ABSTRACT

In this paper, the branch-switch characteristics of coupled flutter is studied basing on SBS(Step-by-step) flutter analysis [Matsumoto 1996,1997,1997,2007,2008] in contrast with the results obtained by conventional CEV(Complex Eigen-Value) analysis. In coupled flutter, there are two different branches, those are heaving branch (HB) and torsional branch(TB). As far as the flutter characteristics, those are frequency-velocity, damping-velocity, amplitude ratio-velocity and phase difference-velocity, the results obtained by both analysis are perfectly coincide, exceptionally branch switching. In CEV analysis, any branch switch is so far never observed. On the contrary, in SBS analysis, at slightly higher reduced velocity after flutter-onset, two branches switch mutually[Matsumoto 1996,1997,1997,2007,2008]. However, it has been not clarified whether which results obtained by SBS and CEV is correct. This paper certifies that the branch switch can really appears by analysis the particular case of thin plate with the frequency ratio between torsional and heaving natural frequencies, fT/fH, of close to 1.0. Furthermore, basing flutter fundamental mode, at higher reduced velocity after flutter-onset, the peculiar flutter in which heaving motion dominates and it is generated by pitching angle in quasi-steady way. This fact indicate that the HB flutter occurs at higher reduced velocity.

KEYWORDS: SBS/CEV FLUTTER ANALYSIS, BRANCH SWITCH IN COUPLED FLUTTER, THIN PLATE

Introduction

The authors have for long terms investigated on coupled flutter instability in relation to aerodynamic stabilization of super long spanned bridges. In particular they proposed a new analytical method so called by SBS(Step-by-step) analysis [Matsumoto 1996,1997,1997,2007,2008]. In this method the coupling between heaving and torsional motion is analized in the step-by-step procedure taking into account of the role of coupling forces and uncoupling forces separately, and iterative calculation is carried out till the flutter-frequency and flutter-damping at certain velocity converges. The four flutter characteristics, those are frequency-velocity, damping-velocity, amplitude ratio-velocity and phase difference-velocity, are obtained by CEV(Complex Eigen-Value)analysis and SBS analysis, individually. The both flutter characteristics obtained by CEV and SBS coincide,
exceptionally branch switching at the certain velocity. In SBS analysis, this branch switch appears but in CEV no appearance. In both analyses, the flutter onset occurs by torsional branch (TB) at the identical velocity. In SBS results, the branch switch from TB to HB appears at a little bit higher velocity after flutter-onset. The study on the branch switch is definitely important to not only in precise clarify the flutter mechanism but also utilize the flutter for power generation in a future. At high reduced velocity after flutter onset, the coupling mode between heaving and torsion is mainly controlled by heaving mode and it is produced by quasi-steady lift force by pitching angle. Therefore, from the view point of flutter mode, the branch switch is thought to must occur, however, from the another clear evidence is expected. Looking on the flutter characteristics of thin plate with the peculiar dynamics of torsional/heaving frequency ratio of close to 1.0, The flutter branch at almost whole velocity should be torsional branch (TB), because its flutter characteristics in coupled DOF, those are frequency-velocity and damping-frequency, are significantly close to those of torsional single DOF. SBS results precisely show the TB in this velocity range. Nevertheless, CEV results show HB. This is apparently contradiction. The reason of this contradiction is no appearance of branch switch in CEV. As far as why in CEV no appearance of branch switching might to be caused in bifurcation issue in non-linear complex number equations, so called “Hopf Bifurcation” issue.

Revised Step-by-Step analysis

From 2DOF spring supported wind tunnel tests, it proved that the damping of SBSA has difference from the test results. Thus, the cause of numerical difference should be investigated by considering the damping and we should revise SBSA. In SBSA, harmonic vibration is assumed in step1. Therefore, it is supposed that damping is not taken into account in the system of step2 generated by the coupled terms of the system of step1. So, in step1, the motion should be assumed as the form which also has the damping.

Then, revised SBSA is illustrated as follows. In TB, as step1, torsional motion is assumed taking the damping in consideration as follows;

$$\phi = \phi_0 e^{-\zeta t} \sin \omega t$$

(5)

In step2, heaving motion is generated by torsional motion as forced vibration, with the certain amplitude ratio and phase difference. As the results, in step3, torsional motion is also generated by heaving motion as free vibration. Finally, the flutter logarithmic damping and the flutter circular frequency are calculated with the following equations;

$$\delta_\tau = -\omega_\tau \delta_\tau - \omega_\tau \Omega_\tau \left[ \frac{A_0}{H_0} (1 + \zeta^2) (\zeta \sin \theta \cos \theta + A_0) \sin \theta \right]$$

(6)

where,

$$\Omega_\tau : \left( \frac{\partial \phi}{\partial \theta} \right), \Omega_\tau : \left( \frac{\partial \phi}{\partial \theta} \right) \eta (\omega_\tau) t$$

Similarly, in HB, heaving motion is assumed as follows in step 1;

$$\eta = \eta_0 e^{-\zeta t} \sin \omega t$$

(7)

And then, the flutter characteristics are calculated with the following equations;

$$\delta_\tau = -\omega_\tau \delta_\tau - \omega_\tau \Omega_\tau \left[ \frac{A_0}{H_0} (1 + \zeta^2) (\zeta \sin \theta \cos \theta + H_0) (\zeta \sin \theta \cos \theta + H_0) (\zeta \cos \theta \sin \theta + A_0) \sin \theta \right]$$

(8)

where,

$$\Omega_\tau : \left( \frac{\partial \phi}{\partial \theta} \right), \Omega_\tau : \left( \frac{\partial \phi}{\partial \theta} \right) \eta (\omega_\tau) t$$
Figure 1 shows certain coupled flutter characteristics of $B/D=20$ rectangular cylinder obtained by CEVA and revised SBSA. The results of both analyses show completely good agreement even though they have different branch definition. Besides, by using revised SBSA, the branch switch characteristics are shown and effects of each unsteady aerodynamic derivative is clear. These characteristics are almost in agreement with former SBSA.

**Coupled Flutter Characteristics of Thin Plate with $f_T/f_H=1.0$ to 1.3**

The flutter characteristics, those are flutter frequency-velocity property and damping-velocity property, of each branch of thin plate with the dynamics of natural frequency ratio, $f_T/f_H$, in the range of 1.0 to 1.3 have been analyzed by SBS method and CEVA method, respectively.
Fig.2 Coupled Flutter Characteristics of HB and TB of Thin Plate, $f_\phi/f_\eta = 1.03$, analyzed by CEV and SBS, respectively (red-circle:SBS(HB), blue-circle:SBS(TB), blue-square:CEV(HB), blue-square: CEV(TB), red-dotted line: Heaving 1DOF, blue-solid line: Torsional 1DOF)

The calculated results are shown in Fig.2. In these figures the calculated results obtained for torsional single DOF are also indicated. The damping –velocity property and frequency-velocity property of TB obtained by SBS method coincide with those of HB obtained by CEV method, indicated by circle and dotted line, individually. The branch difference in the results must be caused by branch switch in SBS analysis and non-branch switch in CEV analysis. On the other hand, those obtained by torsional single DOF indicated by shaded solid line are similar with TB obtained by SBS method. This result clearly indicates that the significant change of damping and frequency with increase of velocity must be for TB. This similar property of flutter characteristics are also observed at higher frequency ratio of 1.3. Therefore, in coupled flutter, at higher velocity than flutter onset velocity branch switch must occur, and SBS method can re-produce the branch switch.

Flutter Mode at Higher Velocity after Flutter Onset

The flutter mode immediately after flutter onset of plate-like bodies, such as rectangular cylinder with B/D=20, truss-stiffened bridge girder of Akashi-Kaikyo Bridge and so on, is like fundamentally torsional motion with the rotational axis at near leading edge[see Fig.3.1], on the other hand, at sufficiently higher velocity after flutter-onset it looks like mainly heaving motion with large amplitude with accompany of torional motion with small amplitude. This heaving motion is excited by lift force induced by pitching angle caused by torsional motion, therefore, its heaving dominated motion can be quasi-steady flutter mode[see Fig.3.2]. In other words, taking into account of these flutter modes, at immediate after flutter-onset, flutter should be controlled by TB, and at sufficiently high velocity by HB, respectively. Nevertheless, basing on the change of flutter-modes at different velocity, CEV solution consistently indicates TB as flutter mode.
Fig.3.2 Quasi-steady flutter where heaving mode generated by quasi-steady lift induced by torsional displacement at high velocity after flutter-onset

On the other hand, SBS solution shows properly change of flutter mode from TB to HB. The reason of branch switch re-production is the analytical procedure in SBS method, where flutter branch is clearly identified as final governing differential equation in 2DOF coupling equations, it means if its governing equation would be heaving equation, the flutter solution corresponds to HB, and if it would be torsional equation, to TB, respectively. On the contrary, in CEV method, flutter differential equations are expressed by complex frequencies and complex amplitude, but these complex equations and their solutions are not classified by flutter branches. Flutter branch is only identified by the frequency-velocity property. In detail, the branch of flutter solutions is only identified by whether the continuous frequency solutions for HB or TB starts at zero-velocity from torsional natural frequency or heaving one, respectively.

**Fundamental Flutter Modes in torsional and heaving 2DOF coupled flutter**

Torsional fundamental mode is defined as substantially torsional vibration around certain point apart from mid-chord point. In this mode, the phase difference, between torsional (noses-up positive) and heaving response (downward positive) at mid-chord point, is 0° or 180° and torsional twist center is upstream point or downward point from the mid-chord point, respectively. These fundamental modes are expressed by T₀ or T₁₈₀. As shown in Fig.4(a), (b), in these modes, torsional response would be excited by the pitching moment \( A_1^* \frac{d\eta}{dt} \) induced by relative pitching angle by \( \frac{d\eta}{dt} \), in which \( \eta \) is heaving displacement at mid-chord point.

On the other hand, heaving fundamental mode is defined as prominent heaving response induced by lift generated by slight pitching angle in quasi-steady sense with -90° or 90° as phase lag of heaving to torsional displacements. These two fundamental modes correspond to \( \frac{dC_L}{d\alpha} > 0 \) or \( \frac{dC_L}{d\alpha} < 0 \) and are expressed by H₀₀ or H₀₉₀, respectively. In heaving mode, heaving response is excited by lift force \( H_3^* \phi \) by slight torsional response \( \phi \) in the quasi-steady state. H₀₀ and H₀₉₀ are illustrated as Fig.4(c), (d).

By taking into account of fundamental mode definition, the flutter mode in coupled flutter can be resolved into two fundamental modes by using phase difference \( \Psi \), in which \( \Psi \) is defined as heaving lag to torsional response as Eq. (6). And the contribution of each fundamental mode is expressed by Eq. (7).

\[
\phi = \phi_0 \sin \omega t, \quad \eta = \eta_0 \sin (\omega t - \Psi)
\]

\[
T_{180} = -\cos \Psi, \quad H_{90} = \sin \Psi \quad (90° \leq \Psi \leq 180°)
\]

\[
T_0 = \cos \Psi, \quad H_{90} = \sin \Psi \quad (0° \leq \Psi \leq 90°)
\]

\[
T_0 = \cos \Psi, \quad H_{-90} = -\sin \Psi \quad (-90° \leq \Psi \leq 0°)
\]

\[
T_{180} = -\cos \Psi, \quad H_{-90} = -\sin \Psi \quad (-180° \leq \Psi \leq -90°)
\]
Relation between flutter branch and fundamental flutter modes

The flutter fundamental modes are closely related to flutter branch of SBSA explained as follows [2].

Torsional branch

Step1: torsional oscillation.
\[ \phi(t) = \phi_0 e^{i\omega t} e^{-i\delta_\phi t} \]  

(8)

Step2: Unsteady lifts, \( H_2^* \frac{d\phi}{dt} \) and \( H_3^* \phi \) act on \( \eta \) system as forced vibration-forces. Then, \( \eta \) response with \( \omega_\eta \) can be excited. In this step, amplitude ratio and phase difference between heaving and torsional response can be characterized.

Step3: Unsteady moment, \( A_1^* \frac{d\eta}{dt} \) and \( A_4^* \eta \), generated by heaving response at step2, act on torsional system as self excited moments, then flutter frequency \( \omega_\phi = \omega_\phi \) and flutter damping \( \delta_\phi = \delta_\phi \) can be characterized, if \( \delta_\phi \) and \( \omega_\phi \) in step1 are identical to those in step3. In which, \( H_1^* \frac{d\eta}{dt} \) is self excited pitching moment induced by \( \frac{d\eta}{dt} \), therefore, the torsional response should correspond fundamental flutter modes \( T_0 \) and \( T_{180} \).

Heaving Branch

Step1: heaving oscillation.
\[ \eta(t) = \eta_0 e^{i\omega t} e^{-i\delta_\eta t} \]  

(9)

Step2: Unsteady moments, \( A_1^* \frac{d\eta}{dt} \) and \( A_4^* \eta \) act on torsional system as forced vibration-forces. Then, \( \phi \) response with \( \omega_\phi \) can be excited. In this step, amplitude ratio and phase difference between heaving and torsional response can be characterized.

Step3: Unsteady lift, \( H_2^* \frac{d\phi}{dt} \) and \( H_3^* \phi \), generated by torsional response at step2, act on heaving system as self excited lift forces, then flutter frequency \( \omega_\phi = \omega_\phi \) and flutter damping \( \delta_\phi = \delta_\phi \) can be characterized, if \( \delta_\phi \) and \( \omega_\phi \) at step1 are identical to those at step3. In which, \( H_3^* \phi \) is unsteady lift induced by relative pitching angle due to torsional response \( \phi \) and heaving response induced by self excited action of \( H_3^* \phi \) should correspond the flutter fundamental modes \( H_{-90} \) and \( H_{90} \).

Thus it is verified that in TB, \( T_0 \) and \( T_{180} \) are classified to self-excite term on the other hand \( H_{-90} \) and \( H_{90} \) to forced term. As contrast, in HB, \( H_{-90} \) and \( H_{90} \) are subjected to self-excited term and \( T_0 \) and \( T_{180} \) to forced term.
Flutter onset velocity branch switch related to fundamental flutter modes

By using Eq. (7) and phase difference $\phi$, flutter modes are resolved as shown in Fig.4. As shown, for the case frequency ratio of $f_\phi/f_\eta=1.3$, flutter onsets in TB at $V=9.6$ [m/s] and the flutter major branch switches from TB to HB at $V=11.1$ [m/s]. Comparing these characteristics, velocities and the flutter fundamental modes, it is clarified that for TB when the self excited term $T_0 (=\cos \phi)$ becomes large, flutter might onset. Furthermore, when the self-excited-term $H_{\phi 0} (=\sin \phi)$ becomes large, branch switch seems to occur. However, there remains some questions in TB, at the maximum value of $T_0$ does not correspond the flutter onset velocity. On the other hand, when $H_{\phi 0}$ becomes large enough in HB, the branch switch from TB to HB seems to occur. Therefore, more details should be studied, taking into account the flutter fundamental modes. And these give us some hints about the physical generation mechanism and branch switch of coupled flutter.

![Fig.5: Flutter major mode in thin plate with frequency ratio 1.3](image)

Conclusions

The conclusions of this study on flutter branch of coupled flutter are summarized as follows.

1. Comparing the flutter characteristics in torsion/heaving coupling state of thin plate with frequency ratio of close 1.0 with those of torsional single state, if the solution of the branch in coupling state is close to flutter properties of torsional single state, this branch should be torsional branch(TB). Nevertheless, complex Eigen-value (CEV) analysis shows consistently HB, on the other hand Step-by-step(SBS) analysis shows properly TB. In the solution by SBS, at significantly low velocity, HB starts but at certain velocity HB is replaced by TB, it means branch switch appearance.

2. Basing on flutter mode, at sufficient higher velocity than flutter onset velocity, flutter mode must HB, but CEV solution indicate consistently TB, because of no re-production of branch switch by CEV method.

3. As far as flutter properties, those are frequency-velocity, damping-velocity, amplitude ratio-velocity and phase difference-velocity, both methods, CEV and SBS, gives us the identical solutions, therefore, there is no serious problem in utilization of conventional CEV method, including the estimation of critical flutter onset-velocity. However, SBS can explain the more precise flutter characteristics than conventional CEV, from the point of branch switch.
References

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