ALPHA-STABLE DISTRIBUTION FOR PREDICTION OF NEGATIVE PEAK WIND PRESSURES ON ROOFS OF LOW-RISE BUILDING

K. Balaji Rao¹, M.B. Anoop¹, P. Harikrishna¹, S. Selvi Rajan¹, Nagesh R. Iyer²
¹Scientist, CSIR-SERC, CSIR Campus, Chennai 600 113, India, balaji@serc.res.in
²Director, CSIR-SERC, CSIR Campus, Chennai 600 113, India, nriyer@serc.res.in

ABSTRACT

In this paper, an attempt is made to study the applicability of alpha-stable distribution for modeling the negative peak wind pressures on low-rise building roofs. The required wind pressure data is obtained from the aerodynamic database of Tokyo Polytechnic University. The generality and flexibility offered by alpha-stable distribution makes it a candidate distribution as a single model for predicting the extreme values of negative peak wind pressure coefficients at different regions on the building roof. The results suggest that for the prediction of extreme negative wind pressure coefficients, alpha-stable distribution is a better candidate distribution than the Gumbel minimum.

Keywords: Peak wind pressure coefficients, Low-rise buildings, Alpha-stable distribution

Introduction

It is known that when the small-scale turbulence associated with the incident wind interacts with low-rise buildings, it will result in large scale fluctuations of pressures induced on the roofs, especially nearer to regions of separation/vortex flow formation. Negative peak wind pressure coefficient of about -20 have been observed on the roof of the full scale experimental building at Texas Tech University [Tieleman (2003); Banks and Meroney (2001)]. Typically, the stochastic process associated with pressure fluctuations in the near portion of separation can exhibit wide-banded nature compared to the band width of incident wind turbulence spectrum [Kumar and Stathapoulos (2000)]. Depending on the mechanism of eddy formation (flow separation/vortex cone formation), location of pressure tap and angle of attack, for a given roof angle, generally negative pressure coefficients follow negatively skewed distributions. The large pressure fluctuations observed can be attributed to the flow separation [Bienkiewicz and Sun (1992)] or initiation of large vortices [Bienkiewicz and Sun (1992); Kawai and Nishimura (1996)] depending on wind and roof angles.

Determination of an appropriate distribution for predicting the negative peaks of wind pressure coefficients on low-rise building roof is an active area of research [see for instance, Holmes and Cochran (2003); Cope et al. (2005); Yang et al. (2013)]. Cope et al. (2005) identified that different probability density function (pdf) models need to be used for wind pressures on different regions of the structure. Recent efforts are towards identification of a single model for the negative tails. Yang et al. (2013) proposed the use of Hermite polynomial model.

In the present investigation, applicability of alpha-stable distribution [Nolan (2009)] for modeling the negative peak wind pressure coefficients on the roof corner regions of a low-rise gabled roof building is examined, using wind tunnel experimental data obtained from the aerodynamic database of Tokyo Polytechnic University [TPU (2013)].
Negative Peak Wind Pressure Fluctuations on Roofs of Low-Rise Buildings

Different investigators have put forward hypotheses for explaining the large negative, intermittent peak pressures observed on the roofs of low-rise buildings [Melbourne, 1980; Melbourne, 1993; Tieleman, 2003]. The reattachment of shear layer to the roof (which occurs for certain conditions of oncoming turbulence and angle of attack) causes the formation of separation bubble. The separation bubble is not steady and has two modes of pseudo-periodic unsteadiness [Blazewicz, 2007], namely, the shedding mode and the flapping mode. The shedding mode is associated with the shedding of large-scale vortices downstream from the separation bubble with a frequency of about $0.65U/x_R$ (where, $U$ is the mean free stream flow velocity, and, $x_R$ is the mean length of separation bubble). The flapping mode is associated with the enlargement and contraction of the separation bubble and the flapping motion of the shear layer with a frequency of about $0.12U/x_R$, accompanied by the shedding of much larger vortices. The large negative pressures observed on the roofs are caused by the large pressure drops beneath the vortices as they convect downstream. The intermittent nature of shedding of vortices leads to jumps in the pressure time history at any given point on the roof. It has been also identified that both small- and large- scale oncoming turbulence influence the development of negative peak pressures [Tieleman (2003)]. This is because, in the presence of only small-scale turbulence, the vortices do not attain full maturity before they are convected downstream and hence maximum peak pressures are not felt. However, when large-scale turbulence is also present, the vortices attain maturity before being shed downstream leading to peak suction pressures of larger magnitudes. The presence of multiple turbulence length scales contributing to the pressure fluctuation phenomenon at the roof corner is also evident from the spectrum of wind pressure presented in Fig. 7 of Kumar and Stathopoulos (2000). To account for these fluctuations in large negative peak pressures, attempts have been made to fit a suitable pdf for peak negative pressure coefficients, and propose an equation for design peak pressure that can be used in the design of components such as claddings [Holmes and Cochran (2003); Cope et al. (2005); Yang et al. (2013)]. Narasimha et al. (2007), in a different context, studied the statistical characteristics of positive- and negative- fluxes in wall bounded flow turbulence in a nearly neutral atmospheric boundary layer, and found that the flow form coherent structures and exhibit burstiness, calling for a separate distribution other than that follows Fourier description. The intermittency/burstiness causes jumps and thus large local pressure fluctuations. Hence, there is a need to use probability distributions with heavy tails to model these pressure fluctuations. It may be better to use distributions with power law decaying tails than exponential tails (which gives low values of probability to the tail regions) to predict very large pressures [Tieleman (2003); Banks and Meroney (2001)]. One candidate distribution with power law decaying tails, which has found applications in different fields [Nolan (2009)], is the alpha-stable distribution. In this paper, an attempt is made to study the applicability of alpha-stable distribution for modeling the negative peak wind pressure fluctuations on low-rise building roofs. The required wind pressure data is obtained from the aerodynamic database of Tokyo Polytechnic University.

Justification for Use of Alpha-Stable Distributions

The motivation for the study is based on the following observations:

- The probability density functions of fluctuating pressures measured on a building model in a boundary layer wind tunnel are found to be negatively skewed with heavier negative tails and sharper peaks than normal distributions [Li et al. (1999)]. Hence, there is a need to use probability distributions with heavy tails to model the large pressure fluctuations and to estimate the extreme values. While there are number of heavy-tailed probability
distributions, such as alpha-stable distribution, log-normal distribution, Student’s t-distribution, hyperbolic distribution, the use of alpha-stable distribution is supported by the generalized central limit theorem. Log-normal distribution is another heavy tailed distribution which is found to be useful in large number of applications, and has also been used for modeling wind pressure fluctuations measured in wind tunnel experiments [Li et al. (1999)]. However, the applicability of log-normal distribution for pressure fluctuations consisting of both positive- and negative- pressures need to be carefully examined.

- Alpha-stable distributions are a rich class of probability distributions, which can accommodate fat tails and asymmetry [Nolan (2009)]. The normal-, Cauchy-, and Levy-distributions are special cases of alpha-stable distributions, suggesting the generality of alpha-stable distributions. Therefore, alpha-stable distribution will be a suitable candidate in the efforts towards identifying a single model for the peak wind pressure coefficients in different regions of the roof of a structure.

- For wind pressure measurement on a building model, the flow is assumed to be steady (i.e. long run behavior of fluctuations is same) and the process of pressure fluctuation is ergodic. In such a condition, the environment around the building envelope vary slowly but, the aerodynamic interaction provides a heterogeneous environment that would result in trapped and hopping behavior (observed in such systems) of pressure field. This is clear from typical time traces of pressure coefficients reported in literature. As has been pointed out by Wang et al. (2012) the phenomenon underlying the observed stochastic pressure process can be modeled using Fickian non-Gaussian process. Their study considered a Gaussian central and non-Gaussian (exponential) tails for describing the probabilistic variations in displacements. It has also been pointed out that the application of non-Gaussian distributions such as alpha-stable distribution is going to be the future area of research. An exhaustive review of recent research needs in bluff body aerodynamics and the need to develop different modeling philosophies have been lucidly brought out by Kareem (2010).

**Alpha-Stable Distribution - some Preliminaries**

For alpha-stable distribution, an explicit expression for pdf generally does not exist. The characteristic function of alpha-stable distribution is given by [Nolan (2009)]:

\[
L_{\alpha,\beta}(t) = E[\exp(itX)] = \begin{cases} 
\exp \left(-c^\alpha \left[ 1-i\beta \text{sign}(t) \tan \left( \frac{\pi \alpha}{2} \right) \right] + i\delta \right); & \text{for } \alpha \neq 1 \\
\exp \left(-c^\alpha \left[ 1+i\beta \text{sign}(t) \frac{2}{\pi} \ln |t| \right] + i\delta \right); & \text{for } \alpha = 1 
\end{cases}
\]

where, \(X\) - random variable, \(i\) - imaginary unit, \(t\) - argument of the characteristic function \((t \in \mathbb{R})\), \(E[\cdot]\) - expected value, \(\alpha\) - characteristic exponent or stability parameter \((\alpha \in (0,2])\), \(\beta\) - skewness parameter \((\beta \in [-1,1])\), \(c\) - scale parameter \((c > 0)\), \(\delta\) - location parameter \((\delta \in \mathbb{R})\), \(\ln\) - natural logarithm, and, \(\text{sign}(t)\) - a logical function having values -1, 0, 1 for \(t < 0, t = 0\) and \(t > 0\), respectively.

The lack of closed form expressions for probability density functions and/or probability distribution functions for general alpha-stable distributions have been pointed as the major drawback for the application of the same in practice. However, as noted by Nolan (2009), computer programs are now available for computation of probability densities, probability distributions and quantiles. Also, number of methods have been proposed by different researchers for the estimations of the parameters (namely, \(\alpha, \beta, c\) and \(\delta\)) of the alpha-stable distribution (see Balaji Rao et al. (2013) for a brief survey). These facilitate the practical applications of alpha-stable distributions.
Wind Pressure Coefficient Data (from TPU(2013))

In this study, an attempt is made to examine the applicability of alpha-stable distribution to model the extreme wind pressure coefficients on the roof corner regions of a low-rise gabled roof building, using wind tunnel experimental data from the aerodynamic database of Tokyo Polytechnic University [TPU (2013)]. Salient information regarding the experimental investigations are as follows. The test model considered is gabled roof with eave height (H0) of 80 mm, breadth (B) of 160 mm and depth (D) of 400 mm (see Fig. 1). In the present study, the roof pitch angles considered are 0°, 4.8°, 9.4° and 14°, and, the wind angle of attack (θ) considered are 0°, 45° and 90°. The tested wind field was the suburban terrain corresponding to terrain category III of AIJ [TPU (2013)]. The turbulence density and the test wind velocity at a height of 100 mm were about 0.25 and 7.4m/s, respectively. The wind pressure taps were placed as shown in Fig. 1. Total number of pressure taps was 210 for all roof pitch angles except for 14°. The sampling frequency was 500Hz and the sampling period was 18 seconds for each sample (which corresponds to 15 Hz and 600 seconds, respectively, in full scale, based on simulated length scale of 1:100 and assumed velocity scale of 1:3). The time history of wind pressure coefficients (ratio of the measured wind pressure to the reference wind pressure of the approaching wind velocity at the average roof height, H) for each of the taps were moving averaged at every 0.006s (corresponding to 0.2s in full scale), and, are given in a database. The same are considered in the present study.

Statistical Analysis of Peak Wind Pressure Coefficients

The time histories of wind pressure coefficients corresponding to the taps 72, 32, 82 and 51 for θ = 0° and 90°, and, taps 72, 52, 32, 73 and 74 for θ = 45°, are considered in the study. It can be noted from Fig. 1 that most of these pressure taps correspond to roof corner where either flow separation would take place or along which conical vertices initiate. The 1-second peak negative wind pressure coefficients (\(\tilde{C}_{p}\)) are obtained, and the alpha-stable distribution and Gumbel minimum distribution are fitted to \(\tilde{C}_{p}\). Here, 1-second peak corresponds to the 1 second peak in the full-scale. In the present study, the parameters of alpha-stable distribution are determined using the procedure proposed by Koutrouvelis (1980).

Results and Discussion

The statistical properties (namely, mean, standard deviation, skewness and kurtosis) of 1-second peak negative wind pressure coefficients (\(\tilde{C}_{p}\)) have been computed, and are given in Table 1. The values of parameters of the alpha-stable- and Gumbel minimum- distributions, fitted to \(\tilde{C}_{p}\), are also given in Table 1.
Table 1 Statistical properties of 1-second peak wind pressure coefficients and parameters of fitted alpha-stable and Gumbel minimum distributions

<table>
<thead>
<tr>
<th>Measurement</th>
<th>θ (°)</th>
<th>Roof pitch angle (°)</th>
<th>wind pressure coefficient</th>
<th>statistical properties</th>
<th>parameters of alpha-stable distribution</th>
<th>parameters of Gumbel minimum distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>mean</td>
<td>SD 1</td>
<td>Skew 2</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>-1.31 0.49 -1.08 1.59</td>
<td>1.84 -1 0.30 -1.35</td>
<td>0.38  -1.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-1.31 0.57 -1.64 4.76</td>
<td>1.67 -1 0.31 -1.38</td>
<td>0.44  -1.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-1.44 0.59 -1.06 0.97</td>
<td>1.76 -1 0.36 -1.50</td>
<td>0.46  -1.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-1.24 0.52 -1.09 1.27</td>
<td>1.74 -1 0.31 -1.30</td>
<td>0.41  -1.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>-1.31 0.35 -1.18 1.81</td>
<td>1.66 -1 0.20 -0.92</td>
<td>0.28  -0.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-0.81 0.27 -1.15 3.08</td>
<td>1.87 -1 0.17 -0.82</td>
<td>0.21  -0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-0.96 0.30 -0.99 1.98</td>
<td>1.82 -1 0.19 -0.98</td>
<td>0.24  -0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-1.17 0.38 -1.01 1.56</td>
<td>1.78 -1 0.24 -1.19</td>
<td>0.30  -0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>0</td>
<td>-0.26 0.11 -0.36 -0.13</td>
<td>2.00 - -0.08 -0.26</td>
<td>0.09  -0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-0.25 0.11 -0.43 0.62</td>
<td>1.91 -1 0.07 -0.25</td>
<td>0.09  -0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-0.26 0.11 -0.31 0.32</td>
<td>1.96 -1 0.08 -0.26</td>
<td>0.09  -0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-0.25 0.10 -0.28 0.32</td>
<td>1.96 -1 0.07 -0.25</td>
<td>0.08  -0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>-1.02 0.38 -1.02 1.79</td>
<td>1.83 -1 0.23 -1.05</td>
<td>0.30  -0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-0.86 0.29 -0.80 0.72</td>
<td>1.89 -1 0.19 -0.88</td>
<td>0.22  -0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-0.91 0.30 -1.56 6.15</td>
<td>1.81 -1 0.18 -0.93</td>
<td>0.24  -0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-0.88 0.26 -0.88 0.92</td>
<td>1.88 -1 0.17 -0.90</td>
<td>0.20  -0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>0</td>
<td>-1.57 0.65 -1.29 2.27</td>
<td>1.75 -1 0.39 -1.63</td>
<td>0.51  -1.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-1.59 0.72 -1.12 1.64</td>
<td>1.78 -1 0.44 -1.66</td>
<td>0.56  -1.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-1.68 0.74 -1.58 3.78</td>
<td>1.63 -1 0.39 -1.76</td>
<td>0.58  -1.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-1.49 0.70 -1.47 1.37</td>
<td>1.69 -1 0.39 -1.57</td>
<td>0.54  -1.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>-2.04 0.78 -1.51 3.47</td>
<td>1.67 -1 0.43 -2.12</td>
<td>0.61  -1.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-2.21 0.74 -0.67 0.37</td>
<td>1.95 -1 0.51 -2.23</td>
<td>0.58  -1.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-2.33 0.73 -0.88 1.48</td>
<td>1.86 -1 0.46 -2.38</td>
<td>0.57  -2.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-1.86 0.62 -1.03 1.52</td>
<td>1.66 -1 0.35 -1.94</td>
<td>0.48  -1.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>0</td>
<td>-0.27 0.15 -1.14 3.07</td>
<td>1.86 -1 0.09 -0.28</td>
<td>0.12  -0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-0.26 0.14 -0.96 2.44</td>
<td>1.83 -1 0.08 -0.27</td>
<td>0.11  -0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-0.27 0.14 -1.20 4.28</td>
<td>1.85 -1 0.09 -0.27</td>
<td>0.11  -0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-0.25 0.12 -0.65 1.46</td>
<td>1.93 -1 0.08 -0.25</td>
<td>0.09  -0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>-1.84 0.77 -1.26 2.20</td>
<td>1.72 -1 0.45 -1.92</td>
<td>0.60  -1.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>-2.37 0.89 -0.59 0.10</td>
<td>2.00 - -0.63 -2.37</td>
<td>0.69  -1.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.4</td>
<td>-2.24 0.82 -1.45 4.81</td>
<td>1.81 -1 0.49 -2.27</td>
<td>0.64  -1.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>-1.91 0.61 -0.56 0.31</td>
<td>1.96 -1 0.42 -1.92</td>
<td>0.48  -1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>45</td>
<td>-1.64 0.48 -0.46 -0.20</td>
<td>1.99 -1 0.34 -1.65</td>
<td>0.37  -1.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>45</td>
<td>-0.22 0.80 -0.52 0.09</td>
<td>1.99 -1 0.56 -2.22</td>
<td>0.62  -1.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>45</td>
<td>-0.94 0.49 -2.80 12.29</td>
<td>1.47 -1 0.19 -1.00</td>
<td>0.38  -0.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>45</td>
<td>-2.41 0.75 -0.75 1.26</td>
<td>1.90 -1 0.49 -2.45</td>
<td>0.59  -2.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>74</td>
<td>45</td>
<td>-1.82 0.51 -0.76 0.79</td>
<td>1.92 -1 0.34 -1.85</td>
<td>0.40  -1.59</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Note: 1: SD - standard deviation, 2: Skew - skewness, 3: Kurt – kurtosis; 4: the distribution function of Gumbel minimum is given by \( F_G(x) = \exp\left(-\exp\left(-\frac{x-\mu}{\gamma}\right)\right) \). When \( \alpha = 2 \), the parameter \( \beta \) loses its significance)

From Table 1, it is noted that, in almost all the cases considered, the value of skewness parameter (\( \beta \)) of the alpha-stable distribution is equal to the lower bound value of -1, indicating that the distribution is maximally negatively skewed. This indicates that while the lower tail of the distribution is Pareto (behaves like \( |x|^{-\alpha} \) for large \( |x| \)), the upper tail has no
Paretian component. The values of location parameter ($\delta$) of the alpha-stable distribution are found to be in good agreement with the mean (see Fig. 2), which is expected since $\delta$ is the mean of the alpha-stable distribution for cases where $\alpha > 1$.

![Fig. 2 Comparison of location parameters of alpha-stable distribution with mean values of $\hat{C}_p$ (estimated from wind tunnel records)](image_url)

The frequencies of experimentally determined wind pressure coefficients, typically for measurement tap 72, for roof pitch angle of $0^\circ$ and $\theta = 90^\circ$, are shown in Fig. 3. In Fig. 3 and also in following portion of the paper, ‘observed’ denotes the wind pressure coefficients computed from the measured wind pressures in the wind tunnel experiments. The frequencies determined assuming wind pressure coefficients follow normal distribution is also shown in Fig. 3, for the purpose of comparison. From Fig. 3, it is noted that, as expected, the distribution of wind pressure coefficients is negatively skewed and the assumption of normal distribution for wind pressure coefficients is not appropriate. The cumulative distribution functions (cdf) of 1-second peak wind pressure coefficients, typically for taps 72 ($\theta=0^\circ$) and 32 ($\theta=90^\circ$) for roof pitch angle of $14^\circ$ are shown in Figs. 4 and 5, respectively. Based on these plots and other such plots (not presented here) for different taps for different combinations of angle of wind attack and roof pitch angle, and, making visual observations, the best fitting probability distributions to the different portions (namely, lower tail, middle and upper tail) of the probability distribution of observed 1-second peak wind pressure coefficients are identified and are given in Table 2.

![Fig. 3 Frequency distribution of wind pressure coefficients for measurement tap 72 (roof pitch angle = $0^\circ$, $\theta = 90^\circ$)](image_url)
From these results, the following points are noted:

- While the Gumbel minimum gives a better fit to the lower tail portion in a number of cases, in general, the estimates are unconservative (compared to observed cdf).
- In most of the cases, alpha-stable distribution provides a conservative estimate of the lower tail portion of peak wind pressure coefficients, which is a desirable feature for the design of roofing elements.
- In almost all the cases, the Gumbel minimum distribution gives a better fit to the middle portion. Gumbel minimum has the limitation that the skewness and kurtosis values are constant (and are equal to 1.14 and 2.4, respectively). From Table 1, it is noted that the skewness and kurtosis values for the observed \( C_p \) vary over a large range. It has been pointed out by Li et al. (1999) that the values of skewness and kurtosis for fluctuating pressures depend on several factors including oncoming turbulence intensity, and hence it may be unreasonable to consider skewness and kurtosis, as constants. Thus, it is felt that alpha-stable distribution is a satisfactory choice for modeling the peak wind pressure coefficients.

To study the level of conservativeness of alpha-stable distribution in modeling the lower tail portion of peak wind pressure coefficients, the values of \( C_p \) typically corresponding to 1% and 33% probability of non-exceedance are determined. The ratio between observed- and model- of 1-second peak wind pressure coefficients corresponding to 1% and 33% probability of non-exceedance are shown in Figs. 6 and 7. In Figs. 6 and 7, ‘model’ denote the
values of 1-sec peak wind pressure coefficients corresponding to the probability of non-exceedance considered assuming $\hat{C}_p$ follows alpha-stable- or Gumbel minimum- distribution. From these figures, it is noted that:

i. For 1% probability of non-exceedance (Fig. 6), the Gumbel minimum distribution gives conservative estimates of $\hat{C}_p$ when the values of $\hat{C}_p$ are low (less than 1.0). For higher values of $\hat{C}_p$, the alpha-stable distribution gives conservative estimates of $\hat{C}_p$.

ii. For 33% probability of non-exceedance (Fig. 7), the alpha-stable distribution gives conservative estimates of $\hat{C}_p$ regardless of the value of $\hat{C}_p$. The Gumbel minimum distribution gives unconservative estimates of $\hat{C}_p$ for most of the cases.

Table 2 Best fitting probability distributions for the different portions of probability distribution of observed wind pressure coefficients

<table>
<thead>
<tr>
<th>Measurement tap (º)</th>
<th>$\theta$</th>
<th>Lower tail</th>
<th>Middle</th>
<th>Upper tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>0</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Gumbel minimum</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>Gumbel minimum</td>
<td>Alpha-stable (overestimates)</td>
<td>Gumbel minimum</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>Gumbel minimum</td>
<td>Alpha-stable (overestimates)</td>
<td>Gumbel minimum</td>
</tr>
<tr>
<td>82</td>
<td>0</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Gumbel minimum</td>
</tr>
<tr>
<td>51</td>
<td>0</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Gumbel minimum</td>
</tr>
<tr>
<td>32</td>
<td>45</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
</tr>
<tr>
<td>52</td>
<td>45</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
</tr>
<tr>
<td>72</td>
<td>45</td>
<td>Alpha-stable</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
</tr>
<tr>
<td>73</td>
<td>45</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
<td>Alpha-stable</td>
</tr>
<tr>
<td>74</td>
<td>45</td>
<td>Gumbel minimum</td>
<td>Alpha-stable</td>
<td>Alpha-stable</td>
</tr>
</tbody>
</table>

(Note: * - judged based on visual inspection of plots similar to Figs. 4 and 5; for $\theta = 0^\circ$ and $90^\circ$, the best fitting probability distributions are common for roof pitch angles of $0^\circ$, $4.8^\circ$, $9.4^\circ$ and $14^\circ$; for $\theta = 45^\circ$, the best fitting probability distributions are for roof pitch angle of $0^\circ$)

![Fig. 6 Ratio of observed- to model- 1-second peak wind pressure coefficients corresponding to 1% probability of non-exceedance](image-url)
The above results suggest that to predict maximum suctional wind pressure coefficients, alpha-stable distribution is a better candidate distribution than the Gumbel minimum.

Conclusions

From the brief review of literature, it is found that fitting a suitable probability distribution to the tails of negative peak wind pressure coefficients to predict the extreme values is an area of active research. In this paper, an attempt is made to propose the alpha-stable distribution for 1-second negative peak wind pressure coefficient. The use of this distribution is justified through its generality (generalized central limit theorem) and the physics of non-Gaussian diffusion process. From the comparison of cdfs of 1-second peak wind pressure coefficients for gabled roof of low-rise building estimated by using Gumbel minimum- and alpha-stable- distributions for different roof pitch angles and angles of incident wind, it is found that alpha-stable distribution can be used as a candidate distribution for obtaining the conservative estimates of extreme values of negative wind pressure coefficients.

Acknowledgements

This paper is being published with the kind permission of the Director, CSIR-SERC, Chennai. The authors are grateful to Prof. R. Narasimha, Chairman, Engineering Mechanics Unit, IIT Madras and Director, National Institute of Advanced Studies, Bangalore, and, an internationally renowned expert in fluid dynamics, for providing relevant literature that helped the authors to gain fluid dynamics expert insights into the phenomenon considered in this paper. The MATLAB programs developed and made available in the public domain by Mark Veillette, Ph.D. Scholar, Department of Mathematics and Statistics, Boston University, Boston, USA, have been used in the present study for determining the parameters and cdfs of the alpha-stable distributions.

References


