Application of wind tunnel measurements in the time domain vibration control analysis

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ABSTRACT

Owing to the limited storage capacity and computational power of computers in the 1980s, analyses for aerodynamic model studies, such as high-frequency base balance (HFBB) or synchronous multi-pressure sensing system (SMPSS), have been traditionally conducted in the frequency domain. As computer technology has improved and the necessity to conduct the vibration control analysis in the time domain grows, this paper investigates the feasibility of implementing vibration control analyses in the time domain using measurements of aerodynamic model studies. A series of wind tunnel tests was conducted to determine the wind forces exerted on a benchmarking building. The base overturning moments and torque experienced by the benchmark building were analysed in the time domain from which the controlled building tip displacement and acceleration responses were determined. The building peak responses were substantially alleviated by about 25%.

INTRODUCTION

The high-frequency base balance (HFBB) testing technique was developed in the early 1980s...
(Davenport and Tschanz, 1981) and has become one of the most common techniques for predicting wind-induced forces for tall building design. Predictions of mean and dynamic loads and responses are interpreted analytically using the modal forces estimated from the overturning and torsional moments experienced by a lightweight and stiff model. The building responses can be obtained in the time domain using random vibration theory or frequency domain using spectral analysis for stationary random loads. Owing to the limited storage capacity and computational power of computer in the 1980s, it is common practice in wind engineering to conduct the analysis in the frequency domain.

Recent advancements in computer technology have provided an alternative to conduct the HFBB analysis in the time domain to obtain more accurate results. In addition there were sometimes cases in which the HFBB tested buildings were found to exceed the design value, such as allowable acceleration, stipulated in design codes. Feasible mitigation solutions, such as incorporating extra damping via vibration control devices, need to be examined within a limited timeframe, during which additional wind tunnel test was often difficult to arrange.

Traditionally, the effects of the control device were routinely approximated by the modal damping values which were subsequently incorporated in the HFBB frequency domain analysis. The approximation of modal damping values in the frequency domain analysis, on one hand, does not precisely reveal the alleviation capacity of the control device and is applicable only to passive-type dampers. On the other hand, it is difficult to characterize the effects of some damping devices as only a modal damping ratio because their damping forces may be coupled with the inter-storey displacement and velocity where they are installed.

The primary objective of this study is to investigate the feasibility of conducting vibration control analysis in the time domain using wind force information obtained from aerodynamic model studies. The second generation wind-excited benchmark building was wind tunnel tested and employed to investigate the accuracy of the analysis method. The results of the vibration analysis in the time domain were evaluated. The analysis procedures are also outlined in detail in this paper.

**FORMULATION OF EQUATIONS OF MOTION**

The concept of rigid floor diaphragms was introduced by Clough (1963) nearly 40 years ago as a means of increasing the efficacy in the solution process associated with the structural dynamics and have been commonly adopted in the finite element modelling of the majority of building structures. A general matrix formulation of the equation of motion for a tall structure with rigid floor systems subject to random wind loads can be expressed as,

\[ M \ddot{x} + C \dot{x} + Kx = W \]  

where \( M \), \( C \), and \( K \) are the structural mass, damping, and stiffness matrices respectively; \( x \) is the displacement vector in metres or radians; and \( W \) is the wind excitation vector in N or N-m.

In general, the simultaneous solution of the coupled equations of motion is not practical for systems with many degrees of freedom, for example typical building structures, that are subjected to random wind loads. By means of modal superposition with mode shapes computed at the mass centres, Eq. (1) is transformed into uncoupled equations as follows,

\[ m_j \ddot{\xi}_j(t) + c_j \dot{\xi}_j(t) + k_j \xi_j(t) = w_j(t) \]  

where:
- generalised mass, \( m_j = \sum_i \left[ m(z_i)\phi_{ix}^2(z_i) + m(z_i)\phi_{iy}^2(z_i) + I(z_i)\phi_{i\theta}^2(z_i) \right] \); 
- generalised damping, \( c_j = 2m_j\omega_j\xi_j \); 
- generalised stiffness, \( k_j = \omega_j^2 m_j \); and
generalised force, \( w_j = \sum_i [w_x(z_i, t)\phi_{jx}(z_i) + w_y(z_i, t)\phi_{jy}(z_i) + w_\theta(z_i, t)\phi_{j\theta}(z_i)] \)

where \( m(z_i), I(z_i), w(z_i), \) and \( \phi(z_i) \) denote the mass, mass moment of inertia, wind forces, and mode shape values for the \( i \)th storey at height of \( z_i \), respectively; and \( \omega_j, \zeta_j \) are the natural frequency and damping ratio for the \( j \)th mode.

**FREQUENCY DOMAIN DYNAMIC ANALYSIS PROCEDURES**

For wind engineering, it has been found convenient to express and analyse the effects of wind on a given tall building in the frequency domain, assuming that the wind loads are stationary and normally distributed. The power spectral density (PSD) of the generalised coordinate for the \( j \)th mode is determined according to Eq. (3).

\[
S_{\xi_j}(\omega) = \frac{1}{k_j^2}|H_j(\omega)|^2 S_{w_j}(\omega)
\]

where: mechanical admittance function, \( |H_j(\omega)|^2 = \frac{1}{1 - \left( \frac{\omega}{\omega_j} \right)^2 - (2\omega\zeta_j \frac{\omega}{\omega_j})^2} \)

PSD of generalised wind load, \( S_{w_j}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{w_j}(\tau)e^{-i\omega\tau} d\tau \)

autocorrelation of generalised wind forces, \( R_{w_j}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T w_j(t)w_j(t + \tau)dt \)

The standard deviation of the modal response for the \( j \)th mode is determined by integrating its PSD as follows:

\[
\sigma_{\xi_j} = \left[ \int_0^\infty S_{\xi_j}(\omega)d\omega \right]^{1/2}
\]

The resonant component of the modal response may then be estimated by subtracting the background fluctuating component from the total dynamic modal response:

\[
\sigma_{\xi_j, res} = \sqrt{\sigma_{\xi_j}^2 - \sigma_{\xi_j, bg}^2}
\]

where standard deviation of background component, \( \sigma_{\xi_j, bg} = \frac{1}{k_j} \left[ \int_0^\infty S_{w_j}(\omega)d\omega \right]^{1/2} \)

The building tip dynamic acceleration responses of \( j \)th mode are computed from:

\[
\begin{pmatrix}
\sigma_{\dot{x}_j} \\
\sigma_{\dot{y}_j} \\
\sigma_{\dot{\theta}_j}
\end{pmatrix} = \omega_j^2 \begin{pmatrix}
\phi_{jx}(h) \\
\phi_{jy}(h) \\
\phi_{j\theta}(h)
\end{pmatrix} \begin{pmatrix}
\sigma_{\xi_j} \\
\sigma_{\xi_j} \\
\sigma_{\xi_j}
\end{pmatrix} = \sigma_{\xi_j} \begin{pmatrix}
\phi_{jx}(h) \\
\phi_{jy}(h) \\
\phi_{j\theta}(h)
\end{pmatrix}
\]

\( (6) \)
Traditionally, the effects of the control device were routinely approximated by the modal damping values which were subsequently incorporated in the mechanical admittance function in the HFBB frequency domain analysis, as expressed in Eq. (3) – (6). The approximation of modal damping values in the frequency domain analysis, on one hand, does not precisely reveal the performance of the control device and is applicable only to passive-type dampers. On the other hand, it is difficult to characterize the effects of some damping devices as only a modal damping ratio.

**TIME DOMAIN ANALYSIS PROCEDURES**

For the current study, the time domain dynamic analysis procedures use a state-space technique in which the modal system is initially solved to determine the generalised coordinate time histories (i.e. $\xi_j(t)$, $\ddot{\xi}_j(t)$, and $\dot{\xi}_j(t)$). The physical responses (i.e. $\ddot{x}(t)$, $\dot{x}(t)$, and $x(t)$) can be computed subsequently in a mode-by-mode manner by multiplying the generalised coordinates with the corresponding mode shape. The state-space form of Eq. (2) is:

\[
\dot{Z} = AZ + Bw_j(t)
\]

where:  
state vector, $Z = \begin{bmatrix} \xi_j(t) \\ \dot{\xi}_j(t) \end{bmatrix}$;  

system matrix, $A = \begin{bmatrix} 0 & 1 \\ -k_j/m_j & -c_j/m_j \end{bmatrix}$; and  

location vector, $B = \begin{bmatrix} 0 \\ 1/m_j \end{bmatrix}$

The output modal response is governed by a so-called observer, $Y$, defined in Eq. (8), by regulating the output matrix, $E$, and feedforward matrix, $D$, as summarised in Table 1.

\[
Y = EZ + Dw_j(t)
\]

<table>
<thead>
<tr>
<th>Modal</th>
<th>$E$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$[1 \ 0]$</td>
<td>0</td>
</tr>
<tr>
<td>ii</td>
<td>$[0 \ 1]$</td>
<td>0</td>
</tr>
<tr>
<td>iii</td>
<td>$[-k_j/m_j \ -c_j/m_j]$</td>
<td>$1/m_j$</td>
</tr>
</tbody>
</table>

By adopting the equivalent static force concept of earthquake engineering, the internal forces associated with the displacement along the x-axis for the $j$th mode are formulated in Eq. (9).

\[
Fx_j(z_i,t) = \omega_j^2 \ddot{z}_j(t)m(z_i)\phi_{jx}(z_i)
\]

These equivalent static forces, by definition, will cause precisely the $j$th mode displacements when imposed externally and were employed as the external forcing functions for the subsequent vibration control analysis in the time domain.
EXAMPLE BUILDING AND EXPERIMENTAL SETUP

The building model considered in this study is the second generation wind-excited benchmark building (Tse et al., 2007), which undergoes 3D lateral-torsional motions under excitations and represents majority of modern tall buildings. The first two modes have dominant sway components along the $y$- and $x$-axes, respectively. The third mode is a predominantly torsional mode of vibration with modest translational components. The natural frequencies of the first 3 modes are 0.231, 0.429 and 0.536 Hz, respectively.

A 1:400 scale rigid model of the benchmark building, as shown in Fig. 1, was constructed from acrylic and wind tunnel tested to measure surface wind pressures acting on the building. The model was installed with 14 layers of pressure-taps over its height, with 32 pressure-taps in each layer. The surface pressures measured from the test were converted into 14 layers of alongwind, crosswind, and torsional wind load distributions, from which the base overturning and torsional moments corresponding to a HFBB test were subsequently synthesised and displayed in Fig. 2.

![Figure 1: Pressure-tapped benchmark building model inside wind tunnel.](image)

![Figure 2: The base overturning moments and torque time series.](image)
NUMERICAL RESULTS

Passive control devices, such as tuned mass damper (TMD), tuned liquid damper (TLD), and tuned liquid column damper (TLCD), have been shown by many researchers as being capable of mitigating the dynamic motions of civil structures and having the advantages of reliability and comparatively low operating and maintenance costs (Spencer and Sain 1997). However, the performance of a passive device is very difficult to optimise due to uncertainties in the structural dynamic properties and the excitation, as they are generally designed for a specific condition (Housner et al. 1997).

In contrast, semi-active control devices have been shown capable to accommodate the reliability of passive devices and the adaptability of a fully active system, but with lower input energy demands than a fully active system (Spencer and Sain, 1997). One example of a semi-active control device is to equip a tuned mass damper (TMD) with a magnetorheological (MR) damper to provide variable stiffness and damping, referred to as smart tuned mass damper (STMD).

A 20 tonne MR damper, which was built at the University of Notre Dame to study its possible application to real structures (Yang, 2001 and 2002), was mounted in each orthogonal direction of a bi-directional TMD as a semi-active system to numerically illustrate the vibration control of the benchmark building, as shown in Fig. 3. The effective mass of the STMD is about 0.4% of the total mass of the building, and its undamped natural frequency in the \( y \)-direction and \( x \)-direction are set at 0.22 Hz and 0.42 Hz, respectively. The damping ratios of the STMD for both directions are 5% of critical. The STMD’s undamped natural frequencies are slightly less than the first two fundamental natural frequencies of vibration of the benchmark building, which are 0.231 Hz and 0.429 Hz, so that the MR dampers provide additional controllable stiffness and damping forces to optimize the natural frequencies of the STMD when operating in its passive mode.

Because the benchmark building underwent 3D lateral-torsional motions, the STMD was offset from the geometrical centre of the building at a distance of 32 m in the \( x \)-direction and 8 m in \( y \)-direction, as shown in Fig. 3, to suppress both translational and torsional motions. It should be noted that it was assumed that the STMD did not induce any additional wind forces on the building although it was installed on the roof.

In this study, the linear-quadratic regulator (LQR) was used for the semi-active control. The control force \( f_c \) was determined by minimizing the following quadratic cost function over a period of time to reduce the tip accelerations, with regards to the maximum available control force generated by the damping device.

\[
J = \int_0^T (Z^T N Z + f_c^T R f_c) dt
\]

where \( N \) and \( R \) are the weighting matrices and \( Z \) is the state vector.

In principle, accelerations and velocities of all floors can be measured although this is likely to be impractical and unnecessary in a real application. Therefore, in the current study, it was assumed that
only the accelerations in x- and y-directions at the corner of the refuge floors and the roof were available for computing the feedback control forces. The response state vector of the building model, including displacement and velocity vectors, was first estimated through a reduced-order observer (Shahian and Hassul 1993) for computing the control forces. The state-space realization for the dynamic system and the measurement of the building model installed with the control device was expressed as,

\[ \dot{Z} = AZ + BW + Ef_c \] (11)

\[ Y = CZ + DW + Ef_c \] (12)

where \( Z \) and \( Y \) are the state vector and the measured output, respectively. \( W \) is the wind excitation vector in N or N-m. \( A, B, C \) and \( D \) are system matrices. \( E \) and \( F \) are location matrices for the control forces, which are determined as following,

\[ f_c = -k\dot{Z} \] (13)

where \( k \) is the feedback gain matrix and is manipulated from the cost function of the LQR in Eq. (10) and \( \dot{Z} \) is the reduced-order observer.

The state-space realization for the dynamic system and the measurement of the building model installed with the control device was numerically simulated. The structural responses (i.e. accelerations at refuge floors and roof), the observer to estimate other response state vectors of the building model, and the controller comprising a control algorithm and an inverse model were digitally implemented with a time step of 0.001 s. To make the simulation realistic, measurement noises modelled as Gaussian rectangular pulse processes were introduced to the building responses and a computational time-delay of 1 milli-s was considered for the building responses simulation.

Figure 4: The uncontrolled and controlled tip displacement and acceleration responses of the wind-excited benchmark building for an incident wind angle of 0°.
The building responses with control were compared with the uncontrolled responses, as shown in Figs. 4 and 5. In general, the STMD shows its capability to mitigate the wind-induced orthogonal translation responses as well as the torsion responses. The reduction of acceleration responses are more pronounced than the displacement responses as the LQR was designed to compute control forces to minimize the acceleration responses in this study. In other words, the STMD is also capable of reducing the displacement responses, if the LQR is designed for that purpose, by changing the parameters of the cost function.

CONCLUSIONS

The vibration control analysis in time domain using wind force information obtained from aerodynamic model tests, such as the high-frequency base balance, was successfully exercised and demonstrated in this paper. A series of wind tunnel tests were conducted to determine the wind loads experienced by the second generation wind-excited benchmark building, from which the base overturning moments and torsional torque were synthesized. The base moment data were used in the subsequent time domain vibration control analyses to determine the building tip displacement and acceleration responses.

The bi-directional TMD was designed with natural frequencies slightly less than the building’s first two natural frequencies and equipped with MR dampers to provide additional controllable damping forces to significantly mitigate both the acceleration and displacement responses in the translation directions and slightly smaller reductions in torsion. The maximum generated control forces were 35kN with a maximum stroke of 42 mm in x-direction and 476 kN with a maximum stroke of 526 mm in y-direction. Correspondingly, the standard deviation and peak responses were reduced by over 50% and about 25%, respectively.
ACKNOWLEDGEMENTS

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REFERENCES