THEORETICAL AND EXPERIMENTAL ANALYSIS OF DYNAMIC CHARACTERISTIC OF TRANSMISSION TOWER-LINE SYSTEM

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ABSTRACT
The evaluation of dynamic characteristics of electrical transmission tower-line system (ETTLS) is a key step for its wind resistant and seismic resistant design. A calculation method for analyzing the dynamic characteristics of ETTLS is founded on the basis of multi-degree-of-freedom model (MDOFM), and the credibility of this method is verified by comparing the calculation results with test results of a full aero-elastic model of an engineering example. In the mean time, through analyzing the dynamic characteristics of an electrical transmission tower with lines and without lines, the influence of coupling between the tower and lines of ETTLS on the dynamic characteristics of the tower is discussed.

KEYWORDS: ELECTRICAL TRANSMISSION TOWER-LINE SYSTEM; DYNAMIC CHARACTERISTIC; AERO-ELASTIC MODEL; COUPLED SYSTEM

Introduction
Nowadays, wind-induced accidents of ETTLS take place from time to time all over the world, and wind-resistant design of this kind of structures is a major research topic which is long-term concerned by the wind engineering and so far has not been solved (Loredo-Souza and Davenport 2001; Homles 1994 1996; H. Yasui 1999; Battista 2003). The evaluation of dynamic characteristics of ETTLS is a key step for its wind-resistant and seismic-resistant design. Nevertheless, the ETTLS is a very complicated coupled system because of the non-linear characteristic of the lines and coupling between electrical transmission tower and lines, hence the dynamic characteristics of ETTLS are very difficult to be evaluated precisely due to above mentioned factors. Up to now, some methods have been developed to evaluate the dynamic characteristics of ETTLS. The continuous type method were proposed to evaluate dynamic characteristics of cables in Cable Structures(1981), and S. Ozono introduced two series of calculation models to evaluate the in-plane dynamic characteristics of ETTLS(1988 1992). In China, Hong-Nan Li founded a MDOFM to evaluate the dynamic characteristics of ETTLS (1990 1996). On the basis of Li’s work, Shu-Guo Liang developed the MDOFM and adapted it to the ETTLS with big spans(1999 2003).
In light of previous works based on MDOFM, this paper develops an improved MDOFM to evaluate dynamic characteristics of ETTLS, in which the stretch deformation of the lines is considered, and the 3-D vibrations of ETTLS could be well simulated. Then the validation of this method was testified by comparison of the calculated results with test results of aero-elastic model of a tower-line engineering example. Finally, through analyzing the dynamic characteristics of transmission tower with lines and without lines, the influence of coupled vibrations of transmission tower-line system on the dynamic characteristics of tower is discussed.

**Calculation Method to Evaluate Dynamic Characteristics**

**Dynamic characteristics of line for in-plane vibration**

When the stretch deformation of the lines is considered, the MDOFM of lines, which consists of links connected by hinges and lumped masses, is sketched as Fig. 1.

To simplify the calculation process, some assumptions are applied as shown in Eq. (1):

\[
\begin{align*}
(l_{i0} + l_i) \cos \theta_{i0} = (l_{20} + l_s) \cos \theta_{20} = (l_{30} + l_s) \cos \theta_{30} &= (l_{40} + l_s) \cos \theta_{40} \\
(l_{50} + l_s) \cos \theta_{50} &= \frac{L}{5} = S \\
H_1 &= (l_{10} + l_s) \sin \theta_{10}, H_2 = (l_{20} + l_s) \sin \theta_{20}, H_3 = (l_{30} + l_s) \sin \theta_{30} \\
H_4 &= (l_{40} + l_s) \sin \theta_{40}, H_5 = (l_{50} + l_s) \sin \theta_{50}
\end{align*}
\]

(1)

Where, \( \theta_{10} \) is the initial angle of link \( l_1 \) in static condition; \( l_{i0} \) is the initial length of link \( l_i \); \( l_s \) is static deformation of link \( l_i \) under weight forces. In this paper the angle \( \theta \) is defined positive when it is in clockwise rotation. There is the constraint equation:

\[
\begin{align*}
[l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 + l_4 \cos \theta_4 + l_5 \cos \theta_5] &= L \\
[l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3 + l_4 \sin \theta_4 + l_5 \sin \theta_5] &= 0
\end{align*}
\]

(2)

As shown in Fig. 1. Assuming that \( \xi_i = \delta \theta_i = \theta_i - \theta_{i0} \) and \( \delta_i = \delta l_i = l_i - l_{i0} - l_s \) are generalized coordinates respectively, \( \xi_1 \) and \( \xi_5 \) can be expressed by the other generalized coordinates.
\[
\begin{align*}
\gamma_i &= \frac{\partial \theta}{\partial \varphi^i} \xi_2 + \frac{\partial \theta}{\partial \varphi^i} \xi_3 + \frac{\partial \theta}{\partial \varphi^i} \xi_4 + \frac{\partial \theta}{\partial \varphi^i} \xi_5 + \frac{\partial \theta}{\partial \varphi^i} \xi_6 + \frac{\partial \theta}{\partial \varphi^i} \delta_2 + \frac{\partial \theta}{\partial \varphi^i} \delta_3 + \frac{\partial \theta}{\partial \varphi^i} \delta_4 + \frac{\partial \theta}{\partial \varphi^i} \delta_5 \\
\gamma_5 &= \frac{\partial \theta}{\partial \varphi^1} \xi_2 + \frac{\partial \theta}{\partial \varphi^1} \xi_3 + \frac{\partial \theta}{\partial \varphi^1} \xi_4 + \frac{\partial \theta}{\partial \varphi^1} \xi_5 + \frac{\partial \theta}{\partial \varphi^1} \delta_2 + \frac{\partial \theta}{\partial \varphi^1} \delta_3 + \frac{\partial \theta}{\partial \varphi^1} \delta_4 + \frac{\partial \theta}{\partial \varphi^1} \delta_5 
\end{align*}
\] (3)

Where
\[
\begin{align*}
\frac{\partial \theta}{\partial \varphi^i} &= -l_5 \cos \theta \delta_i \sin \theta - l_5 \sin \theta \delta_i \cos \theta \\
\frac{\partial \theta}{\partial \varphi^i} &= l_5 \cos \theta \delta_i \sin \theta - l_5 \sin \theta \delta_i \cos \theta \\
\frac{\partial \theta}{\partial \varphi^i} &= l_5 \sin \theta \delta_i \cos \theta - l_5 \cos \theta \delta_i \sin \theta \\
\frac{\partial \theta}{\partial \varphi^i} &= \cos \theta \delta_i \cos \theta_i + \sin \theta \delta_i \sin \theta_i \\
\frac{\partial \delta_i}{\partial \varphi^i} &= l_5 (\sin \theta \delta_i \cos \theta_i - \cos \theta \delta_i \sin \theta_i) \\
\frac{\partial \delta_i}{\partial \varphi^i} &= - \cos \theta \delta_i \cos \theta_i + \sin \theta \delta_i \sin \theta_i \\
\frac{\partial \delta_i}{\partial \varphi^i} &= l_5 (\sin \theta \delta_i \cos \theta_i - \cos \theta \delta_i \sin \theta_i) 
\end{align*}
\] (4)

Then the dynamic differential equations can be founded based on Lagrange equations. The velocities of u and v directions of the masses on lines can be expressed as below.

\[
\begin{align*}
\dot{u}_B &= -H_1 \ddot{x}_1 + \cos \theta_1 \ddot{\delta}_1 \\
\dot{u}_C &= -H_1 \ddot{x}_2 + \cos \theta_2 \ddot{\delta}_2 \\
\dot{u}_D &= H_2 \ddot{x}_3 - \cos \theta_2 \ddot{\delta}_3 + H_2 \ddot{x}_4 - \cos \theta_4 \ddot{\delta}_4 \\
\dot{u}_E &= H_2 \ddot{x}_5 - \cos \theta_3 \ddot{\delta}_5 \\
\dot{v}_B &= -S_1 \ddot{\delta}_1 - \sin \theta_1 \ddot{\delta}_1 \\
\dot{v}_C &= -S_2 \ddot{\delta}_2 - \sin \theta_2 \ddot{\delta}_2 + S_3 \ddot{\delta}_3 - \sin \theta_3 \ddot{\delta}_3 \\
\dot{v}_D &= S_4 \ddot{\delta}_4 + \sin \theta_3 \ddot{\delta}_3 + S_5 \ddot{\delta}_5 + \sin \theta_4 \ddot{\delta}_4 \\
\dot{v}_E &= S_5 \ddot{\delta}_5 + \sin \theta_4 \ddot{\delta}_4 
\end{align*}
\] (5)

And then the kinetic energy can be derived

\[
T = \sum_{i=A,B,C,D,E} \frac{1}{2} m_i (\ddot{x}_i^2 + \ddot{y}_i^2) = T(\ddot{x}_1, \ddot{x}_2, \ddot{x}_3, \ddot{x}_4, \ddot{x}_5, \ddot{\delta}_1, \ddot{\delta}_2, \ddot{\delta}_3, \ddot{\delta}_4, \ddot{\delta}_5) 
\] (7)

Neglecting second and higher order terms, the potential energy can be expressed by

\[
U = -4 mg(l_{10} + l_{1s} + \delta_1) \sin \theta_1 - 3 mg(l_{20} + l_{2s} + \delta_2) \sin \theta_2 \\
- 2 mg(l_{30} + l_{3s} + \delta_3) \sin \theta_3 - mg(l_{40} + l_{4s} + \delta_4) \sin \theta_4 \\
+ \frac{EA}{2l_{10}} (l_{1s} + \delta_1)^2 \ddot{x}_{1s} + \frac{EA}{2l_{20}} (l_{2s} + \delta_2)^2 \ddot{x}_{2s} + \frac{EA}{2l_{30}} (l_{3s} + \delta_3)^2 \ddot{x}_{3s} + \frac{EA}{2l_{40}} (l_{4s} + \delta_4)^2 \ddot{x}_{4s} \\
+ \frac{EA}{2l_{50}} (l_{5s} + \delta_5)^2 \ddot{x}_{5s} - \frac{EA}{2l_{10}} l_{1s}^2 - \frac{EA}{2l_{20}} l_{2s}^2 - \frac{EA}{2l_{30}} l_{3s}^2 - \frac{EA}{2l_{40}} l_{4s}^2 - \frac{EA}{2l_{50}} l_{5s}^2 
\] (8)

When the kinetic energy and potential energy are obtained, the mass matrix and the stiffness matrix of the lines can be deduced by utilizing partial derivatives of the kinetic energy.
energy and the potential energy by different generalized velocities and generalized coordinates respectively. Then the frequencies of different modes are acquired.

**Dynamic characteristics of line for out-of-plane vibration**

The out-of-plane vibration of a line is equivalent to a vertical chain. Fig. 1(b) shows the MDOFM for out-of-plane vibration. The mass matrix and stiffness matrix for out-of-plane vibration can be deduced in light of general structural mechanics.

\[
[K] = mg \begin{bmatrix}
\frac{1}{H_4} & -\frac{1}{H_4} & 0 & 0 \\
-\frac{1}{H_4} & \frac{1}{H_4} + \frac{2}{H_3} & -\frac{2}{H_3} & 0 \\
0 & -\frac{2}{H_3} & \frac{2}{H_3} + \frac{3}{H_2} & -\frac{3}{H_2} \\
0 & 0 & -\frac{3}{H_2} & \frac{3}{H_2} + \frac{4}{H_1}
\end{bmatrix}
\]  

(9)

Where \(m\) is mass, \(g\) is acceleration of gravity.

**Dynamic characteristic of independent tower**

The mass matrix and stiffness matrix of an independent tower are acquired by finite element model. The tower can be simplified as MDOFM which is shown in Fig. 2. The displacement \(\delta_1\), \(\delta_2\) and \(\delta_3\) of particle 1, 2 and 3 can be obtained when the unit force was subjected to particle 1. Similarly the \(\delta_1\), \(\delta_2\) and \(\delta_3\) can be gotten through acting the unit force to particle 2, and the \(\delta_1\), \(\delta_2\) and \(\delta_3\) also can be obtain by acting the unit force to particle 3. Then the flexibility matrix of tower was formed. The torsional flexibility matrix could be acquired by the same way.

![Fig. 2 The calculation schematic of flexibility matrix of tower](image)

**Dynamic characteristics of ETTLS for in-plane and torsional coupled vibrations**

The MDOFM of one tower with two spans of line for in-plane and torsion coupled vibration is shown in Fig. 3. For this model, the stiffness matrix of ETTLS is not coupled, while the mass matrix is coupled. When the coupled movement of tower and lines are considered, the velocities of \(u\) and \(v\) directions of the masses on lines can be expressed as below.

\[
\begin{align*}
\dot{u}_b &= -H_1\dot{\xi}_j + \cos \theta \dot{\delta}_j \\
\dot{u}_c &= -H_1\dot{\xi}_i + \cos \theta \dot{\delta}_i - H_2\dot{\phi}_2 + \cos \theta \dot{\delta}_2 \\
\vdots & \quad \vdots \\
\dot{u}_{j} &= \dot{u}_4 + L \times \dot{\theta}_4 + H_4\dot{\xi}_4 - \cos \theta \dot{\delta}_4 \\
\dot{u}_k &= \dot{u}_4 + L \times \dot{\theta}_4 + H_4\dot{\xi}_4 - \cos \theta \dot{\delta}_4
\end{align*}
\]  

(10)
The kinetic energy of coupled lines is
\[ T = \frac{1}{2} m \left[ \sum_{i=1}^{n} (\dot{u}_i^2 + \dot{\theta}_i^2) \right] + \frac{1}{2} m_A [\ddot{u}_A + L \dot{\theta}_A]^2 \] (12)

Then the coupled terms of mass matrix between tower and lines can be acquired through partial derivatives of kinetic energy by \( \dot{u}_A \) and \( \dot{\theta}_A \) of the tower-line system. Accordingly, total mass and stiffness matrix can be obtained through partial derivatives of kinetic energy and potential energy of the tower-line system respectively.

**Dynamic characteristic of ETTLS for out-of-plane vibration**

Fig. 4 shows the sketch of a MDOFM for out-of-plane vibration of ETTLS. For this model, the mass matrix of ETTLS is not coupled, while the stiffness matrix is coupled. The mass matrix and stiffness matrix of this model also can be deduced in light of general structural mechanics. The coupled terms of stiffness matrix of tower and lines is as follows.

\[ k_{cs} = -\frac{(m_1 + m_2 + m_3 + m_4)g}{L_1} \] (13)

**Production of Aero-Elastic Model**

The prototype of the electrical transmission tower for the wind tunnel test is 48.50m in height, with 400m horizontal span. The design of the tower model, not only satisfy the geometry similarity of structure and various components strictly, but also satisfy the most important parameter similarity for a full aero-elastic model, such as similarity of Strouhal number, Froude number, Cauchy number, inertia parameter, and damping ratio. Similarity of Reynolds number obviously can not be met, but because the components of the electrical transmission tower are angle sheet, difference of Reynolds number does not influence the separation points of the encircle flow field, thus relaxing the requirements of Reynolds number similarity will not induce considerable error for test results. The fundamental similarity ratios of the full aero-elastic model of the tower are tabulated in table 1.

The geometry similarity ratios of aero-elastic model of conductor(ground wire) should be reduced so as to put the aero-elastic model of one tower and two spans of lines into the wind tunnel. Then, the geometry similarity ratios of the spans of cables are reduced to 1: 60,
and the related scales of the cables are correspondingly distorted according to a novel approach proposed by Davenport (2001). The fundamental scales of the aero-elastic model of the cables are tabulated in table 2. Figure 5 shows the aero-elastic model of ETTLS. When the aero-elastic model of one tower with two spans of line was accomplished, the dynamic characteristics of the tower and lines were evaluated based on the data of free vibration tests and wind tunnel tests. Figs. 6–7 are the acceleration response spectrum curves of the tower with and without lines in different directions. By analyzing the frequency spectrum characteristics of the acceleration responses of the tower with and without lines, the frequencies of various modes of tower with and without lines in different directions are distinguished.

Table 1 Similarity ratios of aero-elastic model of transmission tower

<table>
<thead>
<tr>
<th>Similarity ratio</th>
<th>geometry</th>
<th>Wind speed</th>
<th>frequency</th>
<th>Stiffness</th>
<th>mass</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>λ_L</td>
<td>λ_U</td>
<td>λ_n</td>
<td>λ_{EA}</td>
<td>λ_m</td>
<td>λ_ξ</td>
</tr>
<tr>
<td>Value</td>
<td>1:30</td>
<td>1:5.48</td>
<td>5.48:1</td>
<td>1:27000</td>
<td>1:27000</td>
<td>1:1</td>
</tr>
</tbody>
</table>

Table 2 Similarity Ratios of Aero-Elastic Model of Lines

<table>
<thead>
<tr>
<th>Similarity ratio</th>
<th>span</th>
<th>area</th>
<th>sag</th>
<th>frequency</th>
<th>stiffness</th>
<th>mass</th>
<th>damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>λ_L^l</td>
<td>λ_F</td>
<td>λ_s</td>
<td>λ_n^l</td>
<td>λ_{EA}^l</td>
<td>λ_M</td>
<td>λ_ξ</td>
</tr>
<tr>
<td>Value</td>
<td>1:60</td>
<td>1:900</td>
<td>1:30</td>
<td>5.48:1</td>
<td>1:54000</td>
<td>1:27000</td>
<td>1:1</td>
</tr>
</tbody>
</table>

Fig. 4 MDOFM for out-of-Plane vibration

Fig. 5 Aero-Elastic Model of ETTLS

(a) Weak-Axial center point  (b) Strong-Axial center point  (c) Weak-Axial side point

Fig. 6 The acceleration spectrum at the top of tower model without lines
Calculation and Analysis

On the basis of MDOFM, the natural frequencies of the prototype of the cable and the tower with and without lines were calculated. The results obtained by prototype calculation and aero-elastic model tests are tabulated in table 3 and table 4.

Table 3  The Frequencies of Cable

<table>
<thead>
<tr>
<th></th>
<th>Prototype</th>
<th>Model</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Continuous Method $f_c$/Hz</td>
<td>MDOFM $f_m$/Hz</td>
<td>Test Results $f_a$/Hz</td>
</tr>
<tr>
<td>In-plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st mode</td>
<td>0.306</td>
<td>0.298</td>
<td>1.760</td>
</tr>
<tr>
<td>2nd mode</td>
<td>0.401</td>
<td>0.394</td>
<td>2.295</td>
</tr>
<tr>
<td>3rd mode</td>
<td>0.540</td>
<td>0.524</td>
<td>2.850</td>
</tr>
<tr>
<td>4th mode</td>
<td>0.612</td>
<td>0.562</td>
<td>3.515</td>
</tr>
<tr>
<td>Out-of-plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st mode</td>
<td>0.153</td>
<td>0.152</td>
<td>0.880</td>
</tr>
<tr>
<td>2nd mode</td>
<td>0.306</td>
<td>0.298</td>
<td>1.760</td>
</tr>
<tr>
<td>3rd mode</td>
<td>0.459</td>
<td>0.437</td>
<td>2.445</td>
</tr>
</tbody>
</table>

Table 4  The Frequencies of Tower With and Without Lines

<table>
<thead>
<tr>
<th></th>
<th>Prototype</th>
<th>Model</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MDFM $f_m$/Hz</td>
<td>Test results $f_a$/Hz</td>
<td>$f_a/f_m$</td>
</tr>
<tr>
<td>Tower without lines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weak axial</td>
<td>1st mode</td>
<td>1.689</td>
<td>8.154</td>
</tr>
<tr>
<td></td>
<td>2nd mode</td>
<td>4.428</td>
<td>20.94</td>
</tr>
<tr>
<td>Strong axial</td>
<td>1st mode</td>
<td>2.221</td>
<td>11.426</td>
</tr>
<tr>
<td></td>
<td>2nd mode</td>
<td>6.471</td>
<td>25.39</td>
</tr>
<tr>
<td>Torsional direction</td>
<td>1st mode</td>
<td>3.931</td>
<td>14.89</td>
</tr>
<tr>
<td>Tower with lines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weak axial</td>
<td>1st mode</td>
<td>0.667</td>
<td>3.076</td>
</tr>
<tr>
<td></td>
<td>2nd mode</td>
<td>1.553</td>
<td>6.549</td>
</tr>
<tr>
<td>Strong axial</td>
<td>1st mode</td>
<td>1.790</td>
<td>10.15</td>
</tr>
<tr>
<td>Torsional direction</td>
<td>1st mode</td>
<td>1.189</td>
<td>3.900</td>
</tr>
</tbody>
</table>

As shown in table 3, the calculation results on the basis of MDOFM are in good agreement with the results of continuous type model proposed by Irvine. The error range between the two sets of results is approximately within 8%. Furthermore, by comparing the calculation results with those obtained by aero-elastic model test, the ratios are approximately equal to $\sqrt{30}$, which is the due frequency ratio of the model. As it is shown in table 4, the frequency ratios of model to prototype of the tower with and without line are almost smaller than $\sqrt{30}$, but all the frequency ratios are still close to $\sqrt{30}$ except those of torsional vibration. On the other hand, by comparing the correspondent frequencies between tower with and without lines we get the conclusion that the frequencies of tower with lines are much lower than those without lines.
Conclusions

The calculation method of 3-D dynamic characteristics of ETTLS are established based on MDOFM. Through comparison of the calculation results of prototype with the test results of aero-elastic model of ETTLS, some conclusions are obtained as follows.

1. By comparing the calculation results with test results of a full aero-elastic model of an engineering example, the credibility of the MDOFM of ETTLS developed in this paper is verified.

2. The influence of the coupled vibration on the characteristics of ETTLS is obvious and can not be neglected. The frequencies of tower with lines are much lower than those without lines. So it is necessary to consider coupled vibration between tower and lines in the design of ETTLS.

References


