



## Potential output power from the 20 kW Jacobs wind turbine generator across 19 sites in northern Nigeria.

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### ABSTRACT

A method, which combines the power output characteristics of a wind turbine generator and the wind characteristics of a location was employed for the estimation of output power from the 20 kW Jacobs wind turbine (model 29-20) generator at 19 locations in northern Nigeria. The method assumes a wind turbine generator system which is characterised by a cut-in wind speed, a rated wind speed and cut-out wind speed. The generator power output was assumed to vary according to a third-degree polynomial with wind speed between cut-in and rated wind speeds and to be constant between rated and cut-out wind speeds. The Weibull distribution model was used to characterise the wind speeds of the 19 locations. The results of the evaluation indicate that at a hub height of 50 m, the 20 kW Jacobs wind turbine generator is expected to have a maximum annual capacity factor of 23.48 % or 41,119 kWh/year output power across the 19 locations. Most of the locations have very low capacity factors. Also, there is sufficient correspondence of higher mean wind speeds and higher output powers.

### INTRODUCTION

Previous estimates of potential output were theoretical maximum extractable power densities from

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hypothetical wind turbine generators, operating with a constant rotor efficiency of 59.3 % for various locations in Nigeria (Ojosu, 1990; Isirienu, 1991; Iheonu, 2002). The power output from a wind turbine generator at a particular location depends on the mean wind speed and the standard deviation of wind speeds about the mean wind speed. Estimating the power output from a wind turbine generator is complex because of the variability of the wind speed with time and the dependence of the output power from the wind-powered turbine on the wind speed. This paper applies a method for computing expected output power from a wind turbine generator, given the observed wind speed distribution at a location and the power output characteristics of the wind turbine generator. The output power is evaluated at 50 m hub height for the 20 kW Jacobs wind turbine (model 29-20) generator. The wind speed data used for the analysis are those of 19 locations across northern Nigeria. The results of the output power are expressed as capacity factors. Table 1 shows the locations considered in the analysis.

## METHODOLOGY

The mean output power,  $\overline{P}_o$ , from a wind turbine generator at a location can be estimated from its power output characteristics and the wind characteristics as represented by the probability distribution of wind speeds  $p(V)$ . This is given by Manwell et al. (2002) as:

$$\overline{P}_o = \int_0^{\infty} P(V)p(V)dV, \quad (1)$$

where  $P(V)$  is the power output function of the wind turbine generator. Fig. 1 shows a typical power output versus wind speed curve for a wind turbine generator (Ge Power System 1.5SL). The Ge Power System 1.5SL, for example, begins generating electricity at its cut-in wind speed  $V_i = 3$  m/s. It has a rated wind speed  $V_r = 11.8$  m/s at which maximum possible power of 1,500 kW is generated. The unit must be shut down at wind speeds greater than cut-out wind speed  $V_o = 25$  m/s, so wind speeds greater than the cut-out wind speed generate no power ([www.gepower.com](http://www.gepower.com), July, 2003).

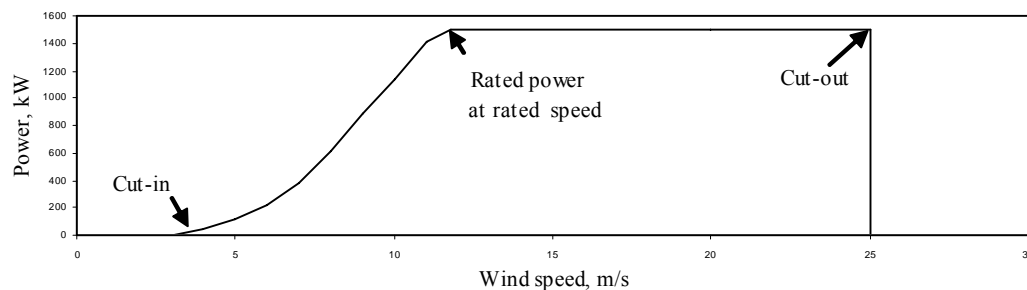


Figure 1. Typical Power Output Curve of a Wind-Powered Turbine (Ge Power System-1.5SL).

The 20 kW Jacobs wind turbine generator on the other hand, has cut-in wind speed  $V_i = 3.58$  m/s, rated wind speed  $V_r = 11.62$  m/s at which maximum possible power of 20 kW is generated, cut-out wind speed of  $V_o = 53.63$  m/s ([www.windturbine.net](http://www.windturbine.net), July, 2003).

Bala and Pam (2004) described an analytical method for representing the power output function,  $P(V)$  of a wind turbine generator if the cut-in wind speed, rated wind speed, cut-out wind speed and rated power are known, as:

$$P(V) = \begin{cases} 0, & V \leq V_i \\ A + BV + CV^2 + DV^3, & V_i < V \leq V_r \\ P_r, & V_r \leq V \leq V_o \\ 0, & V > V_o, \end{cases} \quad (2)$$

assuming a third-degree polynomial for the power output of a wind turbine generator, between cut-in and rated wind speeds. Where  $V$  is hub height wind speed,  $P_r$  is the rated power. The constants  $A$ ,  $B$ ,  $C$  and  $D$  are evaluated based on the following conditions:

$$\begin{aligned} A + BV_i + CV_i^2 + DV_i^3 &= 0 \\ A + BV_r + CV_r^2 + DV_r^3 &= P_r \\ A + BV_m + CV_m^2 + DV_m^3 &= P_r (V_m / V_r)^3 \\ A + BV_x + CV_x^2 + DV_x^3 &= P_r (V_x / V_r)^3 \end{aligned} \quad (3)$$

where,

$$V_m = (V_i + V_r)/2 \quad (4)$$

and

$$V_x = 2(V_i + V_r)/5. \quad (5)$$

Gaussian elimination technique (Stroud, 1995) was used to evaluate  $A$ ,  $B$ ,  $C$  and  $D$  from equation (3) for the 20 kW Jacobs wind turbine generator. Its simulated power output curve was then plotted.

The two – parameter Weibull wind speed probability distribution,  $p(V)$ , is expressed by Justus et al. (1978) as:

$$p(V) = \left(\frac{k}{c}\right) \left(\frac{V}{c}\right)^{k-1} e^{-\left(\frac{V}{c}\right)^k}, \quad (6)$$

where  $c$  is the scale parameter and  $k$  is the shape parameter. The scale parameter  $c$  has units of speed and is related to the mean wind speed. The shape parameter  $k$  is dimensionless and is inversely proportional to the variance of the wind speeds about the mean wind speed.

The Weibull wind speed probability distribution model because of its accepted application in wind energy studies for a long time (Baynes, 1974; Hennessey, 1977; Dorvlo, 2002) was selected for the study. The mean wind speed and standard deviation method for the estimation of Weibull parameters was found to give the best fit of the Weibull distribution model to observed wind speed data for Kano, Maiduguri, Sokoto and Zaria (Pam et al., 2005). It was used to estimate the Weibull parameters for the 19 locations.

If the mean wind speed  $\bar{V}$  and standard deviation  $\sigma$  are known, then  $k$  and  $c$  can be computed from the approximate relations (Justus et al., 1978):

$$k = \left(\frac{\sigma}{\bar{V}}\right)^{-1.086} \quad (7)$$

and

$$c = \frac{\bar{V}}{\Gamma(1+1/k)}, \quad (8)$$

where  $\Gamma$  is the gamma function.

Lysen (1983) used the following approximation to find  $c$ ,

$$\frac{c}{\bar{V}} = (0.568 + 0.433/k)^{-\frac{1}{k}}. \quad (9)$$

Weibull parameters  $c$  and  $k$  at 50 m height were determined for the 19 locations by means of equation (7), equation (9) and summaries of five year (1998-2002) observed daily mean wind speed data at 10 m height obtained from the Nigerian Meteorological Agency, Federal Ministry of Aviation, Oshodi, Lagos, Nigeria. The observed wind speed data was projected to 50 m by means of the  $1/7^{\text{th}}$  power law velocity extrapolation formula due to the absence of information on wind shear for locations in northern Nigeria. The mean wind speeds and standard deviations were then obtained. The summaries of the wind speed data for the 19 locations are shown in table 1.

Table 1. Summaries of wind speed data at 50 m height for the 19 locations: mean wind speed  $\bar{V}$  and standard deviation  $\sigma$ .

Location	Abbreviation	Lat. ° N	Long. ° E	$\bar{V}$	$\sigma$
Abuja	Abu	9.25	7	2.6	0.9
Bauchi	Bau	10.28	9.82	1.1	0.66
Bida	Bid	9.1	6.02	1.66	0.52
Gusau	Gus	12.17	6.7	3.87	1.52
Ilorin	Ilo	8.48	4.58	3.63	1.14
Jos	Jos	9.87	8.88	6.59	2.16
Kaduna	Kad	10.6	7.45	3.32	1.45
Kano	Kan	12.05	8.53	5.22	1.94
Katsina	Kat	13.02	7.68	3.69	2.09
Lokoja	Lok	7.78	6.74	1.5	1.08
Maiduguri	Mai	11.85	13.08	3.44	1.57
Makurdi	Mak	7.73	8.53	3	1.76
Minna	Min	9.62	6.53	4.59	2.22
Nguru	Ngu	12.9	10.47	1.6	1.14
Potiskum	Pot	11.67	11.2	3.27	1.34
Sokoto	Sok	13.02	5.25	5.07	2.08
Yelwa	Yel	10.88	4.75	1.6	0.81
Yola	Yol	9.23	12.47	1.19	0.74
Zaria	Zar	11.13	7.68	3.65	1.38

From the third-degree variation of power output with wind speed between cut-in and rated wind speeds and the Weibull distribution, defined by the estimated  $c$  and  $k$  values, equation (1) becomes

$$\bar{P}_o = \int_{V_i}^{V_r} (A + BV + CV^2 + DV^3) \left(\frac{k}{c}\right) \left(\frac{V}{c}\right)^{k-1} e^{-\left(\frac{V}{c}\right)^k} dV + P_r [p(V_r \leq V \leq V_o)]. \quad (10)$$

This comes from substitution of the power function  $P(V)$  from equation (2) into equation (1), with the distribution  $p(V)$  in the integral of equation (1) being given by equation (6). Constants  $A, B, C, D$  for the 20 kW Jacobs wind turbine generator and Weibull parameters  $c$  and  $k$  for the 19 locations were substituted into equation (10) and solved via numerical integration. The numerical integration technique used to solve equation (10) is the Simpson's rule (Stroud, 1995). The results are mean output values of the power in kW. These results were then expressed as capacity factors. Capacity factor is the ratio of the mean output power,  $\bar{P}_o$ , to the rated power,  $P_r$ , of a wind turbine generator. The capacity factor in equation form is given by,

$$CF = \frac{\overline{P_o}}{P_r}, \quad (11)$$

## RESULTS AND DISCUSSION

The constants for the 20 kW Jacobs wind turbine (model 29-20) generator were found to be  $A = -3.713$  kW,  $B = 1.402$  kN,  $C = -0.1716$  kNm<sup>-1</sup>s and  $D = 0.0195$  kNm<sup>-2</sup>s<sup>2</sup>. Figure 2 shows the simulated power output curve for the 20 kW Jacobs wind turbine generator obtained from the analysis.

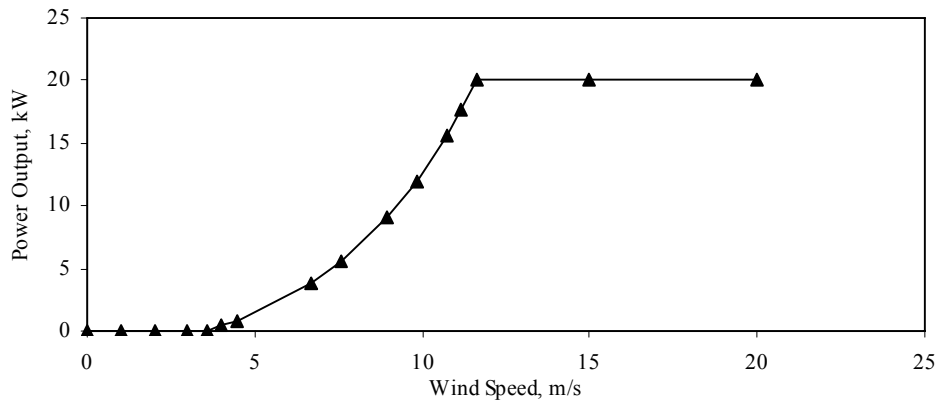


Figure 2. Power Output Curves for Jacobs Wind Turbine (model 29-20).

Figure 3 shows the results of expected output power expressed as annual capacity factors from the 20 kW Jacobs wind turbine (model 29-20) generator operating at a height of 50 m for the 19 locations. Figures 4 and 5 show annual scale and shape parameters, annual capacity factors from the 20 kW Jacobs wind turbine (model 29-20) generator at a height of 50 m for the 19 locations. For some of the locations the output power is so small that their capacity factors fail to appear due to the scale chosen for the figures.

From the results of figure 3, the 20 kW Jacobs wind turbine (model 29-20) generator is expected to operate with a maximum capacity factor of 23.47 % in the 19 locations. Most of the locations have very low capacity factors. A capacity factor of 23.47 % from a 20 kW wind generator means a mean output power of 4.694 kW or an annual power output of 41,119 kWh. The average American household consumes about 10,000 kWh of electricity annually (Annual Energy Review 2000, 2001). Much less is expected for a Nigerian household. Nigeria had a consumption of 200 kWh/capita in 2005, which translates to 1,000 kWh for a household of 5 individuals. This was for grid connected electricity which was inadequate. If three times this amount, i.e. 3,000 kWh is assumed for a Nigerian household at a rural setting which will make power supply satisfactory, then the 20 kW Jacob wind turbine (model 29-20) generator can be used to power about 3, 14, 7, 3, 2, 6, 7 and 2 households in Gusau, Jos, Kano, Katsina, Maiduguri, Minna, Sokoto and Zaria respectively.

Figures 4 and 5 show the capacity factor as it relates to annual mean wind speed (or Weibull  $c$  values) and standard deviation (Weibull  $k$  values) for the 19 locations. It is seen that there is a trend of increasing output with increasing mean wind speed. However, there is variation due to influence of standard deviation of wind speeds about the mean wind speed. It is possible to find pairs of locations for which the higher output occurs at the location with the lower mean wind speed (because of this influence of standard deviation), for example Gusau and Katsina.

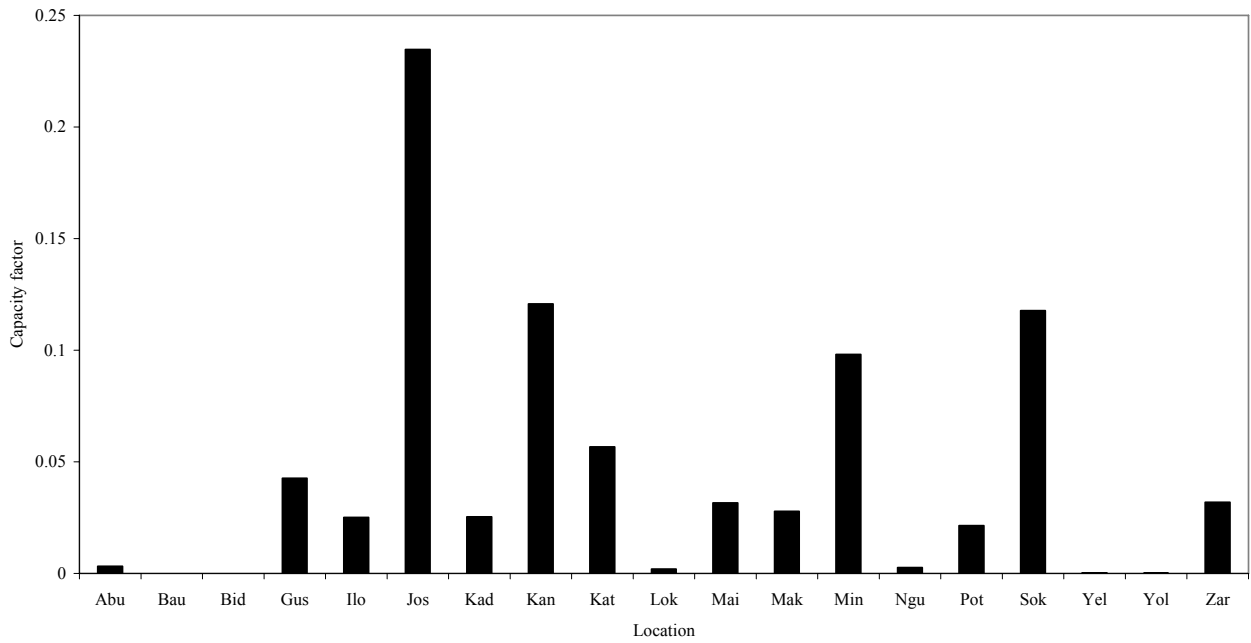


Figure 3. Annual capacity factors at a height of 50 m for the 19 locations.

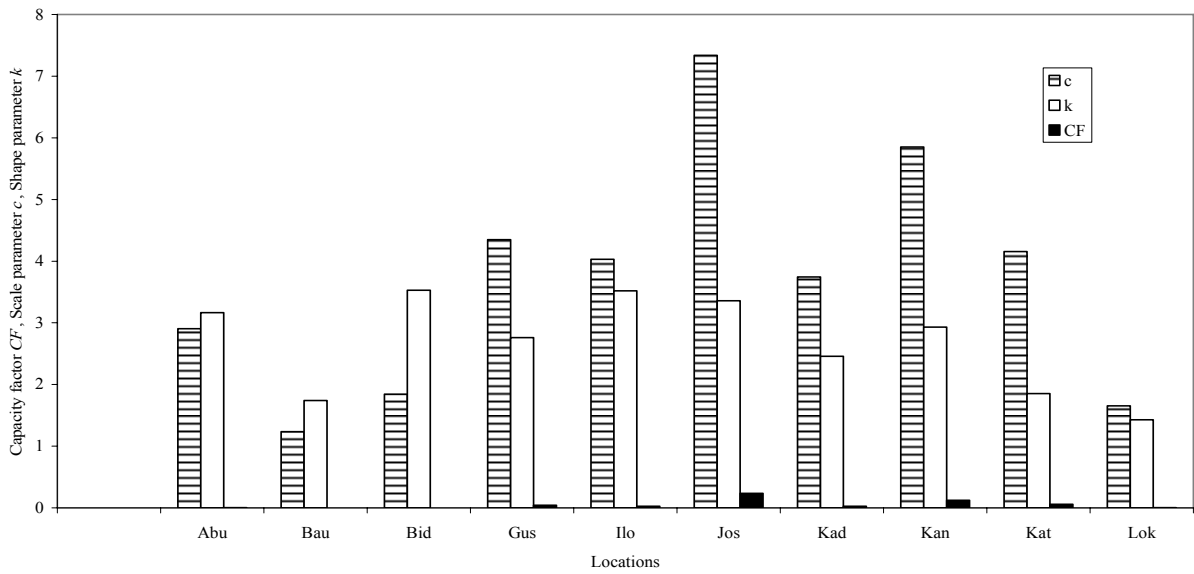


Figure 4. Annual scale and shape parameters, annual capacity factors at a height of 50 m for Abuja, Bauchi, Bida, Gusau, Ilorin, Jos Kaduna, Kano, Katsina and Lokoja.

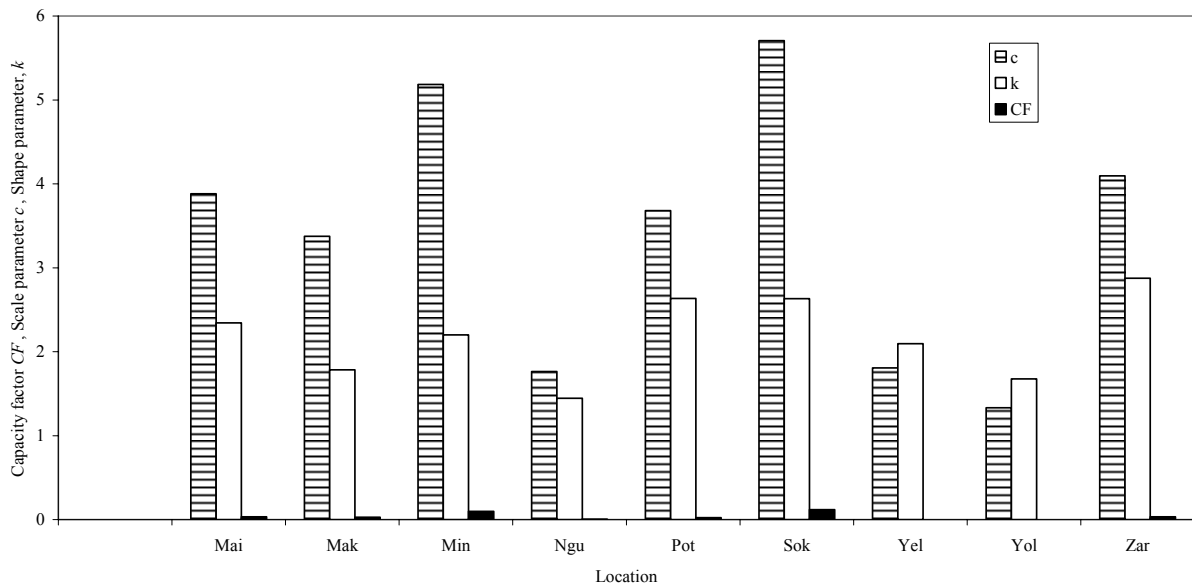


Figure 5. Annual scale and shape parameters, annual capacity factors at a height of 50 m for Maiduguri, Makurdi, Minna, Potiskum, Sokoto, Yelwa, Yola and Zaria.

## CONCLUSION

The 20 kW Jacob wind turbine (model 29-20) generator operating at 50 m height has very low power output for most of the locations. It is expected to operate with a maximum capacity factor of 23.47 % across the 19 locations in northern Nigeria.

## REFERENCES

- Ojosu, J.O. and Salawu, R.I. (1990). "An evaluation of wind energy potential as a power generation source in Nigeria", *Solar & Wind technology*, 7 (6), 663-673.
- Isirisena, U.A., Alfa, B., Bakwa, D.D. and Tsor, J.O. (1991). "Assessment of the wind power potential at Heipang, near Jos," *Nigerian Journal of Solar Energy*, 10, 44-55.
- Iheonu, E.E., Akingbade, F.O.A. and Ocholi, M. (2002). "Wind resource variations over selected sites in the West African sub-region", *Nigerian Journal of Renewable Energy*, 10 (1&2), 43-47.
- Manwell, J.F., McGowan, J.G. and A.L. Rogers, A.L. (2002). *Wind Energy Explained*. John Wiley & Sons Ltd, West Sussex, England.
- [www.gepower.com](http://www.gepower.com). (July, 2003).
- [www.windturbine.net](http://www.windturbine.net) (July, 2003).
- Bala, E.J. & Pam, G.Y. (2004). "Mathematical representation of actual power output of four wind turbine generators", *Nigerian Journal of Renewable Energy*, 12 (1 & 2), 128 – 133.
- Stroud, K. A. (1995). *Engineering Mathematics*. Fourth Edition. The Bath Press, Bath.
- Justus, C.G., Hargraves, W.R., Mikhail, A. and Graber, D. (1978). "Methods for estimating wind speed distributions", *Journal of Applied Meteorology*, 17, 350-353.
- Baynes, C.G. (1974). *The statistics of strong winds for engineering applications*. University of Western Ontario Report, BLWT-4-1974.
- Hennessey JR., J. P., (1977). "Some aspects of wind power statistics," *Journal of Applied Meteorology*, 16 (2), 119-128.
- Dorvlo, A.S.S. (2002). "Estimating wind speed distributions", *Energy Conversion and Management*,

43 (17), 2311-2318.

Pam, G.Y., Bala, E.J. & Aku, S.Y. (2005). "Estimating wind speed probability distributions for Kano, Maiduguri, Sokoto and Zaria in northern Nigeria", *Nigerian Journal of Renewable Energy*, 13 (1 & 2), 60 -67.

Lysen, E.H. (1983). *Introduction to Wind Energy*. SWD Publication SWD 82-1, the Netherlands.  
Annual Energy Review 2000 (2001), Washington D.C.: Energy Information Administration, DOE/EIA-0384 (2000).