NUMERICAL SIMULATION OF VORTEX INDUCED VIBRATION OF THREE CYLINDERS IN REGULAR TRIANGLE ARRANGEMENT

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ABSTRACT

This paper presents a two-dimensional simulation of the vortex induced vibrations of three elastic cylinders in regular triangle arrangement at low Reynolds number. The motion of every single cylinder, which is free to oscillate in two degrees-of-freedom in a uniform flow and has the same mass and natural frequency in both X and Y directions, is modeled by a mass-spring-damping system. The displacement, mean and fluctuating aerodynamic forces, Strouhal number (St) and vortex shedding pattern in the wake for each cylinder are analyzed with eight spacing ratios L/D changing from 1.5 to 6.0. The results indicate that the mean drag forces of upstream cylinders are larger than that of downstream cylinder, and the downstream cylinder is usually undergone serious fluctuating lift and drag forces. It is found that the simultaneous resonance in the x- and y-directions may occur for the downstream cylinder and the frequencies of the streamwise and transverse responses are same but with a phase shift. The streamwise oscillation of downstream cylinder could be as large as 1.0D, and the maximum transverse amplitude of three cylinders can reach to 0.85D. It is indicated that the cross-flow oscillation amplitude of three cylinders significantly increased compared with the flow-induced vibration of a single elastic cylinder and the streamwise oscillation of downstream cylinder is unneglectable for vortex-induced vibration of multi-cylinder system.

KEYWORDS: THREE CYLINDERS, VORTEX INDUCED VIBRATION, SPACING RATIO, SIMULTANEOUS RESONANCE

Introduction

Vortex induced vibrations (VIV) of cylinder group are a very common phenomenon in engineering, such as high-rise building groups, suspended cables of long-span bridges, marine cables and subsea pipelines in offshore platform, and so on. The resulting vibration has significant influence of the fatigue life of structures and could cause disastrous failure of industrial facilities with heavy financial losses. Therefore, engineers and researchers have spent much effort to investigate the VIV of cylindrical structures [Sarpkaya (2004), Govardhan (2001), Jauvtis (2003), Williamson (2004)]. Nevertheless, the studying on VIV of the cross-flow past more than one or two cylinders is still relatively scarce, especially numerical simulation [Lin (2005), Liu (2001), Lam (2006)]. Computational fluid dynamics (CFD) has become a powerful tool for solving complex fluid flow problems in the last decade and has been used to calculate the flow around single and multiple cylinders cover a wide range of Reynolds number (Re). In this paper, numerical simulations are presented for the
VIV of three cylinders in triangle arrangement at Re=200. The 2-D Navier-Stokes equations are solved by a finite volume method (FVM) with an industrial CFD code in which a coupling procedure has been implemented in order to obtain the cylinder response. The spacing ratio L/D is set as 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0 and 6.0 in turn. The main objective of this study is to examine the effect of L/D on the flow pattern, aerodynamic forces and response of the three elastically mounted cylinders.

**Numerical method**

*Computational models and boundary conditions setting*

The numbering and arrangement form of three equal diameter cylinders are shown in Figure 1. The L/D is defined as spacing ratio, here, L is the center-to-center distance, D is the cylinder diameter and equal to 0.01 m. The unstructured grids are employed to discretize the flow field as shown in Figure 2. The rectangle computational region is 45D x 30D with 15D upstream, 30D downstream and 15D on either side, respectively. The flow direction is from left to right, left side is set as *velocity-inlet*, right side is set *pressure-outlet*, the relative pressure is set as 0, the upper and lower free slip boundaries are set as *symmetry* and the model surface is set as *wall*.

![Figure 1: Calculation Model and arrangement form of three elastic cylinders](image)

![Figure 2: Computational region and Mesh division at L/D=3.0](image)

*Solution process of fluid-structure interactions*

The fluid-structure interaction system was solved by loosely coupled method, and the dimensionless time-step was set as 0.06 for the all computations. The unsteady flow field is solved by CFD code (Fluent) based on a finite volume method (FVM) with a Pressure-Based algorithm. The Newmark-β method is manually written into the User-Defined-Function (UDF), during the calculation process, it is linked with solver to obtain the response of cylinders, the grid domain is updated by a dynamic mesh model. Utilizing rigid motion macro of Fluent to transfer cylinder’s velocity to mesh, when the mesh iteration converged, the whole fluid domain is updated and the next time-step started. The loop continues until the stable solution is achieved.

*Results analysis*

Current studies confirm that the ratio between the natural frequency $f_n$ of elastic cylinder and the vortex shedding frequency $f_s$ of flow around rigid cylinder (hereinafter referred to as “frequency ratio $f_n/f_s$”), the dimensionless mass ratio $M^*=m/\rho D^2l$ and reduced damping $\zeta = 8\pi S_S^2 M^* \zeta$ are important parameters which have significant influence on the structural vibration. The parameters $S_S=0.01, M^*=1.0$ and $f_n/f_s=1.30$ are chosen for each of three cylinders, and a single cylinder with the same parameter subjected to VIV confined in the resonance band such as described by Zhou (1999).

*Validation*

Firstly, the flow around a rigid cylinder at Re=200 is carried out in order to ensure the reliability of numerical calculation. Figure 3 (a) denotes the dimensionless time histories of lift and drag coefficients of single cylinder. Figure 3 (b) indicates spectral analysis of lift and
drag coefficients. The present calculations agree well with the published results are tabulated in Table 1. $C_l$ and $S_t$ calculated in this paper are larger than that of the empirical formula summarized by Norberg (2003), $C_l$ is relatively exact and slightly smaller than that of the cell boundary element method by Farrant (2000). $C_l$, $C_l'$ and $S_t'$ are smaller than the previous results of simulations and are close to the results of Lam (2008). It is shown that the grid resolution, time step and numerical solving form at chosen in present paper are proper for numerical simulation of the flow around the cylinder.

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<th>$C_d$</th>
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<th>$C_l'$</th>
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Figure 3: Lift and drag coefficient time histories (a) and spectral analysis (b) for the flow around a rigid cylinder

**Aerodynamic forces and Responses**

Figure 4 shows the force coefficients vary as the spacing ratio for three cylinders in regular triangle arrangement, which include the mean and RMS value of lift coefficient $C_l$ [Figure 4(a)] and $C_l'$ [Figure 4(b)], the mean and RMS value of drag coefficient $C_d$ [Figure 4(c)] and $C_d'$ [Figure 4(d)], respectively.

Figure 4(a) indicates that the mean lift coefficients of upstream cylinders 2 and 3 are equal to each other in magnitude with opposite direction. It is seen that $C_l'$ of upside is positive and $C_l'$ of underside is negative, and the $C_l$ of downstream cylinder 1 has positive-negative value for $L/D=2.0~5.0$. All lift coefficient mean values gradually trend to zero as $L/D$ increasing, and it indicates that the flow interference effects between upper and lower rows cylinders is gradually weakened as spacing increasing. In Figure 4(b), the RMS lift coefficients of cylinders 2 and 3 are close to each other, and the $C_l'$, $C_l'$ are less than $C_l$ of cylinder 1 for the all $L/D$ range. The $C_l'$, $C_l'$ rapidly increase form 0.23 to 0.77 at $L/D=1.5~2.5$, approaching to 0.65 for $L/D>3.5$. When $L/D$ increases up to 2.0, the $C_l'$ increases form 0.58 to 0.93, the variation of $C_l'$ represents a concave characteristic for $L/D=2.5~4.0$, the $C_l'$ reaches maximum value of 1.06 at $L/D=4.0$ and gradually decreases with $L/D$ increasing. In Figure 4(c) and (d), the $C_d$, $C_d'$ of cylinders 2 and 3 are keep coincident with each other. The $C_d'$, $C_d'$ have little change when $L/D$ varies from 1.5 to 2.0, then increase slowly with $L/D$ increasing and keep constant of 2.2 when $L/D>3.5$. In the region of $L/D=1.5~4.0$, the $C_d'$ is less than $C_d'$ and $C_d'$, this is because of the downstream cylinders are shielded in the wake of upstream cylinders. In the small region of $L/D<2.0$, the $C_d'$, $C_d'$, $C_d'$ of the three cylinders are approximately equal to each other. But the $C_d'$, $C_d'$ have little change, the $C_d'$ has a convex characteristic for $L/D=2.0~6.0$ and reaches the maximum value of 0.55 at $L/D=3.5$. 

Figure 4: Mean and RMS values of lift and drag coefficients versus $L/D$
The ratio of cross-flow displacement mean and RMS values to diameter is shown in Figure 5(a) and Figure 5(b), respectively. The ratio of the in-flow displacement mean and RMS values to diameter is shown in Figure 5(c) and Figure 5(d), respectively.

In Figure 5(a), the cross-flow displacement mean $Y$ of each cylinder express same trend with the lift coefficients as shown in Figure 4(a). When the L/D changes from 2.5 to 3.0, the $Y$ of each cylinder obviously increase. Such an abrupt increase is due to the flow pattern transformation which will be discussed in the subsequent section later. Figure 5(b) shows the cross-flow displacement RMS value $Y'$ of each cylinder behave similarly. That is, when the L/D=1.5~2.5, the $Y'$ of each cylinder obviously increase. Such an abrupt increase is due to the flow pattern transformation which will be discussed in the subsequent section later. Figure 5(b) shows the cross-flow displacement amplitude of each cylinder reach their maximum values with 0.85 times of cylinder diameter. In Figure 5(c), it can be seen that the in-flow displacement mean $X$, of upstream cylinder 2, 3 have the same changing trend and increase by 1.5 times in the all L/D range. The $X$ of downstream cylinder 1 gradually increases from 0.18 to 0.6 as L/D increases and reaches the maximum value at L/D=6.0. Figure 5(d) shows the in-flow displacement RMS value $X'$ of downstream cylinder 1 reaches maximum value also at the L/D=3.5, where the maximum value is about 1.0D. It can be seen that the variation trend of $X'$ is different from $Y'$ and the order of magnitude of $X'$ is about an order larger than that of $Y'; this means that the flow experiences a significant flow pattern transformation there.

![Figure 5: Mean and RMS values of x- and y-directions displacemet versus L/D](image)

Figure 6 shows the frequency analysis of the force and response of cylinders, which include the Strouhal number $St$ [Figure 6(a)], dimensionless frequency of drag coefficient $f_{cd}$ [Figure 6(b)], the dominant frequency of y-direction displacement $f_Y$ [Figure 6(c)] and x-direction displacement $f_x$ [Figure 6(d)].

In the subgraphs of Figure 6, the changing trend of parameters which include $St$, $f_{cd}$, $f_Y$, $f_x$ of cylinder 2,3 become consistent with each other. In general, $St$ is calculated only when vortex shedding occurs behind the cylinder. If there is no vortex shedding behind the upstream cylinder, $St$ is not calculated. In Figure 6(a), the $St$ of cylinder 2,3 increases from 0.121 to 0.184 when L/D increases up to 2.0 and increases slowly when L/D $\geq$ 2.5; the maximum $St$ of cylinder 2,3 occurs near L/D=4.0, where the maximum value is about 0.194. In the region of L/D $\leq$ 2.5, the $St$ of cylinder 1 increases from 0.121 to 0.182 and has a concave characteristic for L/D between 2.0 and 6.0, namely, a slight increase in value from L/D=2.0 to a maximum 0.23D at L/D=3.5 and then descends again. The computed in-flow displacement amplitude of downstream cylinder 1 reaches maximum value also at the L/D=3.5, where the maximum value is about 1.0D. It can be seen that the variation trend of $X'$ is different from $X$, $Y'$ and the order of magnitude of $X'$ is about an order larger than that of $X$, $Y'; this means that the flow experiences a significant flow pattern transformation there.
flow frequency of vortex shedding approach to the in-flow frequency of vortex shedding for the downstream cylinder 1. The dominant frequencies of the transverse displacement $f_y$ and the streamwise displacement $f_x$ are shown in Figure 6(c) and (d), respectively. When $L/D \leq 3.5$ the $f_{y2}^1$, $f_{y3}^1$ are very close to the $f_{y2}^2$, $f_{y3}^2$ for the downstream cylinder 2,3, on the other hand, the $f_{x2}^1$, $f_{x3}^1$ are about 2 times of $f_{y2}^2$, $f_{y3}^2$ when $L/D \geq 4.0$. In the region of $L/D=2.5\sim6.0$, the $f_{x1}^1$ approaches to the $f_{y1}^1$ for the downstream cylinder, it is indicated that both transverse and streamwise resonances have been occurred simultaneously when $f_x$ is equal to $f_y$. The maximum transverse oscillation amplitude of each cylinders can be reached 0.85D which is much larger than 0.57D of the single cylinder undergoing VIV reported by Zhou (1999) using a DVM with the same parameters setting. The amplitude of the fluctuating part of the streamwise vibration increased evidently and maximum value approaches to 0.23D for the cylinder 1. It is means that the transverse oscillation amplitude of upstream cylinders significantly increased and the streamwise oscillation of downstream cylinders is unneglectable for vortex-induced vibration of multi-cylinder system.

![Figure 6: FFT spectrum analysis of force and displacement versus L/D](image)

The motion trajectories of each cylinder centroid for spacing ratio $L/D=1.5$, 2.5, 3.5 and 5.0 are shown in Figure 7(a-d). These plots clearly show that the oscillations are self-limiting process. When $L/D$ increases from 1.5 to 2.5, the motion trajectories of upstream cylinder 2,3 express oblique and erect ellipses, it indicated that the frequency of streamwise oscillation is equal to the frequency of the transverse oscillation of two upstream cylinders and there is a certain phase difference between two oscillations, which is similar to reported by Lin (2005). As $L/D$ increases further, the orbits of cylinders 2,3 change into a full “figure of 8” shape, and then the “figure of 8” shape is more and more distinct with continuously increasing of the spacing ratio as shown in Figure 7(c-d). The reason could be attributed to the frequency of streamwise oscillation is two times of that of transverse oscillation for two upstream cylinders; furthermore, the RMS of streamwise vibration is less than RMS of transverse vibration, so, the trajectory represents a vertical “figure of 8” shape. In the all subgraph of Figure 7, it can be seen that the motion trajectory of downstream cylinder 1 represent superposition of downward and upward oblique ellipses, this signifies the frequency of streamwise oscillation is equal to the frequency of the transverse oscillation, but the phase difference between two oscillations changed continuously with time advancing.

![Figure 7: Locus of the vibrating cylinders in regular triangle arrangement at some L/D](image)

Figure 8 indicate the time histories of lift and drag coefficients, the ratio of the cross-flow displacement to diameter D and the ratio of the in-flow displacement to diameter D of every cylinder for the $L/D=1.5$, 2.5, 3.5 and 5.0.
In the all subgraph of Figure 8, it can be seen that the lift coefficient and transverse displacement of every cylinder represent in-phase for the all L/D range, so are those of the drag coefficient and streamwise displacement of downstream cylinder 1. The lift coefficient and transverse displacement of upstream cylinders are comparatively regular as shown in Figure 8(b-d) and it indicates that the vortex shedding is controlled by a dominant frequency. It is evidently noticed that the fluctuating part of the streamwise displacement of upstream cylinders is very small. The irregularity of the aerodynamic force and the response of three cylinders indicate that there are several frequency components as shown in Figure 8(a), so the intense interference of the upstream cylinders to the downstream cylinder induce complex vortex shedding from the downstream cylinder for the small spacing ratio L/D=1.5. As shown in Figure 8, the following discussion will mainly concentrate on cylinder 1, in the all L/D range, the computed aerodynamic force and the response of cylinder 1 behave irregular shape and it can be obviously observed that the phase difference between cross-flow and in-flow range, the computed aerodynamic force and the response of three cylinders is very small. The irregularity of the aerodynamic force and the response of three cylinders as L/D increases and the vortices shedding from downstream cylinder are tempestuously merged in the wake. In the region of L/D≥4.0, the flow pattern is about close

Flow pattern

Figure 9 shows the instantaneous vorticity contours in the wake of three elastic cylinders in regular triangle arrangement at the time when the transverse oscillating amplitude of the downstream cylinder 1 reaches to the positive maximum value.

In Figure 9(a-b), at small L/D=1.5~2.0, two inner side free shear layers from the upstream cylinders reattach onto the downstream cylinder surface, and the flow pattern is similar to the wake of flow past a single body. When L/D=2.5~3.5, the shear layer of upstream cylinders rolls up in the wake, enters into the wake of downstream cylinder, combines with the wake vortex of downstream cylinder and induces the wake of downstream cylinder presenting different scale and disturbed vortices as shown in Figure 9(c-e), this figures denote that the vortex shedding pattern approaches to “in-phase” for upstream cylinders as L/D increases and the vortices shedding from downstream cylinder are tempestuously merged in the wake. In the region of L/D≥4.0, the flow pattern is about close
to “anti-phase” for two upstream cylinders as shown in Figure 9(f-h). Figure 9(f) shows that the vortices from upstream cylinders surface are merged after a certain distance and then changed into single vortex street at L/D=4.0, where the wake structure is more complex and the fluid-solid coupling effect is also significant, therefore, the abrupt increase of aerodynamics force and displacement maybe occurred near L/D=4.0 as discussed in above. Figure 9(g-h) denote that the interference effect between three cylinders is gradually weakened and the upstream cylinders have an approximate independent vortex street, but the wake of downstream cylinder had little influenced by the upstream cylinders when L/D ≥5.0.

![Figures](a) L/D=1.5 (b) L/D=2.0 (c) L/D=2.5 (d) L/D=3.0 (e) L/D=3.5 (f) L/D=4.0 (g) L/D=5.0 (h) L/D=6.0

**Figure 9:** Instantaneous vorticity contour in the wake of three cylinders

**Conclusions**

This paper presents a feasible research method for numerical simulation of vortex-induced vibration of elastic multi-cylinder oscillating system. The fluid domain simulation is completed by Fluent, the structure response is achieved using the Newmark-β method and the grid domain updating is accomplished through a dynamic mesh method. The mass ratio, reduced damping and frequency ratio are kept invariant and the emphasis analysis is carried out for influence of the spacing ratio variety to aerodynamical forces, vibrating responses and wake vortex modes of three cylinders in regular triangle configuration. The following conclusions are obtained from this study:

1. As the spacing ratios increasing, the mean lift coefficient of each cylinder trends to 0. The fluctuating drag and lift coefficients of downstream cylinder are larger than those of upstream cylinders for all range of spacing ratios. The $\bar{C}_l$ of downstream cylinder is less than $\bar{C}_l$, $\bar{C}_l$, $\bar{C}_l$ of upstream cylinders when L/D ≤4.0 and it is larger than that when L/D ≥5.0, the process bearing in mind the difference in Re.

2. In all L/D range, the fluctuation of the transverse displacement (Y') is very close to each other, and the fluctuating streamwise displacement of downstream cylinder (X') is larger than that of upstream cylinders.

3. When L/D increases up to 2.5, the motion trajectories of upstream cylinders represent oblique and erect ellipses, as the L/D increases further, the orbits of upstream cylinders change into a full “figure of 8” shape. For the downstream cylinder, the motion trajectory represents superposition of downward and upward oblique ellipses.

4. The maximum transverse oscillation amplitude of three cylinders can be reached 0.85D which is much larger than that of the single cylinder undergoing VIV with the same parameters setting. On the other hand, the maximum streamwise fluctuating amplitude of the downstream cylinders reaches 0.22D. It is indicated that the transverse oscillation amplitude of upstream cylinders significantly increased and the streamwise oscillation of downstream cylinders is unneglectable for vortex-induced vibration of multi-cylinder system.

**References**


