Applications of Generalized Coordinate Synthesis Method in Wind Engineering

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ABSTRACT

Generalized Coordinates Synthesis (GCS) method is a convenient approach to predict wind-induced responses of buildings and structures based on the wind pressure time histories obtained by wind tunnel tests. Unlike traditional ways in frequency domain, GCS method calculates the wind-induced responses in time domain. The wind pressure time histories are firstly transformed into modal forces by matrix multiplication and consequently the single-degree-of-freedom generalized coordinate equations are solved readily by Fast Fourier Transform in frequency domain. Finally the responses can be obtained by the mode superposition. GCS method generally takes less than 10% of time consuming of traditional methods while the results are mathematically equivalent with them. In addition, further applications can be developed easily since the time histories of modal forces have been explicitly given using GCS.

Keywords: Generalized Coordinates Synthesis method, wind-induced response, mode superposition, High Frequency Pressure Integration

Introduction

The High Frequency Pressure Integration (HFPI) method is widely adopted to determine the overall wind-induced response with the development of the synchronous pressure measurement technique [Steckley et al. (1992), Cermak (2003)]. The traditional ways to predict the response is usually based on the complete quadratic combination (CQC) method. They require a large calculation amount since the wind-induce vibration are often involved with thousands of degrees of freedom (DOF) and a great number of point excitations.

In recent years, Lin (1992) proposed a new algorithm so-called Pseudo-Excitation Method (PEM) for random seismic responses. It significantly decreased the calculation amount by introducing pseudo-excitations derived from the decomposition of the excitation power spectrum distribution (PSD) matrices. The PEM has also been extended to the wind engineering field [Sun et al. (1998)] and applied for the wind-excited high-rise buildings [Xu et al. (1999)] and bridges [Zhu et al. (2005)]. However, most of the wind-response analysis for large civil engineering structures is carried out based on the wind pressure time history from the wind tunnel tests. Under such conditions, the advantages of the PEM are remarkably diminished by the large computational task of the estimation and decomposition of the wind-pressure PSD matrices so that PEM is still time-consuming for wind-induced response analysis. It is suggested that the PEM is more appropriate for the random vibration characterize by the PSD matrices of excitation-source.

The authors proposed a distinctive approach to calculate the wind-induced responses [Chen & Qian (2012)]. Unlike traditional ways in frequency domain, GCS method calculates the wind-induced responses in time domain on the basis of modal analysis. It directly solves...
the generalized coordinate’s dynamic equation using the discrete numerical method in the frequency domain. And the standard derivations of the response are consequently derived from the covariance matrices of the generalized coordinates. It reduces the calculation amount dramatically while keeping the same accuracy as the conventional CQC method. Moreover, a few of further applications can be developed easily since the time histories of modal forces have been explicitly given using GCS.

This paper describes the principal framework of GCS method and its applications in wind engineering.

Framework of GCS method

Mode decomposition of the dynamic equation for wind-induced response

Supposing the random wind excitations obtained from wind tunnel test are given at \( M \) positions, the dynamic equation of a linear and elastic structure with \( N \) DOF is expressed as

\[
[M] \ddot{x}(t) + [C] \dot{x}(t) + [K] x(t) = [R] P(t)
\]

Where \([M]\), \([C]\) and \([K]\) are the mass, damping and the stiffness matrices with the order of \( N \times N \) respectively; \([P(t)]\) is the \( M \times 1 \) time-varying wind load vector and the \( N \times M \) matrix \([R]\) expands the wind load \([P(t)]\) to all nodes of the structure where the wind load acts; \([x(t)]\) is the \( N \times 1 \) displacement vector. Adopting the mode superposition and only taking the first \( r \) modes into consideration, the structural displacements \([x(t)]\) can be expressed as the sum of the generalized coordinates

\[
[x(t)]=[\Phi] \{q(t)\}
\]

where \([\Phi]\) represents the \( N \times r \) modal matrix and \([q(t)]\) are the generalized coordinates. Substituting the above expression into Eq. (1) and under the assumption of the proportional damping, Eq. (1) is reduced into \( r \) SDOF equations. Taking the modal matrix normalized by the mass matrix, the equation of the \( j \)th generalized coordinate is written as

\[
\ddot{q}_j(t) + 2 \zeta_j \omega_j \dot{q}_j(t) + \omega_j^2 q_j(t) = f_j(t)
\]

where \( \zeta_j \) and \( \omega_j \) are the modal damping and circular natural frequency of the \( j \)th mode. The modal force \([f(t)]\) is determined by the mode shape and excitations

\[
[f(t)]=[\Phi]^T[R] \{P(t)\}
\]

CQC method establishes the relationship between PSD matrices of the response and the excitation on the basis of above equations. The covariance matrix of the response is subsequently obtained from the integral of PSD matrices.

However the calculation amount becomes very large when CQC method is applied in the wind-induced response analysis for a complex building or structure with thousands of DOF. GCS method provides a different way to calculate the response.

Calculation of the modal forces

There are usually hundreds of pressure taps on the scaled model in the wind tunnel test. The first step will be to expand the wind pressure from the measurement points to all nodes on which wind load act, as the matrix \([R]\) in Eq. (1) shall do. \([R]\) depends on the methods of expanding the original data. The inverse distance weighted (IDW) interpolation method is an optional method to employee. It’s defined as:

\[
w_j(t) = \frac{\sum p_k(t) / d_{kj}}{\sum 1 / d_{kj}}
\]

Where \( p_k(t) \) and \( w_j(t) \) is the wind force acting on the \( k \)th measurement point and the \( j \)th freedom of the structure, respectively. The \( d_{kj} \) represents the distance between them. Three
measurements points nearest the \( j \)th freedom should be picked up prior to the application of Eq. (5). In addition, the tributary area should be taken into account in Eq.(5). The matrix \([R]\) is a sparse matrix with only three non-zero elements in each row once the IDW interpolation method is employed. Consequently Eq. (4) can be rewritten as,

\[
\{f(t)\} = [\Phi]^{T}[R]\{P(t)\} = [\Phi]\{P(t)\}
\]

(6)

The transfer matrix \([T]\) only depends on the mode shape and the space distribution of the structural freedoms and the measurement points. So it can be computed priorly and such calculation needs to perform only once. As \([T]\) has a order of \( r \times M \) which is far less than the order of \([\Phi]\), the computational efficiency is remarkably improved.

Solution of the responses and the efficiency of GCS method

While the modal forces are determined, Eq. (3) can be solved by Duhamel integration in time domain or by Fourier transform in frequency domain. Although the Fourier transform doesn’t exist for the stationary random process, the discrete Fourier Transform (DFT) still can be performed on Eq.(3) because \( f_j(t) \) is a finite discrete series. Thus the generalized coordinate of the \( j \)th mode is given as:

\[
q_j(t) = \tilde{F}<H_j(i\omega)f_j(\omega)>
\]

(7)

where \( \tilde{F} < \cdot > \) denotes the inverse DFT and \( H_j(i\omega) \) and \( f_j(\omega) \) represent respectively the frequency response function of the \( j \)th mode and FFT result of the \( j \)th modal force.

Once the generalized coordinates of the first \( r \) modes are obtained, the mean response can be expressed as the supermotion of the average value of the generalized coordinates

\[
\{\bar{x}\} = [\Phi]\{\bar{q}\}
\]

(8)

In addition, the \( r \) should be large enough to obtain the satisfying results for the mean response. Subtracting the mean response and making some manipulations, the covariance matrix of the response can be derived as follows,

\[
[V_{xx}] = [\Phi][V_{qq}][\Phi]^T
\]

(9)

where \([V_{qq}]\) is the \( r \times r \) covariance matrix of the generalized coordinates.

Eq. (9) gives the relationship between \([V_{xx}]\) and \([V_{qq}]\). The standard derivation of the structural response at any freedom can be obtained conveniently from this equation. The equivalence of GCS method and CQC can be proved easily through Parseval energy integral equation.

In short words, GCS method is a high-efficiency approach appropriate for the wind-induced response analysis on the basis of the wind pressure time history. Although it’s similar to the traditional mode superposition method, some important steps, such as the calculation of the modal forces and the covariance matrix, have been optimized according to the analysis processes. Besides, CQC method as well as its improved forms is the most common and maybe only approach which can be found in previous studies on the wind-induced response. GCS provides a much better option to predict the wind-induced response.

Example and comparison of different methods

The efficiency of the GCS method has been verified by the wind-induced response calculation of a large-scale railway station. The time history of the wind pressure is directly obtained from the wind tunnel test. The rigid model test is carried out in the wind tunnel of China Academy of Building Research. The test section is 4m wide and 3m high. The photograph of the model is shown in Fig. 1.

There are 998 pressure taps and 8981 structural nodes on the surface where the wind acts. All of the pressure is sampled simultaneously. The DOF of the structural systems reaches 26943 if only the translation movement freedoms are counted in. The sampling rate is 400Hz corresponding to 7.3Hz in the full scale. The sampling time lasts 21s which results in
8200 time steps. The first 600 modes are taken into considered. It should be interpreted that the natural frequency of the 600th mode is about 7.1Hz which is well beyond the frequency resolution of the original data from wind tunnel test. Therefore only the background responses are obtained for those modes with natural frequency greater than 3.75Hz. However the more modes are included the more accuracy will be achieved, especially for the mean response calculation.

It takes only 13s by GCS to obtain the standard deviations of all structural nodes on which the wind acts while the calculation time will be 246s for PEM. The efficiency has been improved significantly. The standard deviations of the vertical displacement on some nodes are shown in Fig. 2.

![Wind tunnel model of the rail-way station](image1)

![Standard deviations of the vertical displacement (mm)](image2)

Further applications of GCS method and example

GCS is a method in time domain so that some applications can be developed readily.

Firstly, since the time history of generalized coordinates are obtained, the response time history can be determined consequently according to Eq.(2). However, Eq.(2) is invalid for the velocity and acceleration. They should be determined by mode superposition with the first and second derivative of the generalized coordinates. The second derivative of the generalized coordinates is given as follows,

\[
\dddot{q}_j(t) = -\tilde{F} \left\{ \omega^2 H_j (i\omega) f_{c\rho} (\omega) \right\}
\]

Secondly, it has therotical and practical importance to decompose the structural response into three components, i.e., the mean, background and resonant components. The mean response is expressed as the Eq. (8). And the background component is the response that exluded the mean response and the effects of the structural vibration. In other words, if the fluctuating part of the excitation at each time step is regarded as static force, the background response can be determined accordingly by the static structural analysis. The time history of the generalized coordinates corresponding to the background response is given as,
where \( \omega_j \) and \( f_j'(t) \) represent the \( j \)th natural frequency and the fluctuating part of the \( j \)th generalized force, respectively. The background component of the structural response is consequently obtained by the application of mode superposition similar with Eq.(2). After subtracting the mean and background components from the total response given by Eq.(2), the remain part is exactly the resonant component.

Fig. 3 presents the wind-induced displacement, velocity and acceleration on the top of a super high-rise building with a height of 436m (as shown in Fig.4). The quasi-static displacement, i.e., the mean response plus the background response is also given in this figure. All these time histories are solved using GCS method based on the wind tunnel pressure test.

\[
q_{bj}(t) = \frac{1}{\omega_j^2} f_j(t)
\]  

(11)

For responses other than displacement, their time histories and statistical results can be obtained by the mode influence matrix \([E]\) instead of mode shape matrix \([\Phi]\) in Eq.(2) and Eq.(8). Fig. 5 presents a typical cable-membrane structure. The traditional wind-resistant design method with equivalent static wind load is obviously not suitable for such structures because of the involvement of several vibration modes. The best approach is to provide the envelope of internal force of cables under the wind action to the structural designer. The internal force can be combined directly with the corresponding effects induced by other loads.
It will make the design process more convenient and more accurate. However, CQC method requires large comfort to carry out such analysis while the GCS method makes it practicable and more efficient. Fig. 6 presents the envelop of internal force of all 100 cables among different wind directions.

![Wind Tunnel Model and Structural Model](image)

**Fig. 5 A typical cable-membrane structure**

![Graph of Internal Force](image)

**Fig. 6 The envelope of the internal force of the cables**

**Conclusions**

GCS provides a distinctive and efficient approach to predict the wind-induced response. Although it is in essence an improved forms based on modal analysis, some important steps have been optimized according to the analysis processes. More critically, there are plenty of wind-induced response analyses based on CQC method as well as its improved forms but the approaches based on time domain are hardly found. GCS method provides a different idea to solve the problem and demonstrate its value by its concision and high efficiency.

GCS method has several remarkable advantages. Firstly, it improves the calculation efficiency significantly in contrast with the traditional ways in frequency domain. It generally takes less than 10% of time consuming of traditional methods while the results are mathematically equivalent with them. Secondly, responses other than displacement can be obtained readily once the mode influence coefficient is determined. Finally, it solves the response in time domain so that the response time history can be obtained in case of need.
References


