity goes into the lock-in region. Some values of instantaneous amplitude $D_{CL}(t)$ and the corresponding phase angle $\beta_{CL}(t)$ with varying reduced velocity $V_r$ at several excitation amplitudes are compared with previous experiment and CFD results at the corresponding excitation amplitudes, as shown in Figure 8 and 9. The most noticeable feature is the sudden increase from negative to positive phase angle through the lock-in region. This feature is perfectly reproduced by the present method. The reduced velocity corresponding to the resonance speed is adjacent to 8.0 from Figure 8 and 9. This observation is supported by the previous experiment results, such as that given by Otsuki et al. [7]. The simulated general tendency of instantaneous amplitude $D_{CL}(t)$ and phase angle $\beta_{CL}(t)$ is in good agreement with the previous experiment and CFD results. Nevertheless, two typical discrepancies can be found. One lies in the lock-in region where the instantaneous amplitudes $D_{CL}(t)$ of the present and previous data are distributed in a large range. The other lies in below or above lock-in region where the phase angle $\beta_{CL}(t)$ of the present and previous data are also dispersed. These discrepancies will be discussed in Section 4.4.

4.4 Discussions

Since the instantaneous flow field around the square cylinder is highly three-dimensional (3D), the 2D CFD simulation cannot fully reproduce this feature. The previous CFD study [24] has obviously demonstrated that the spectrum shape of lift force from 2D simulation shifts from the spectrum of lift force obtained from the experiment [29] to a relative high vortex-shedding frequency. This may induce the peak of amplitude $D_{CL}(t)$ from the CFD simulation appearing at a relative low reduced velocity while that from the experiment data lies at a relative high reduced velocity, as shown in Figure 8.

The spanwise correlation of the fluctuating surface pressure on the square cylinder has been early studied by wind tunnel experiments [26, 29, 30]. The experimental lift force on the square cylinder is actually a space-averaged value. The present 2D simulated lift force does not take the influence of spanwise correlation into account. This may cause large discrepancy between the simulation and experiment results. The experimental lift forces may be mainly affected by the spanwise correlation of the fluctuating surface pressure on the side surface of the square cylinder.

In the following, only the frequency response component $L(t)$ of lift force on the square cylinder will be discussed. For two points A and B on the top or bottom side surface of the square cylinder along the axial direction, assume that the correlation coefficient $R_{AB}(z,t)$ is independent on the time $t$ during the square cylinder oscillation.

$$ R_{AB}(z,t) \approx R_{AB}(z) $$

Further assume that the phase angle $\beta_{CL}(t)$ defined in Equation (12) is also independent on the axial position. The frequency response component $L(t)$ can be expressed as

$$ L(t) = 0.5 \rho U^2 H^2 C_{l,t}(t) \int_0^L R_{AB}(z) \, dz $$

Define the axial correlation length scale of the square cylinder $C_R$ as

$$ C_R = \int_0^L R_{AB}(z) \, dz $$

The frequency response component of the experimental lift coefficient $C_{l,t}(t)$ is defined as
Therefore, the experimental \( CL_{\text{L,E}}(t) \) and the present 2D simulation \( CL(t) \) have relationship:

\[
L \cdot CL_{\text{L,E}}(t) = CR \cdot CL(t)
\]  

(17)

The expression of \( CL_{\text{L,E}}(t) \) can be expressed by Eq. (12) and the corresponding instantaneous amplitude is defined as \( D_{CL_{\text{L,E}}}(t) \). Assume that experimental phase angle equals the simulation one, the relationship between the experimental \( D_{CL_{\text{L,E}}}(t) \) and simulating \( D_{CL}(t) \) can be given as

\[
L \cdot D_{CL_{\text{L,E}}}(t) = CR \cdot D_{CL}(t)
\]  

(18)

In this expression, the amplitude \( D_{CL}(t) \) from the 2D CFD simulation cannot be directly compared with the experimental amplitude \( D_{CL_{\text{L,E}}}(t) \) since the experimental square cylinder is a fully 3D body. At the same time, the correlation length scale \( CR \) is dependent on the length \( L \) of the square cylinder. The amplitude \( D_{CL_{\text{L,E}}}(t) \) may vary with length of square cylinder for the same experimental condition.

Otsuki et al. [7] investigated the square cylinder of width \( H=150 \text{mm} \) and length \( L=660 \text{mm} \) in a wind tunnel. Referring to the spanwise correlation distribution of fluctuating pressure of the square cylinder [26, 29], the correlation length scale \( CR \) is approximately \( 3.2H \) for \( L/H=4.4 \). Therefore, the ratio of the experiment \( D_{CL_{\text{L,E}}}(t) \) to the 2D simulated \( D_{CL}(t) \) is about 0.73 from Equation (18). Other factors are also needed to be mentioned. For example, the method of subtraction of the inertia force from the measured data consisting of the aerodynamic force plus the inertia force may introduce some numerical errors, as pointed out by Otsuki et al. [7]. The spanwise correlation of phase angle \( \beta_{CL}(t) \) may also introduce some numerical errors. These factors may further decrease the ratio of the experiment data to the 2D simulation result.

By considering the above correlation coefficient and numerical errors, the present numerical results are acceptable. If 3D CFD simulation and more advance turbulent models are used, better simulation result can be expected.

5 CONCLUSIONS

(1) The proposed CFD simulation integrated with the CWT analysis can be used to determine nonlinear features of the aerodynamic forces on the square section in forced large amplitude oscillation. The simulated results are in good agreement with the previous experimental data and CFD results.

(2) The amplitude of aerodynamic forces on the square section depends on the amplitude of the forced oscillation in addition to reduced velocity.

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REFERENCES
