ACTIVE CONTROL OF ALONG-WIND RESPONSE OF TALL BUILDING USING LQG CONTROLLER

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ABSTRACT

Most of modern tall buildings using lighter construction materials which have high strength and less stiffness are more flexible so could be excessive wind-induced excitations resulting in occupant discomfort and structural unsafety. It is necessary to predict such a wind-induced vibration response and seek to find out the method to mitigate such an excessive wind-induced vibration. Active control method using actuator control force for mitigating along-wind vibration of a tall building with linear quadratic Gaussian (LQG) controller is investigated. Fluctuating along-wind load acting on a tall building which can be treated as a stationary Gaussian white noise process could be simulated numerically in time domain using along-wind load spectra. And using this simulated along-wind load calculated along-wind response of a tall building without and with LQG controller.

Keywords: Along-wind Load, Numerical Simulation, Wind-load Spectra, Along-wind Response

Introduction

Most of modern tall buildings using lighter construction materials which have high strength and less stiffness are more flexible so occur excessive wind-induced vibration resulting in occupant discomfort and structural unsafety. It is necessary to reduce the wind-induced displacement and acceleration at the top floor level of tall building for increasing performance of occupant comfort and structural safety of building.

Although many studies for estimating and mitigating the wind-induced structural vibration of a tall building have been improved significantly, however the complicated mechanism interacting with the fluctuating atmospheric flow and building sides has not been developed successfully [Kareem A. (1982)]. The along-wind response of a tall building can be estimated theoretically by a gust factor method [Davenport A.G. (1967), Vickery B.J. (1972) and Simiu E. (1974)]. However the across-wind response can not be estimated using such a gust factor approach [Kareem A. (1982)]. The characteristics of fluctuating across-wind load is basically different from that of fluctuating along-wind load, which consist of approaching atmospheric turbulent flow of mean wind velocity direction [Davenport A.G. (1967), Vickery B.J. (1972) and Simiu E. (1974)]. The across-wind load consisting of complicated mechanism interacting with the fluctuating atmospheric flow and building sides, that is, the distortion of the mean flow, the flow separation, the vortex shedding and the wake formulated around leeward side of a building, so can not be predicted theoretically [Kareem A. (1982)]. Therefore across-wind load formulation depends on a boundary layer wind tunnel experiment using a scaled model [Kareem A. (1982, 1984)]. However it takes a lot of time and cost to do wind tunnel experiment.
Therefore many studies for mitigating such an excessive wind-induced vibration have been conducted for many decades [Housner G.W et al. (1997) and Soong T.T (1998)]. In recent years, they employed the modern optimal control theory and devices to obtain a required control force and reduced vibration response [Housner G.W et al. (1997) and Soong T.T (1998)].

In 1972, Yao introduced modern control theory into vibration control of civil engineering structures [Yao J.T.P (1972)]. As the fact that modern tall building subjected to fluctuating wind load would be oscillate at the fundamental natural frequency of the building is known to them, modern control theory and auxiliary devices have been applied to control wind-induced excitations of tall building. One of the common control devices is a tuned mass damper (TMD) system. The concept of TMD on reducing structural response was from Den Hartog (1947). It consists of a auxiliary mass with properly tuned spring and damping devices, which increasing damping in the main structure, so reducing the response of the main structure. Since, then, many studies have been conducted on the behavior and effectiveness of TMD. A number of TMDs have been installed in tall buildings for the control of wind-induced vibration [Housner G.W et al. (1997)]. The Center Point Tower in Sydney is one of the first TMD in a building. When wind load is modeled as a stationary Gaussian white noise, the TMD parameter for reducing wind-induced excitations in building could be derived by McNamara (1977). At that time 400-ton TMD has been installed for the Citicorp Center, which is 274m tall office building in New York. Another TMD has been designed in the John Hancock Tower in Boston. Both of TMD have been installed to reduce wind-induced vibrations [Chang J.C.H. (1980)]. It was accepted as a common knowledge that the performance of TMD could be increased by incorporating a feedback controller through the use of an active control force in the design of TMD, which is called active tuned mass damper (ATMD) [Chang J.C.H. (1980), Ankireddi S. et al. (1996) and Ricciardelli F. et al. (2003)]. However, the fact that ATMD is superior to TMD to reduce wind-induced vibration of a tall building is still question [Ricciardelli et al. (2003)]. After that many advanced studies of active control law including linear quadratic regulator (LQR), linear quadratic Gaussian (LQG), H₂ and H∞ have been developed [Yang J.N et al. (2002, 2003), Ankireddi S. (1997), Stavroulakis G.E et al. (2006) and [Housner G.W et al. (1997)].

In modern control theory, fluctuating along-wind load acting on a tall building is treated as a Gaussian white noise process. Therefore linear control system which have a system noise and measurement noise of a Gaussian white noise is considered as a stochastic linear control system.

In this study, LQG controller is used to investigate the effectiveness of reducing along-wind response of a tall building. Fluctuating along-wind load acting on a tall building was simulated numerically in time domain by using the along-wind load spectra proposed by G. Solari (1993). Simulation procedure used in this study is taken from Shinozuka and Deodatis (1987, 1996). And using this simulated fluctuating along-wind load, estimated along-wind responses of a tall building with and without LQG controller at the top floor level and found out the effectiveness of LQG controller in reducing along-wind response of a tall building.

Equations of motion

A tall building under fluctuating along-wind load with an active control force such as an actuator is shown in Figure 1. The building is modeled in this figure as a single degree of freedom system with a mass \( m_1 \), damping constant \( c_1 \), and stiffness constant \( k_1 \), which corresponding to the first mode modal mass, damping, and stiffness of the building, and \( f(t) \) represents the along-wind load, and \( u(t) \) is an active control force.
Figure 1. Building-Active Control force System

The linear dynamic equations of motion for a tall building subjected to fluctuating along-wind load and active control force could be written as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) + u(t)$$  \hspace{1cm} (1)

This equations could be rewritten in terms of the state-space formulation as follows [Soong T.T(1998)];

$$\dot{X}(t) = AX(t) + Bu(t) + Hf(t)$$  \hspace{1cm} (2)

where $X(t)=[x(t) \ x(t)]$ denote state vector with

$$A = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$ \hspace{1cm} (4)

is a system dynamic matrix

$$B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$ \hspace{1cm} (5)

is a location vector of $u(t)$

$$H = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$ \hspace{1cm} (6)

is a location vector of $f(t)$

**Numerical Simulation of Fluctuating Along-wind Load**

Fluctuating along-wind load treated as a random process of stationary Gaussian white noise could be simulated numerically in time domain using along-wind load power spectral density data. This simulated fluctuating along-wind load is particularly useful for some response estimation which is a narrow banded process such as along-wind response of a tall building. The numerical simulation procedure presented in this study is from Deodatis(1996), as follows

$$f(t) = \sum_{k=1}^{N} 2\sqrt{S_f(f_k)\Delta\omega \cos(\omega_k t + \phi_k)}$$  \hspace{1cm} (7)
where \( S_F(f_1) \) is the value of the spectral density of across-wind load corresponding to the first modal resonant frequency.

\[
\Delta \omega = (\omega_u - \omega_l) / N
\]

\[
\omega_k = \omega_l + \left( k - \frac{1}{2} \right) / N
\]

\( \omega_u \) = upper frequency of \( S(\omega) \)

\( \omega_l \) = lower frequency of \( S(\omega) \)

\( \Phi_t \) = uniformly distributed random numbers between 0–2π

\( N \) = number of random numbers

The along-wind load power spectral density used in equation (7) is that by G. Solari (1993) as follows

\[
S_F(n) = \left[ \rho BH C_{x,0} \bar{V}(h) \sigma_v(h) K_b \right]^2 S_{veq}(n)
\]

(8)

where

\[
S_{veq}(n) = \frac{S_v(h; n)}{\sigma_v^2(\eta)} \left[ \frac{n C_x B}{\bar{V}(h)} \right] \left[ \frac{1}{2} \right] \left[ e^{-\frac{1}{2} \eta^2} \right] \left[ 1 - e^{-2 \eta} \right]
\]

where

\[
L(\eta) = \frac{1}{\eta} - \frac{1}{2 \eta^2} (1 - e^{-2 \eta})
\]

\[
K_b = \frac{1}{H \bar{V}(h) \sigma_v(h)} \int_0^H \bar{V}(z) \sigma_v(z) \psi_1(z) \, dz
\]

\[
\frac{n S_v(z; n)}{\sigma_v^2(z)} = \frac{6.868 \frac{f L_v}{z}}{(1 + 10.302 \frac{f L_v}{z})^{5/3}}
\]

where

\[
f = \frac{nz}{\bar{V}(z)}
\]

\( C_{x,0}, C_z = \) lateral and vertical exponential decay coefficients

\( C_v = \) cross-correlation coefficient of pressure acting on the windward and leeward face

\( L_v(h) = \) integral length scale of turbulence at height \( h \)

\( \rho = \) air density

\( B = \) width of building

\( H = \) height of building

\( h = \) reference height of building

\( C_D = \) drag coefficient

\( C_m, C_e = \) absolute values of mean pressure coefficients on windward and leeward face

\( \bar{V} = \) mean wind velocity
\[ \sigma_r = \text{standard deviation of longitudinal turbulence} \]
\[ n = \text{frequency} \]
\[ S_f(n) = \text{power spectrum of first fluctuating modal force} \]

**Linear Quadratic Gaussian Controller**

A tall building which is subjected to fluctuating along-wind load which has a stationary white noise spectra can be considered as the linear dynamic system which have a system and measurement noise. If the system and measurement noises are zero-mean Gaussian white noise which have a constant covariance intensity, which are stationary white noise spectra matrices. If the external random load acting on a tall building such as a fluctuating along-wind load could be considered as a system noise which have a constant power spectral density of Gaussian white noise, we can formulate a plant model dynamics with LQG controller in modern optimal control theory as follows [Dorato P. *et al.* (1995)]

\[ \hat{X}(t) = AX(t) + Bu(t) + w(t) \quad (9) \]
\[ Y(t) = CX(t) + Du(t) + v(t) \quad (10) \]

where we let \( D=0 \) for simplicity. \( X(t) \) and \( Y(t) \) is the state and output vector. And \( w(t) \) and \( v(t) \) are the system and measurement noises respectively, which are assumed to be uncorrelated zero mean Gaussian white noises which have a covariance intensity matrices \( W \) and \( V \) respectively. That is,

\[ E[w(t)w(t + \tau)^T] = W\delta(\tau) \quad (11) \]
\[ E[v(t)v(t + \tau)^T] = V\delta(\tau) \quad (12) \]

and

\[ E[w(t)v(t + \tau)^T] = 0, \quad E[v(t)w(t + \tau)^T] = 0 \quad (13) \]

where \( E \) means the expectation operator and \( \delta(T - \tau) \) is a delta function.

It is known that the optimal controller \( u(t) \) in equation (9) is obtained when all states \( X(t) \) of the system and the output \( Y(t) \) is a combination of all states. However, in practice, all states \( X(t) \) are not available and that system and output measurement are driven by stochastic disturbance called noise. In that situations we need a state estimator or observer to estimate all states of the system. The design of the state observer could be performed using Kalman filter, which is an optimal state observer for a stochastic dynamic system.

Let \( \hat{X}(t) \) be the state estimate and \( Xe(t) = X(t) - \hat{X}(t) \) denote the estimation error, then the state be estimated by using Kalman filter described as

\[ \hat{X}(t) = A\hat{X}(t) + Bu(t) + L(Y(t) - C\hat{X}(t)) \quad (14) \]

where the observer gain matrix \( L \) is given by

\[ L = G C^T V^{-1} \quad (15) \]
where  
\[
J = E[Xe(t)Xe^T(t+\tau)]
\]

And  \(J\) satisfies the filter algebraic Riccati equation (FARE) 
\[
A\Gamma + \Gamma A^T + W - \Gamma C V^{-1}C^T \Gamma = 0
\]  
(16)

In LQG controller problem, the optimal controller \(u(t)\) in equation (9) that minimize the cost function \(J\) subjected to constrained equation (14) is determined by separately as the deterministic linear quadratic regulator (LQR) controller problem.

As with the deterministic LQR problem, the optimal control law that minimizes same quadratic cost functional which have a trade off between the state cost and control cost could be found. However, for wind excitation problem, the cost functional for the deterministic LQR problem can not be employed because of the stochastic nature of the state space formulation of the stochastic differential equation of (9). That is, the ensemble average over all possible realizations of the excitation is considered, so the cost functional \(J\) is given by 
\[
J = E \left\{ \lim_{T \to \infty} \frac{1}{T} \int_0^T [X^T(t)QX(t) + u^T(t)Ru(t)]dt \right\}
\]  
(17)

where \(Q\) and \(R\) are positive semi-definite and positive definite weighting matrices. Minimizing \(J\) with keeping the system response and the control effort close to zero needs appropriate choice of the weighting matrices \(Q\) and \(R\) [J.Suhardjo et al. (1992)]. If it is desirable that the system response be small, then large values for the elements of \(Q\) should be chosen. If it wants the control energy be small, then large values of the elements of \(R\) should be chosen.

The optimal controller \(u(t)\) is determined as follows 
\[
u(t) = -K\dot{X}(t)
\]  
(18)

where 
\[
K = RBP
\]

where \(P\) is the unique symmetric, positive semi-definite solution to the algebraic Riccati equation (ARE) given by 
\[
AP + PA^T + PBR^{-1}B^T + Q = 0
\]  
(19)

It is noted that we can determine the LQR controller feedback gain matrix \(K\) and the observer gain (Kalman filter) matrix \(L\) independently. This is the so called separation principle.

**Numerical Example**

This numerical example is from "Numerical Examples" presented in the reference [Solari G(1993)]. Along-wind rms responses of tall building without and with LQG controller were estimated using the along-wind load spectra proposed by G.Solari(1993).

The tall building’s height \(H=180m\), width \(B=60m\), depth \(D=30m\), first modal natural frequency \(n_1=0.27Hz\), critical damping ratio=0.015, first modal mass=24000000kg, \(h=120m\), \(\bar{V}(h)=40.96m/s\), \(\sigma_r(h)=5.39m/s\), \(L_r(h)=582.48m\), \(C_x=16\), \(C_z=10\), \(C_w=0.8\), \(C_{\nu}=0.5\), \(K_{\nu}=0.5\), etc.
The rms of along-wind response by Solari is 0.027m. Numerically simulated along-wind load is shown in Figure 1 and the along-wind response without LQG controller is in Figure 2, which have the value of rms is 0.021. This is a good approximated to that of the closed form response by Solari.

Estimated reduced responses with LQG controller are shown in Figure 3 and 4, where the values of $Q$, $R$, and $V$ are given by

$$Q = 1.0e + 008 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = 1.0e-012, \quad V = 1.0e-004, 1.0$$

and the values of rms for estimated reduced responses with LQG controller are given in Table 1.

![Figure 1. Time series of along-wind load](image1)

![Figure 2. Time series of along-wind response without LQG controller (rms=0.021)](image2)
As shown above results, as the value of measurement intensity covariance is increase, the reduced responses with LQG controller are decreased.

**Conclusions**

Fluctuating along-wind load acting on a tall building is simulated numerically using along-wind load spectra data and along-wind responses of a tall building without and with LQG controller were estimated using this simulated along-wind load. The estimated responses without LQG controller is a good approximated to that of analytic responses. And the reduced along-wind responses with LQG controller were estimated under varied parameters of LQG control law. Therefore LQG controller is effective and useful for designing active control of wind-induced vibration of a tall building to improve the performance and safety of a tall building.
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