CROSS-CORRELATION OF FLUCTUATING COMPONENTS OF WIND SPEED BASED ON STRONG WIND MEASUREMENT

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ABSTRACT

We report on the structure of atmospheric turbulence, focusing especially on the spatial correlations of three components of fluctuating wind speed using our records taken from measurements at a transmission tower located at the tip of a cape during Typhoon KIRK. Our results show that the non-dimensional cross-spectrum of longitudinal component of fluctuating wind speed between two distantly-positioned points of the Kármán type has the best fit to the measurement results, and the approximate expression to the Kármán type without special functions can sufficiently explain several characteristics of the non-dimensional cross-spectrum of isotropic turbulence. In addition, we show that the cross-correlation coefficient between longitudinal and vertical lateral components of fluctuating wind speed is approximately expressed using the spatial correlation coefficient between the components of fluctuating wind speed in the same direction.

KEYWORDS: TURBULENCE STRUCTURE, CROSS-SPECTRUM, CROSS-CORRELATION

Introduction

For the dynamic response analysis of a structure under strong winds, the spectral response method in a frequency domain or the step-by-step integration of motion equation in a time domain is used. The estimation of the wind response of a slender structure requires the longitudinal and/or lateral components (u- and v-components) of fluctuating wind speed in a horizontal direction, and the vertical lateral component (w-component) is rarely considered. However, we must set up the wind field including w-component in the case of calculating the response of a suspension-bridge or a long-span conductor which is strongly affected by w-component. The spectral response method postulates a linear deformation of a structure, and so for the structure for which deformation is not linear, we generally have to perform the step-by-step integration of motion equation. For example, for a coupling system with cable, such as a transmission tower, an increase in the gradient angle of the conductor-plane with the wind speed enlarges not only the geometrical nonlinearity of the conductor’s rigidity but also the effect of w-component of wind speed on the response as revealed by [Fujimura and Maeda et al. (2007)]. In this case, it is necessary to generate the three-dimensional field of fluctuating wind speed including the three components in a time domain. And then the nonlinear response simulation of the coupled system of tower and conductor is performable using the step-by-step integration method.

In order to generate the three-dimensional wind field with the three components of fluctuating wind speed under strong winds, information about the spatial structure between the three components is needed. When the gust loading factor method was introduced to the
wind load design of structures and buildings, many semi-empirical equations concerning the spatial structure of wind were proposed in studies by [Davenport (1961) and Harris (1971)]. Currently, Kármán’s spectrum, which was derived from the theory of isotropic turbulence in the work by [Kármán (1948)], is often used as the power spectrum of fluctuating wind speed. This spectrum expression is in good agreement with the measurement results as reported by [Maeda and Adachi (1983), Maeda and Maki no (1988)]. [ESDU750 01 (1975)] describes Kármán’s cross-spectrum as being the best expression for non-dimensional cross-spectrum of fluctuating wind speed between two distantly-positioned points, but the exponential expression proposed in a study by [Davenport (1961)] as an approximate expression is used in practice because it is a simpler and more convenient form of expression to use than Kármán’s cross-spectrum. However, the exponential expression is very different from Kármán’s cross-spectrum derived from the theory of isotropic turbulence in that its correlation value is consistently 1 at the point where frequency is zero regardless of the distance between two points, and that it does not have any negative values as pointed out by [Maeda and Makino (1980)]. As for the cross-correlation between the longitudinal component (u-component), horizontal lateral component (v-component) and vertical lateral component (w-component) of fluctuating wind speed, [ESDU74031 (1974)] describes the co-variance $\overline{uv}$ and $\overline{vw}$ as being small and which can be ignored, but this is not so for the $\overline{uw}$ co-variance near the ground, and the formulated value of $\overline{uw}/\sigma_u\sigma_w$ as a function of height. However, there are few earlier reports which describe the co-variances and cross-correlation coefficients between u- and w-components of fluctuating wind speed at two distantly-positioned points, and we cannot find any later reports.
In this paper, the authors estimate the non-dimensional cross-spectrum of fluctuating wind speed between two distantly-positioned points using wind measurement data at a 200m high transmission tower located at the tip of a cape during the passing of Typhoon KIRK (1996) and the estimated values are compared with Davenport’s exponential expression and an approximate expression to the Kármán type proposed by [Maeda and Makino (1980)] without uncontrollable special functions. Additionally, the authors discuss the co-variance and cross-correlation coefficients between $u$- and $w$-components of fluctuating wind speed from their data and propose an approximate expression for the values, which is useful for the three-dimensional simulation of a fluctuating wind field in a time domain.

**Outline of Strong Wind Observation**

*Observed Tower and Terrain around Observed Tower*

The observed tower was a 214.5m tall 500kV transmission tower at the tip of a cape at a height of 50m above sea level, as shown in Photo 1. Figure 1 shows the relation between the observed tower and the azimuth direction. Hills stood to the north of the tower, and it was surrounded by sea on its south, east and west sides.

*Outline of Measurement Data*

Eight ultrasonic anemometers, four for three components of wind speed and four for two components, were positioned on the tower for wind measurement, as shown in Figure 2. Every anemometer was located several meters from a main post in a horizontal direction, as shown in Photo 2. The sampling frequency of measurement was 120 Hz. The strong wind observations were conducted during typhoons and non-typhoons from 1996 to 1997, and in this paper we selected a total of three samples measured at U4, U6 and U8 when the mean wind speed of U2 was more than 20m/s and the wind direction was from the west where the observed tower was downwind during the passing of Typhoon KIRK (No.12 in 1996), whose path is shown in Figure 3. Each sample was 600 seconds long. Table 1 shows the mean wind speed observations at different anemometer positions.

### Table 1: Mean characteristics of the measurement data.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Anemometer No.</th>
<th>$\overline{U}$ (m/s)</th>
<th>$\sigma_u$ (m/s)</th>
<th>$\sigma_v$ (m/s)</th>
<th>$\sigma_w$ (m/s)</th>
<th>$x/L_u$ (m)</th>
<th>$x/L_v$ (m)</th>
<th>$x/L_w$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>U4</td>
<td>26.6</td>
<td>2.6</td>
<td>2.4</td>
<td>1.6</td>
<td>149.9</td>
<td>72.5</td>
<td>43.1</td>
</tr>
<tr>
<td></td>
<td>U6</td>
<td>25.2</td>
<td>2.6</td>
<td>2.5</td>
<td>1.7</td>
<td>143.2</td>
<td>89.5</td>
<td>42.6</td>
</tr>
<tr>
<td></td>
<td>U8</td>
<td>23.4</td>
<td>2.3</td>
<td>3.0</td>
<td>1.6</td>
<td>75.9</td>
<td>81.1</td>
<td>23.0</td>
</tr>
<tr>
<td>No. 2</td>
<td>U4</td>
<td>28.9</td>
<td>3.2</td>
<td>2.9</td>
<td>2.0</td>
<td>195.9</td>
<td>61.6</td>
<td>36.8</td>
</tr>
<tr>
<td></td>
<td>U6</td>
<td>26.8</td>
<td>3.1</td>
<td>3.1</td>
<td>2.0</td>
<td>166.4</td>
<td>73.2</td>
<td>39.2</td>
</tr>
<tr>
<td></td>
<td>U8</td>
<td>25.3</td>
<td>2.8</td>
<td>3.4</td>
<td>1.9</td>
<td>121.0</td>
<td>133.0</td>
<td>34.9</td>
</tr>
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<td>No. 3</td>
<td>U4</td>
<td>22.2</td>
<td>2.5</td>
<td>3.0</td>
<td>1.6</td>
<td>203.9</td>
<td>112.1</td>
<td>42.1</td>
</tr>
<tr>
<td></td>
<td>U6</td>
<td>20.6</td>
<td>2.5</td>
<td>2.8</td>
<td>1.5</td>
<td>241.7</td>
<td>104.8</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>U8</td>
<td>18.8</td>
<td>2.3</td>
<td>2.9</td>
<td>1.3</td>
<td>191.4</td>
<td>145.2</td>
<td>16.3</td>
</tr>
</tbody>
</table>

![Figure 3: Path of Typhoon KIRK.](image)
speed, $\overline{U}$, the standard deviation, $\sigma$, and the longitudinal scale of turbulence, $L$, at every anemometer calculated using every sample, and Figures 4(a), 4(b) and 4(c) show the time histories recorded at anemometers U4, U6 and U8 for sample 1, respectively. Wind records of U1 at the top of the tower were not included in this study because measurements were affected by a position light near anemometer U1.

**Non-dimensional Cross-spectrum of Fluctuating Wind Speed**

*Proposed equations for non-dimensional cross-spectrum*

The non-dimensional cross-spectrum of $u$-component of fluctuating wind speed between vertically-distantly-positioned points A and B, as shown in Figure 5, is written as follows:

$$
\tilde{S}_u^c(\zeta, n) = \frac{S_u^c(\zeta, n)}{\sqrt{S_u(n)}\sqrt{S_u(n)}}, \quad (1a)
$$

$$
S_u^c(\zeta, n) = \int_{-\infty}^{\infty} R_u^c(\zeta, \tau) \exp(-j2\pi n \tau) d\tau. \quad (1b)
$$

Where $R_u^c(\zeta, \tau) = E[u(t)u(t+\tau)]$, $S_u(n)$ and $S_u(n)$ are the power spectra at points A and B respectively, $\zeta$ is the vertical distance between points A and B and $n$ is the frequency. The non-dimensional cross-spectrum of Davenport and Kármán types and an approximate expression to the Kármán type proposed in the work study by [Maeda and Makino (1980)] are written as follows, respectively:

**Davenport:**

$$
\tilde{S}_u^c = \exp\left(-k_r \frac{|n\zeta|}{\overline{U}}\right), \quad (2a)
$$

**Kármán:**

$$
\tilde{S}_u^c = 1.772 \left\{ \frac{\theta}{2} \right\}^{5/6} K_{5/6}(\theta) - \left\{ \frac{\theta}{2} \right\}^{11/6} K_{11/6}(\theta), \quad (2b)
$$

Figure 4: Time histories of wind speed for sample 1.
Maeda and Makino: \( \tilde{S}_{\zeta} = \exp(-k_1\theta) \cdot (1-k_2\theta^2) \). \hspace{1cm} (2c)

Where \( k_1 = 13(\zeta/z)^{1/4} \), \( z_w = 0.5(z_d + z_w) \), \( \theta = (0.747 \zeta/x L_u)^2 + (2m\zeta/U)^{1/2} \), \( k_1 = 1.0, k_2 = 0.2 \) and \( K_\mu(\theta) \) is the modified Bessel function of the second kind of order \( \mu \).

**Measurement Results of Non-dimensional Cross-spectrum**

Figure 6 shows the non-dimensional cross-spectra of every non-dimensional distance, \( \zeta/\zeta L_u \), estimated from the measurement data in the case of \( n=0 \). The spectra and cross-correlation coefficients which are mentioned later are estimated using the Auto-regressive Model and the AR order is decided determined by the minimum FPE criterion as introduced by [Maeda and Makino (1981), Maeda and Makino (1992)]. In the figure, the expressions written with Eqs. (2a), (2b) and (2c) have been added. As shown in the figure, the estimated values of measurement results decrease with an increase in the non-dimensional distance. Kármán’s expression is in good agreement with the measurement results, but Davenport’s expression is not in agreement with them. And the expression proposed by Maeda and Makino is in very good agreement with Kármán’s expression.

Figures 7(a) and 7(b) show the non-dimensional cross-spectra of the lateral components (v- and w-components) of fluctuating wind speed estimated from the measurement data in the case of \( n=0 \), respectively. The non-dimensional distance on the horizontal axis is defined as \( u x L_u/5.0 \zeta \) using the characteristic of the theory of isotropic turbulence that is \( L_u: L_u: L_u = 2:1:1 \). The estimated values of measurement results of v- and w-components decrease with an increase in the non-dimensional distance and trace Kármán’s expression as well as those of \( u \)-component.

Figures 8(a), 8(b) and 8(c) show the non-dimensional cross-spectra of \( u \)-component of every non-dimensional frequency, \( n^x L_u/U \), in the cases that \( \zeta/\zeta L_u \approx 0.28, 0.44 \) and 0.82
respectively. In the figures, the expressions written with Eqs. (2a), (2b) and (2c) have been added. Regardless of non-dimensional distance, Kármán’s expression is in good agreement with the measurement results, and especially in the case of $\zeta/L_u \approx 0.44$ when the measurement results include negative values as shown in Figure 8(b), although it is clear that Davenport’s cross-spectrum cannot express this characteristic. The expression proposed by Maeda and Makino can sufficiently explain the characteristics of the measurement results.

Cross-correlation Coefficient between $u$- and $w$-components of Fluctuating Wind Speed

Definition of Cross-correlation Coefficient between $u$- and $w$-components

The cross-correlation coefficient between $u$- and $w$-components of fluctuating wind speed at vertically-distantly-positioned points A and B, as shown in Figure 5, is written as follows:

$$ R_{uw}^c(\zeta, \tau) = \frac{R_{uw}^c(\zeta, \tau)}{\sigma_u \sigma_w}. $$

(3)

Where $R_{uw}^c(\zeta, \tau) = E[u(t)w(t+\tau)]$ and $\sigma_u$ and $\sigma_w$ are the standard deviations of $u$- and $w$-components at points A and B, respectively.

Measurement Results of Cross-correlation Coefficient between $u$- and $w$-components

Figures 9(a), 9(b) and 9(c) show the values of $\frac{\bar{u}w/\sigma_u \sigma_w}{\sigma_u \sigma_w}$ and $\bar{R}_{uw}^c(\zeta,0)$ estimated from the measured data, respectively. As shown in the figures, the values of $\frac{\bar{u}w/\sigma_u \sigma_w}{\sigma_u \sigma_w}$ are between -0.3 and -0.4 at every height, $z$. The values of $\bar{R}_{uw}^c(\zeta,0)$ are close to zero with an increase in non-dimensional distances, $\zeta/L_u$, and the absolute values of $\bar{R}_{uw}^c(\zeta,0)$ are smaller than 0.1 in the case of $\zeta/L_u > 0.5$.

In order to evaluate the statistical nature of $\bar{R}_{uw}^c(\zeta,0)$ and $\bar{R}_{uw}^c(\zeta,0)$, we define the ratio of the cross-correlation coefficient to $\frac{\bar{u}w/\sigma_u \sigma_w}{\sigma_u \sigma_w}$, and Figures 10(a) and 10(b) show the measurement results of $\bar{R}_{uw}^c(\zeta,0)$ and $\bar{R}_{uw}^c(\zeta,0)$ respectively. In Figure 10(a), the averages of $\bar{R}_{uw}^c(0,\tau)$ estimated from all the measurement data are discretely expressed. The longitudinal
spatial correlation coefficient, \( f(r) \), and the lateral spatial correlation coefficient, \( g(r) \), between the components of fluctuating wind speed in the same direction proposed by \([\text{Kármán (1948)}]\), which are written as Eqs. (4) and (5) respectively, have been added in Figures 10(a) and 10(b) respectively:

\[
\begin{align*}
 f(r) &= f(\overline{U}\tau) = \alpha_1 |a_1 \overline{U}\tau|^{1/3} K_{1/3}(|a_1 \overline{U}\tau|), \\
 g(r) &= g(\zeta) = \alpha_1 |a_1 \zeta|^{1/3} \{K_{1/3}(|a_1 \zeta|) - 0.5 |a_1 \zeta| K_{-2/3}(|a_1 \zeta|)\},
\end{align*}
\]

where \( \alpha_1 = 2^{2/3}/\Gamma(1/3) \approx 0.5926 \) and \( a_1 = \sqrt{\pi} \Gamma(5/6)/\Gamma(1/3) \approx 0.7468 \). As shown in Figure 10(a), Eq. (4) is in good agreement with the measurement results. And as shown in Figure 10(b), the measurement results vary but Eq. (5) traces the median measurement result.

**Derivation of Approximate Expression for Measurement Result**

[Hinze (1987)] showed that the cross-correlation coefficient between \( u \)- and \( w \)-components is defined using the spatial correlation coefficient between the components of fluctuating wind speed in the same direction, \( f(r) \) and \( g(r) \), as follows:

\[
\tilde{R}^c_{uw}(\zeta, \tau) = \frac{f(r_1) - g(r_1)}{r_1^2} |\overline{U}\tau| \zeta. 
\]

Where \( r_1^2 = (\overline{U}\tau)^2 + \zeta^2 \). Eq. (6) was derived by assuming that the co-variance between the mutually-perpendicular components of fluctuating wind speed is zero in the field of isotropic turbulence. However, the measurement results of the co-variance between \( u \) and \( w \) components, \( uw \), are not zero, as shown in Figure 9(a). Additionally, in considering that \( \tilde{R}_{uw}^c(0, r) \approx \overline{uw}/\sigma_u \sigma_w \cdot f(r) \) and \( \tilde{R}_{uw}^c(\zeta, 0) \approx \overline{uw}/\sigma_u \sigma_w \cdot g(r) \) as shown in Figure 10, it is thought that the approximate expression for the cross-correlation coefficient between \( u \)- and \( w \)-components can be written as follows:

\[
\tilde{R}^c_{uw}(\zeta, \tau) = \frac{f(r_1)}{r_1^2} \left\{ |\overline{U}\tau| \zeta + \frac{\overline{uw}}{\sigma_u \sigma_w} (\overline{U}\tau)^2 \right\} - \frac{g(r_1)}{r_1^2} \left\{ |\overline{U}\tau| \zeta - \frac{\overline{uw}}{\sigma_u \sigma_w} (\zeta)^2 \right\}. 
\]

Figures 11(a) and 11(b) show the comparisons of the measurement results of \( \tilde{R}^c_{uw}(\zeta, \tau) \) with Eq. (7) in the cases of \( \zeta/\gamma L_u = 0.25 \) and \( \zeta/\gamma L_u = 1.04 \), respectively. In the case of \( \zeta/\gamma L_u = 0.25 \), the approximate expression is in good agreement with the measurement results. On the other hand, in the case of \( \zeta/\gamma L_u = 1.04 \), the approximate expression includes the positive values and is different from the measurement results. However, as shown previously, there are few correlations between \( u \) and \( w \) components in the case of \( \zeta/\gamma L_u > 0.5 \), and so it is thought that this distinction is not important.

**Conclusions**

Some of our findings concerning the spatial correlation of three components of fluctuating wind speed based on the wind speed measurement at the transmission tower

![Figure 11: Comparison of measurement results of cross-correlation coefficients between \( u \)- and \( w \)-components with Eq. (7) in the cases of \( \zeta/\gamma L_u = 0.25 \) and \( \zeta/\gamma L_u = 1.04 \).](image-url)
located at the tip of a cape during Typhoon KIRK (1996) are summarized as follows.

1. The measurement results of the non-dimensional cross-spectra are in good agreement with Kármán’s expression derived from the theory of isotropic turbulence.

2. The approximate expression to the Kármán type without special functions can sufficiently explain several characteristics of the measurement results of the non-dimensional cross-spectra.

3. The cross-correlation coefficients between the longitudinal and vertical lateral components of fluctuating wind speed are close to zero with an increase in absolute values of non-dimensional time and non-dimensional distances.

4. The cross-correlation coefficients between the longitudinal and vertical lateral components of fluctuating wind speed are approximately expressed using the spatial correlation between the components of fluctuating wind speed in the same direction.

The spatial correlation of the three components of fluctuating wind speed, as described in this paper, can be useful for the three-dimensional simulation of a wind field in a time domain.

Acknowledgement

This work was supported by Grant-in-Aid for JSPS Fellows, 21-4044, Japan Society for the Promotion of Science.

References


