NEW INFLOW BOUNDARY CONDITIONS FOR MODELING THE NEUTRAL EQUILIBRIUM ATMOSPHERIC BOUNDARY LAYER IN SST k-ω MODEL

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ABSTRACT

Modeling an equilibrium atmospheric boundary layer (ABL) in computational wind engineering (CWE) requires the inflow boundary conditions to satisfy the turbulence model equations. Based on this viewpoint, two new sets of inflow turbulence boundary conditions (logarithmic law type/power law type) are presented theoretically for the SST k-ω model for the purpose of practical use in CWE. The capabilities of the proposed inflow boundary conditions in producing a horizontally homogeneous equilibrium ABL are subsequently demonstrated by performing numerical simulations of simple ABL flows of four wind terrain categories.

KEY WORDS: COMPUTATIONAL WIND ENGINEERING, SELF-SUSTAINABLE EQUILIBRIUM ATMOSPHERIC BOUNDARY LAYER, BOUNDARY CONDITIONS, SST k-ω MODEL, RANS

1. Introduction

Being able to model equilibrium atmospheric boundary layers (ABL) in CFD is an important precondition for modeling flow around buildings. Equilibrium ABLs imply horizontal homogeneity, which means that the streamwise gradients of all variables should be zero.

In recent years, the requirements of modeling of an equilibrium ABL for the numerical investigation of flow around buildings has been emphasized by many researches (Richards and Quinn, 2002; Blocken et al., 2007). Both the guidelines for CFD prediction of wind flow in the urban environment by the COST Action 732 group (Franke et al., 2007) and the AIJ (Mochida et al., 2006, Tominaga et al. 2008) emphasized the requirement to model an equilibrium ABL prior to the numerical investigation of flow around buildings.

Richards and Hoxey (1993) proposed inflow boundary conditions of mean wind speed and turbulence quantities for the standard k-ε model that satisfied the transport equations for k and ε. This type of boundary conditions has been widely used in CWE based on the RANS method (Franke et al., 2007). Recently, Hargreaves and Wright (2007) indicated that using Richards and Hoxey’s inlet boundary conditions is necessary but not sufficient to produce an equilibrium ABL for the standard k-ε model.

The problem of simulating an equilibrium boundary layer is further investigated by Yang et al. (2008, 2009) from the point of view of the turbulence model itself. Based on the assumption of local equilibrium of turbulence, the solution of the k equation of the standard k-ε model was derived firstly and a new set of inflow turbulence boundary conditions was
proposed. And the capability of these inflow boundary conditions in the standard k-ε model to produce an equilibrium ABL was numerically demonstrated.

In this paper, the research method originally proposed based on the standard k-ε model is extended to the SST k-ω model, which is more suitable for blunt body flow simulation, and the similar new sets of inflow boundary conditions for modeling the equilibrium ABL are proposed. The different descriptions of the inflow velocity (logarithmic law/power law) and the corresponding new inflow boundary conditions are presented simultaneously for practical use.

2. Equilibrium ABL models for the SST k-ω model

The k-ω based SST (Shear Stress Transport) model developed by Menter (1994) employs the k-ω model near the surface and the k-ε model in the free-shear layers. A blending function is adopted to bridge these two models. The SST k-ω model takes into account of the transport of the turbulent shear stress and gives highly accurate predictions of the onset and the amount of flow separation under adverse pressure gradients (Menter, 1994). For these reasons and relatively high efficiency for numerical solution, the two-equation SST k-ω model is adopted frequently for the numerical simulation of blunt body flow.

The equations of the Wilcox k-ω model are multiplied by a blending function F₁, and the transformed k-ε equations are multiplied by the function 1-F₁. Then the corresponding turbulent kinetic energy k equation and the turbulent frequency ω equation are obtained to form the SST k-ω model:

\[
\frac{\partial \rho k}{\partial t} + \frac{\partial}{\partial x} \left( \rho u k - \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x} \right) = \frac{\partial}{\partial z} \left( \frac{\partial}{\partial z} \left( \frac{k}{\sigma_k} \frac{\partial k}{\partial z} \right) \right) - C_{\mu} \rho \omega k \tag{1}
\]

\[
\frac{\partial \rho \omega}{\partial t} + \frac{\partial}{\partial x} \left( \rho u \omega - \left( \mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x} \right) = \alpha \frac{\omega}{k} P - \beta \rho \omega^2 + 2(1 - F_1) \frac{\rho}{\sigma_\omega \omega} \left( \frac{\partial k}{\partial x} \right)^2 \frac{\partial \omega}{\partial x} \tag{2}
\]

where \( \rho \) is the density of fluid, \( k \) and \( \omega \) are the turbulent kinetic energy and its dissipation frequency, respectively and \( P_k \) is the production of turbulent kinetic energy. The eddy viscosity in the SST k-ω model is given by

\[
\mu_t = \rho \frac{k \omega}{\omega} \tag{3}
\]

Comparing the eddy viscosity in the standard k-ε model:

\[
\mu_t = \rho C_{\mu} \frac{k^2}{\varepsilon} \tag{4}
\]

The following relation between \( \omega \) and \( \varepsilon \) can be obtained:

\[
\omega = \frac{1}{C_{\mu}} \frac{\varepsilon}{k} \tag{5}
\]

We assume a steady, incompressible and homogeneous flow. Homogeneity implies \( \omega, k \) and \( \varepsilon \) are invariant with the streamwise and lateral coordinates \( x \) and \( y \), and only vary with height above ground \( z \). Furthermore, in highly turbulent flows, \( \mu_t \gg \mu \). Eq. (1) then reduces to:

\[
\frac{\partial}{\partial z} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial z} \right) + \frac{\mu_t}{\sigma_k} \left( \frac{\partial u}{\partial z} \right)^2 - C_{\mu} \rho \omega k = 0 \tag{6}
\]

Substituting Eq. (3) into Eq. (6) leads to the following equation:

\[
\frac{1}{\sigma_k} \frac{\partial}{\partial z} \left( k \frac{\partial k}{\partial z} \right) + \frac{k}{\omega} \left( \frac{\partial u}{\partial z} \right)^2 - C_{\mu} \rho \omega k = 0 \tag{7}
\]
Assuming that the turbulence is under local equilibrium condition, i.e., the rate of production of turbulent kinetic energy is equal to the rate of dissipation: \( P_k = \rho \varepsilon = \rho C_\mu k \omega \). Based on the Boussinesq eddy viscosity hypothesis to estimate the Reynolds stress, \( P_k \) can be written as: \( P_k = \mu_r \left( \frac{\partial u}{\partial z} \right)^2 \). Combining the above two expressions of \( P_k \) with Eq. (3) yields

\[
\omega = \frac{1}{\sqrt{\nu}} \frac{C_\mu}{C_w} \left( \frac{1}{\nu} \frac{\partial}{\partial z} \right) \frac{\partial}{\partial z} k
\]

Substituting Eq. (8) into Eq. (7) yields:

\[
\sqrt{\frac{C_\mu}{\nu}} \left( \frac{1}{\nu} \frac{1}{\partial z} \right) \frac{\partial}{\partial z} k = 0
\]

Eq. (9) can be rewritten as:

\[
\frac{k \cdot \partial k}{\partial u} = \text{const.}
\]

2.1 “Log-law” type equilibrium inflow boundary conditions

We assume that the mean velocity profile can be represented by the logarithmic law:

\[
u = \frac{\nu_0}{\kappa} \ln \left( \frac{z + z_0}{z_0} \right)
\]

Where \( \nu_0 \) is the friction velocity, \( \kappa \) is the von Karman constant and \( z_0 \) is the aerodynamic roughness length. Then the solution of Eq.(10) can be expressed in the following form after a simple linear transformation is introduced:

\[
k = \frac{\nu_0^2}{\sqrt{\nu}} \left( C_1 \ln \left( \frac{z + z_0}{z_0} \right) + C_2 \right)
\]

In Eq. (12), \( C_1 \) and \( C_2 \) are two constants that describe the inflow turbulence level, and they could be determined through the nonlinear fitting of the experimental data or the theoretical formula.

In above derivation, the analytical solution to the \( k \) transport equation (Eq. (1)) is obtained. The \( \omega \) transport equation (Eq. (2)) on the other hand is a complex nonlinear partial differential equation, for which no closed-form analytical solution can be obtained.

Next, considering the relationships of \( \omega \) with \( u \) and \( \varepsilon \) in Eq. (8) and Eq. (5), a similar form for \( \omega \) and \( \varepsilon \) could be suggested:

\[
\omega = \frac{u_0}{\kappa \sqrt{\nu}} \left( \frac{1}{\nu} \frac{\partial}{\partial z} \right) \left( \frac{1}{\nu} \frac{\partial}{\partial z} \right) \left( \frac{1}{\nu} \frac{\partial}{\partial z} \right) \left( \frac{1}{\nu} \frac{\partial}{\partial z} \right)
\]

\[
\varepsilon = \frac{u_0^3}{\kappa (z + z_0)} \sqrt{C_1 \ln \left( \frac{z + z_0}{z_0} \right) + C_2}
\]

Modeling an equilibrium boundary layer in CFD requires the inflow boundary conditions to satisfy the turbulence model equations (Yang et al., 2008, 2009). Based on this viewpoint, the new set of the inflow boundary conditions for modeling an equilibrium ABL with the SST \( k-\omega \) model could be proposed (see Eqs. (11), (12) and (13) or (14)) based on the approximate solutions to the turbulence model equations. For the logarithmic law of the velocity profile being adopted, it is referred to as “Log-law” type inflow boundary conditions hereafter. The capability of the “Log-law” type to model an equilibrium ABL have been numerically verified for the standard \( k-\varepsilon \) model (Yang et al., 2009).
2.2 “Power-law” type equilibrium inflow boundary conditions

The mean velocity profile $u$ could also be expressed by the power law as Eq. (15), such as in the Load Code for the Design of Building Structures of China (Load code for the design of building structures, GB 5009-2001, China).

$$u = u_r \left( \frac{z}{z_r} \right)^{\alpha_i}$$  \hspace{1cm} (15)

Where $z_r$ is the reference height, $u_r$ is the mean velocity at the reference height $z_r$ and $\alpha_i$ is the characteristic index describing the corresponding terrain category.

If the power law of the mean velocity is employed, the theoretical solution to the Eq.(10) can be expressed in the following form:

$$k = \sqrt{D_z z^{\alpha_i} + D_i} \hspace{1cm} (16)$$

Considering the relationships of $\omega$ with $u$ and $\varepsilon$ in Eq. (8) and Eq. (5), a similar form for $\omega$ and $\varepsilon$ could be obtained:

$$\omega = \frac{\alpha_i u}{C_{\mu} z}$$  \hspace{1cm} (17)

$$\varepsilon = \alpha_i C_{\mu} \frac{u}{z} \sqrt{D_z z^{\alpha_i} + D_i} \hspace{1cm} (18)$$

Where $u$ is the mean velocity profile described as Eq. (15).

Through above derivation, another set of inflow boundary conditions (Eqs. (15), (16) and Eq.(17) or Eq.(18) ) for modeling the equilibrium ABL with the SST $k$-$\omega$ model is obtained similarly. For the power law of the velocity profile being employed, it is referred to as “Power-law” type inflow boundary conditions.

Comparing the two sets of equilibrium inflow boundary conditions, the “Log-law” type and the “Power-law” type, it is found that the expression of $k$ is similar, while the expression of the $\omega$ or $\varepsilon$ is slightly different-the “Power-law” type contains the variable “$u$”.

3. Verification of the new equilibrium ABL model

In this section, the performances of the two types of inflow turbulence boundary conditions, “Log-law” type and “Power-law” type, are demonstrated by numerically reproducing the simple boundary layer flows of four wind terrain categories A, B, C and D defined in the Load Code for the Design of Building Structures of China (Load code for the design of building structures, GB 5009-2001, China).

3.1 Computational model

A series of full size 3D numerical simulations in a domain without obstacles is carried out to verify the capability of the present two types of inflow turbulence boundary conditions for modeling an equilibrium ABL. The size of the computational domain is $L \times B \times H = 500 \text{ m} \times 400 \text{ m} \times 300 \text{ m}$. The height of the first mesh layer above the ground, $y_{\text{min}}$ is set as 0.1m. The mesh sizes in horizontal and lateral direction are set as 2.5m and 13.3m uniformly. This gives a total amount of cells of 600,000. The simulations are performed with the commercial CFD code CFX5.7.

The boundary conditions for the computation models are listed in Table 1, in which the inflow turbulence boundary conditions for the SST $k$-$\omega$ model take the expressions of “Log-law” type (Eqs.(11)-(14)) and “Power-law” type (Eqs. (15)-(18)) respectively. And Four wind terrain categories A, B, C and D, which are defined in the Load Code for the Design of Building Structures of China, are simulated in the numerical research. Therefore, totally eight computations of ABL flows are performed. In each one, the parameters in the inflow
boundary conditions are taken from the code or through nonlinear fittings of the turbulent kinetic energy.

The ground boundary is modeled as a rough wall and the roughness height are set as \(1/2y_{min}\). The upper face of the computational domain is modeled with the free slip boundary condition. In this study, the shear stress suggested by Richards and Hoxey (1993) is not applied at the upper face in order to demonstrate the simple effect of inflow boundary conditions on the modeling of the boundary layer flow.

According to previous researches (Yang et al, 2008, 2009), the values of the turbulence parameters would heavily influence the numerical simulation results of ABL and the blunt body flows. Therefore, the relevant turbulence parameters, e.g. \(C_\mu\), in the SST \(k-\omega\) model should be changed accordingly so as to consist with the turbulence characteristics being researched.

Table 1 Boundary conditions of computation models

<table>
<thead>
<tr>
<th>Location</th>
<th>Boundary condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflow boundary</td>
<td>“Log-law” type:</td>
</tr>
<tr>
<td></td>
<td>(u = \frac{u_*}{\kappa} \ln \left( \frac{z + z_0}{z_0} \right), \quad v = 0, w = 0;)</td>
</tr>
<tr>
<td></td>
<td>“Power-law” type:</td>
</tr>
<tr>
<td></td>
<td>(u = u_* \left( \frac{z}{z_*} \right)^{\alpha}, \quad v = 0, w = 0;)</td>
</tr>
<tr>
<td></td>
<td>(k = \sqrt{D_z n^2 + D_2})</td>
</tr>
<tr>
<td></td>
<td>(\omega = \frac{\alpha u}{\sqrt{C_\mu} z})</td>
</tr>
<tr>
<td>Downstream boundary</td>
<td>Outflow (\frac{\partial}{\partial x} (u, v, w, k, \omega) = 0)</td>
</tr>
<tr>
<td>Upper face of</td>
<td>Free slip (w = 0, \quad \frac{\partial}{\partial z} (u, v, k, \omega) = 0)</td>
</tr>
<tr>
<td>computational domain</td>
<td>Free slip (v = 0, \quad \frac{\partial}{\partial z} (u, w, k, \omega) = 0)</td>
</tr>
<tr>
<td>Ground surface boundary</td>
<td>Wall Rough wall modification with roughness height (K_S=0.05m).</td>
</tr>
</tbody>
</table>

The computational settings include the SIMPLEC algorithm for pressure-velocity coupling, the second-order upwind scheme for the convective terms and the central differencing scheme for the diffusion terms in the momentum equation and the turbulence model equations. The flow field is initialized by using the values set for the inlet boundary conditions. The convergence criteria of the scaled residuals for all the variables and the continuity equation are set as \(10^{-6}\).

3.2 Numerical Results

Figure 1 to Figure 4 illustrates the numerical simulation results of simple ABL flows of four terrain categories under the two types of inflow boundary conditions.

As can be seen from the figures:

(1) As a whole, under the present two types of turbulence boundary conditions, both the profiles of mean velocity and turbulent kinetic energy of four categories could be sustained well throughout the whole domain except for the small regions near the ground and top. The errors at the top are related to the top boundary treatment (Richards and Hoxey 1993,
Blocken et al. 2007, Hargreaves and Wright, 2007) and the errors at the ground are influenced by the method of rough wall modifications in the numerical models.

(2) With the increase of the terrain roughness, the precisions of simulated ABL show a tendency of decrease. This phenomena exhibit the important influence of wall roughness on the ABL besides the inflow boundary conditions. Therefore, more appropriated rough wall modifications are needed to consider the real larger roughness under terrain categories C or D for obtaining more accurate results. Recently, Fang (2009) proposed a new method of rough wall function modification to improve the simulation accuracies of large roughness terrains.

(3) The differences in performance between the Log-law type inflow boundary conditions model and the Power-law model in modeling an equilibrium ABL could be neglected. Both could generate appropriated equilibrium ABL without any additional modifications, and the precisions exhibited in the numerical simulations are almost the same. The reason could be easily understood: though the final expressions of the two boundary conditions are different due to different expressions for the mean velocity being adopted, both of them are theoretically deduced from the SST k-ω model equations based on the same method.

Fig. 1 Comparisons of velocity and turbulent kinetic energy profiles at inlet, 1/2L and outlet for Category A under Log-law and Power-law equilibrium inflow boundary conditions.
Fig. 2 Comparisons of velocity and turbulent kinetic energy profiles at inlet, 1/2L and outlet for Category B under Log-law and Power-law equilibrium inflow boundary conditions.

Fig. 3 Comparisons of velocity and turbulent kinetic energy profiles at inlet, 1/2L and outlet for Category C under Log-law and Power-law equilibrium inflow boundary conditions.
4. Conclusions

In this paper, based on the viewpoint of modeling an equilibrium atmospheric boundary layer (ABL) requiring the inflow boundary conditions to satisfy the turbulence model equations, two new sets of inflow turbulence boundary conditions (“Log-law” type and “Power-law” type) are presented theoretically for the SST $k-\omega$ model, which is more suitable for blunt body flow simulation. The capabilities of the proposed two types of inflow boundary conditions in producing a horizontally homogeneous equilibrium ABL are subsequently demonstrated by performing numerical simulations of simple ABL flows of four wind terrain categories. More researches on the rough wall modification are still needed to improve the simulation accuracies of large roughness terrains.

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References


