Understanding rapid flow separation with spanwise variation

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ABSTRACT: The current study investigates the influence of spanwise flow on three-dimensional vortex evolution. Using a robotic apparatus, a flapping motion was imposed on high aspect-ratio profiles. The flapping motion produced a spanwise variation in separation, and was reproduced for profiles with and without sweep to vary spanwise velocities. An analytical model for leading-edge vortex growth was proposed, based on the transportation of vorticity-containing mass through the shear-layer. When applied to the flapping motion, it was found that the analytical model predicted similar force coefficients with and without spanwise flow, which was confirmed by experiment. The model also predicted that the rate of vortex growth decreased with time in the presence of spanwise flow, although this could not be immediately verified experimentally.

KEYWORDS: Leading-edge vortex, spanwise flow, vortex modeling.

1 INTRODUCTION

Traditionally it is common to analyse flapping wings and rotating blade systems as a series of quasi-two-dimensional elements [1, 2]. However, flapping and rotating systems can exhibit large spanwise gradients in velocity and effective incidence. These spanwise gradients can couple with rapid flow separation to form highly three-dimensional vortex behaviour. The application of two-dimensional models to such highly three-dimensional flows will typically result in large errors when predicting aerodynamic performance [3]. Consider the case of a rotating blade, as on a wind turbine, experiencing a sharp-edged axial gust. The spanwise distribution of angle of attack \( \alpha(r) \) from the base flow \( u_{\infty} \) takes the form arctan\((u_{\infty}/\omega r)\). As shown in Figure 1, the change in effective incidence for a gust of strength \( \Delta u = u_{\infty} \) would be greatest towards the root. Large, rapid changes in effective incidence towards the root \( (r/R \approx 0) \) can be expected to cause highly separated flow, while this effect would diminish when moving towards the tip \( (r/R \approx 1) \). This can be compared to the root-flapping of a bird’s wing. Changes in effective incidence would in this case grow towards the tip. At low speeds, where highly-separated flows are expected, gradients relating to vortical growth would increase towards the tip instead. In both scenarios, the low-momentum of separated flows would be small relative to coriolis and centripetal accelerations [4]. These accelerations would act to drive flow away from the root. Therefore, the direction of spanwise flow can coincide or oppose the gradient of effective incidence. The relationship between these two
effects, i.e. spanwise separation and coriolis/centripetal acceleration, is not immediately obvious.

![Figure 1: The velocity triangle and resulting forces for a rotating blade (left), and change in effective incidence as a function of span for a rotating blade subjected to a sharp-edged axial gust (right).]

Of particular interest is how the leading-edge vortex (LEV) moves in the spanwise direction [5]. However, in highly three-dimensional cases, conventional blade-element schemes would fail to provide insight into spanwise transportation of vorticity. In the current study, analytical models for two- and three-dimensional vortex growth are proposed. These models are then compared with results obtained experimentally so as to verify the robustness of the model. Comparisons can be drawn from the relative influence of these sources of acceleration to spanwise velocity.

2 BACKGROUND

Consider the vorticity transport equation for an incompressible, barotropic fluid with conservative body forces:

\[
\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla) \vec{\omega} = (\vec{\omega} \cdot \nabla) \vec{u} + \nabla \times \left( \frac{\nabla \cdot \tau}{\rho} \right). \tag{1}
\]

The physical interpretation of each term from left to right can be described as: the changes in vorticity of the fluid due to unsteadiness, convection, vortex stretching through velocity gradients and the viscous diffusion of vorticity. Let us introduce a coordinate system set to a generic profile such that the \(z\)-direction is aligned with the blade span, and the \(x\)-direction with the blade chord, with the remaining \(y\)-coordinate then necessarily being normal to the blade surface, as shown in Figure 2. For the case of a simple plunging motion on a two-dimensional blade, a number of simplifications can be made. Firstly, viscous diffusion can be ignored under the assumption that timescales of diffusion are much larger than vortex growth itself. Furthermore, all gradients in the \(z\)-direction must necessarily be zero. The \(z\)-vorticity transport equation will subsequently reduce to:

\[
\frac{\partial \omega_z}{\partial t} + u \frac{\partial \omega_z}{\partial x} + v \frac{\partial \omega_z}{\partial y} = 0 . \tag{2}
\]
This is not conditional on a spanwise velocity field, caused for instance by a infinitely long wing of finite sweep, or on the existence of \( \omega_z \) or \( \omega_y \). It follows that, after the elimination of \( z \)-gradient term, the remaining vortex stretching terms must sum to zero:

\[
\omega_x \frac{\partial w}{\partial x} + \omega_y \frac{\partial w}{\partial y} = \left( \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \right) \frac{\partial w}{\partial x} + \left( \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right) \frac{\partial w}{\partial y} = \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} - \frac{\partial w}{\partial y} \frac{\partial w}{\partial x} = 0 .
\] (3)

As it is reasonable to assume a leading-edge vortex (LEV) would form parallel to the leading edge, it follows from equation (2) that the \( \omega_z \) feeding the LEV would be confined to the \( x-y \) plane. If the LEV is continually fed, it would grow until the vortex detaches and convects downstream. Thus, it may be conjectured that there is no two-dimensional mechanism from which one may stabilize the LEV. Consider now instead a three-dimensional case that is superficially similar, but where all gradients in the \( z \)-direction are non-negligible. A flapping wing would appear as a spanwise-dependent plunging motion. An order-of-magnitude argument can be made where \( \omega_z \) is much larger than either \( \omega_x \) or \( \omega_y \). In other words, the LEV can still be considered roughly parallel with the \( z \)-axis. The vorticity equation would then reduce to:

\[
\frac{\partial \omega_z}{\partial t} + u \frac{\partial \omega_z}{\partial x} + v \frac{\partial \omega_z}{\partial y} + w \frac{\partial \omega_z}{\partial z} = \omega_z \frac{\partial w}{\partial z} .
\] (4)

Equation (4) introduces the ability for vorticity to be transported and stretched in the \( z \)-direction. In some circumstances, the vortex stretching term may be sufficiently large to cancel the convection terms. Only when those terms balance can \( \partial \omega_z / \partial t \) be brought to zero and equilibrium be obtained. In other words, vortex stabilization must by definition require a three-dimensional flow. Planforms such as delta wings drain vorticity with spanwise flow to provide stable vortex lift under quasi-steady conditions. However, it is not clear what specific three-dimensional features, such as wing kinematics or shape, would result in an appropriate balance of vortex convection and stretching in highly-unsteady motions. This question ultimately motivates the development of the model described in the following section.
3 ANALYTICAL MODEL

3.1 Two-Dimensional Case

An analytical model for the unsteady lift on a plunging flat plate has been developed inspired by the work of Kaden [6]. As described in his model, the growth of a two-dimensional vortex can be attributed to the transport of vorticity-containing mass through the shear layer into the vortex in question. More recently, a model for the steady flux of vorticity in a shear layer has been tested by Sattari et al. [7]. The mass-flux into the LEV may therefore be described through conservation of mass for an incompressible fluid as:

\[ m'(t) = \rho \int_0^d \int_0 u(y, t) \, dy \, dt, \] (5)

where \( m'(t) \) is the vorticity-containing mass in the vortex per unit span, \( u(y, t) \) is the shear-layer velocity profile and \( d \) is the shear-layer thickness. If the area of the vortex described by \( m'(t) \) is then approximated as a roughly semi-circular shape attached to the blade, as in Figure 3, its radius \( R(t) \) would be:

\[ R(t) = \sqrt{\frac{2 m'(t)}{\pi \rho}}. \] (6)

Knowing \( m'(t) \), and thus \( R(t) \), we may then compute the circulation of the vortex by the line integral of velocity around the vortex core:

\[ \Gamma(t) = \oint \vec{u} \cdot d\vec{l} = \pi u(d, t)R(t), \] (7)

where \( u(d, t) \) is the velocity on the outer edge of the shear layer and the velocity along the profile surface is assumed zero. Recently it has been shown by Pitt-Ford and Babinksy that the bound circulation on flat plates in such unsteady conditions is negligible [8]. Thus, the LEV may be assumed to be the sole contributor of circulation. Lift may then be calculated, as a first approximation, by the Kutta-Joukowski equation. Difficulty now arises when attempting to predict the properties of the shear-layer growth from the bulk kinematics of the blade such as the plunge velocity \( \dot{h}(t) \) and the effective incidence, \( \alpha_{\text{eff}}(t) = \arctan(\dot{h}/\dot{u}_\infty) \).

In the current model, the LEV is taken as a semi-circular shape located directly over the blade, while the shear layer exists on the outer radius of this semi-circle at position \( \alpha_{\text{eff}} \), as seen in Figure 3. The outer velocity \( u(d, t) \) is determined by the superposition of three velocities: an acceleration around a cylindrical blockage of radius \( R(t) \) at position \( \alpha_{\text{eff}} \); the induced velocity from the LEV itself; and the projection of the plunge velocity \( \dot{h} \) onto the tangent of the semi-circle at \( \alpha_{\text{eff}} \):

\[ u(d, t) = u_\infty \left( 1 + \frac{R^2(t)}{r^2} \right) \sin(\alpha_{\text{eff}}) + \frac{\Gamma(t)}{2\pi r} + \dot{h} \cos(\alpha_{\text{eff}}), \] (8)

where \( r = R(t) + d \), \( u_b \) is the velocity component from blockage effects, \( u_i \) is the velocity component induced by the LEV, and \( u_p \) is component from the projection of the plunging motion. Furthermore, \( u(0, t) \) is taken as the induced velocity component evaluated at \( r =
$R(t)$. From empirical observations, the shear-layer velocity profile was found to have an approximately sinusoidal transition from $u(0, t)$ to $u(d, t)$ such that:

$$u(y, t) = \frac{u(0, t) - u(d, t)}{2} \cos \left(\frac{\pi y}{d}\right) + \frac{u(0, t) + u(d, t)}{2},$$

which then reduces the integral for vorticity-containing mass to:

$$m'(t) = \rho \int_0^t \frac{u(0, t) + u(d, t)}{2} \, dt.$$  \hfill (10)

Figure 3: Comparison of the instantaneous streamline system (left) with the simplified model of a leading-edge stagnation point and semi-cylindrical blockage (right).

3.2 Extension to the Three-Dimensional Case

Given $\Gamma(z, t)$ is known for each two-dimensional slice, a blade-element scheme can be implemented to compute lift for three-dimensional motions. In the current study, a model was developed for the case of a high aspect-ratio blade far from the tip regions, such that three-dimensional effects were due exclusively to spanwise flow from sweep or centripetal/coriolis accelerations. In three dimensional flows, it is likely that flow near the blade can be broken into spanwise and chordwise components. While the chordwise-component of velocity contributes to vortex growth through the shear layer, the spanwise component serves to transport mass along the span only. We can revise the term for $u(d, t)$ such that:

$$u(d, t) \bigg|_z = u_c \left(1 + \frac{R(z, t)}{r^2}\right) \sin(\alpha_{\text{eff}}) + \frac{\Gamma(z, t)}{2\pi r} + \dot{h}(z) \cos(\alpha_{\text{eff}}),$$  \hfill (11)

where $u_c$ is the chordwise component of $u_\infty$ and $\alpha_{\text{eff}}$ is revised accordingly as $\arctan(\dot{h}/u_c)$. In addition, we must include an extra term for $m'(t)$ to account for the mass transfer through spanwise flow:

$$m'(t) \bigg|_z = \rho \int_0^t \left(\frac{u(0, t) + u(d, t)}{2} + \int_R \vec{u}_s(z, t) \cdot \vec{n} \, dA\right) \, dt,$$  \hfill (12)

where $z$ is the spanwise position, $R$ is the surface swept by a semi-circular disc of radius $R(z, t)$ at the blade element $z$ and $\vec{u}_s(z, t)$ is the spanwise component of velocity. It is worth noting that as gradients in the $z-$direction tend towards zero, equations (11) and (12) reduce to the two-dimensional cases represented by equations (8) and (10).
4 EXPERIMENTAL SETUP

Various blade motions were produced using a six degree-of-freedom hexapod manipulator, as shown in Figure 4. The hexapod was mounted over top of a custom water tunnel, which operated at 0.2m/s. A six-component ATI Gamma force/torque balance was flanged to the hexapod with a high aspect ratio flat blade of varying sweep $\Lambda = 0^\circ$ and $-45^\circ$ below. Each of the flat blades has a chord of 5cm measured normal to the leading edge, and a thickness of 3.12mm. In all test cases, the wing pierced the free-surface, which was considered as a mirror plane. Furthermore, the gap between the blade tip and tunnel floor was maintained under 3mm and therefore was similarly modeled as a mirror plane. The Reynolds number based on chord and freestream flow was $Re = 10,000$. Two test motions were considered. The first and most simple motion was of a sinusoidal-plunging movement as shown in equation (13):

$$\dot{h} = 0.1 \sin(2t) .$$  \hspace{1cm} (13)

where $\dot{h}$ is the plunge velocity. This corresponds to a reduced frequency of:

$$k = \frac{\pi f c}{u_\infty} = 0.25 ,$$  \hspace{1cm} (14)

where $f$ is the frequency of the motion. As a second case, a flapping motion described by equation (15) was considered. Kinematics were such that the plunging motion of case 1 was replicated at the 3/4 span, neglecting centripetal accelerations. The plate rotated about a virtual pin at the tunnel wall, by way of the programmable hexapod controller. The motion of case 2 took the form:

$$\dot{\theta} = 0.15 \sin(2t) .$$  \hspace{1cm} (15)

Figure 4: Experimental setup with high aspect ratio blade (A), 6-component balance (B), and hexapod manipulator (C) mounted to water tunnel.
5 RESULTS AND DISCUSSION

A comparison of the temporal evolution and spacial variation of vortex growth demonstrates the differences arising from spanwise flow in the flapping motion. Modeled vortex radii for flapping motion with and without sweep are compared in Figure 5. It can be seen that the rate of vortex growth slows with the period much more pronouncedly in the swept case than the unswept case. This suggests that vortex feeding and draining are tending towards similar rates. The mass flux through the shear-layer, which is roughly proportional to the shear-layer velocities shown in Figure 6, does not decrease with period. Therefore, mass transfer through spanwise flow becomes increasingly large, limiting vortex growth. If the rate of vortex growth would eventually decrease to zero, this would represent a stable vortex. Furthermore, the vortex is smaller in the swept case than in the unswept case across the entire span. This can be partially explained from the smaller chordwise velocity component $u_c$ relative to the freestream velocity $u_\infty$, limiting the rate of vortex feeding. However, since the vortex radius in the swept case is not simply a multiple of the unswept case as the change in chordwise velocity is, spanwise mass transfer towards the tip may also contribute to the smaller radius.

![Figure 5: Modeled vortex radius as a function of period (left, averaged with span), and as a function of span (right, averaged with period).](image)

To better understand the differences between swept and unswept cases, the individual components of shear layer velocity are compared in Figure 6. Note that the velocity component due to blockage effects is higher in the swept case than in the unswept case despite the smaller blockage radius. For this velocity component $u_b$, the effect of a smaller LEV radius in the swept case is balanced by a larger effective incidence. This effect of effective incidence can also be seen in the plunging component of velocity $u_p$. The equal induced velocity components $u_i$ imply that unswept cases have a higher LEV circulation, given the larger vortex radius in unswept cases.

Measurements of lift coefficients show reasonable agreement with the analytical model for both two- and three-dimensional cases, shown in Figure 7. The largest discrepancies occur
at \( t/T = 0 \), where the analytical model predicts a slope of zero. All measurements show a finite slope at \( t/T = 0 \), as the LEV begins to grow instantaneously. It is speculated that this effect is related to the vorticity-containing mass in the boundary layer contributing to the initial stages of LEV growth. Furthermore, agreement between the analytical model and measurements are also poor after one-quarter cycle, as the model cannot account for the vortex eventually detaching and convecting downstream.

Figure 6: A comparison of terms contributing to \( u(d, y) \) as a function of period, for cases with and without spanwise flow; \( u_b \) is velocity due to blockage, \( u_p \) is velocity due to plunging, and \( u_i \) is velocity induced by the LEV (see eq. (8)).

Figure 7: Lift coefficient as a function of period for plunging motion (left) and for flapping motion (right).
6 CONCLUSIONS

In this study, an analytical model for LEV growth has been developed and tested. When compared to physical measurements, rates of vortex growth between $0.05 \leq t/T \leq 0.2$ were reasonably accurate. Furthermore, the model predicted that the rate of LEV growth decreased across the first quarter period for a flapping motion in the presence of spanwise flow. This may contribute to vortex stability, however the reduction in vortex growth rate could not be immediately confirmed experimentally. The model was shown to be incapable of correctly predicting early-stage vortex growth. The model was also not developed to account for the eventual separation of the vortex from the profile. Nevertheless, the simple analytical description of the model and the ease of implementation are highly desirable. Future work will include verification that vortex growth can be limited by spanwise flow. Furthermore, the influence of an existing boundary layer on early-stage vortex growth and the influence of additional components of spanwise acceleration will be investigated.

REFERENCES


