Dynamic Properties of Stay Cables on the Penobscot Narrows Bridge

Harold R. Bosch¹, James R. Pagenkopf²

¹Federal Highway Administration, McLean, Virginia, USA, harold.bosch@fhwa.dot.gov
²GENEX Systems, McLean, Virginia, USA, james.pagenkopf.CTR@fhwa.dot.gov

ABSTRACT
An experimental study was conducted to investigate the dynamic response characteristics of the stay cables of the Penobscot Narrows Bridge in Maine, USA. The natural frequencies and inherent damping ratios of selected stay cables are estimated based on the decaying free-vibration of the cables. Testing of selected stay cables was performed in two phases, before and after the installation of external viscous dampers. Before the installation of dampers, the range of measured inherent cable damping was 0.10–0.39% for the first mode and 0.17–0.46% for the second mode, which is far below the level required for controlling wind or wind/rain induced vibrations. Post-damper installation the ranges increased to 1.22–2.21% for the first mode and 1.41–2.48% for the second mode with respective Scruton numbers ranging from 7–11, above the target value of five for cables with aerodynamic surface treatments.

INTRODUCTION
Cable-stayed bridges are exposed to natural winds of varying intensity and direction. Of particular importance is the dynamic behavior of the deck’s supporting cables. These structurally critical components are often excited into several vibration modes by the ambient wind conditions. Study of the damped behavior of cables is fundamental to ensure a safe and structurally sound system.

Large amplitude vibration of stay cables under conditions of moderate wind, sometimes in conjunction with light rain, has been observed with increasing frequency in recent years [1]. This problem is not new and has been studied extensively starting in the mid 1980s. With a growing inventory of cable-stayed bridges, we have experienced a significant increase in reports of large amplitude cable vibrations. Some structures have been retrofit to mitigate these vibrations. Cable-stayed bridges under design and/or construction are currently incorporating dampers, cross-ties, and/or aerodynamic surface treatments into the cable system.

A national research project is underway to investigate this cable vibration problem and develop comprehensive guidelines for both design and retrofit [2]. Among other things, the study has involved synthesis of existing information, analysis of the mechanics of wind-induced cable vibration, wind tunnel testing to clarify dry cable galloping, and evaluation of mitigation methods. In conjunction with this project, a number of full scale experimental studies have been conducted to establish dynamic properties of representative bridge stay cables and performance of various mitigation features. In addition to the subject bridge, tests have been performed on new bridges such as Zakim and Emerson, as well as existing bridges such as Luling, Sunshine Skyway, and C&D Canal. Natural vibration frequencies and damping ratios of stay cables are extracted from analysis of freely decaying motion of these cables. When inherent damping of the cables is not sufficient enough to avoid excessive vibrations, mitigation measures in the form of external viscous dampers are often considered and the required damper coefficients are calculated using commonly accepted approaches. As an alternative or compliment to dampers, cross-ties can be considered to stiffen the cable system and enhance aerodynamic performance.
THEORETICAL BACKGROUND

**Vibration of Taut String with Distributed Damping**

The transverse vibration of a taut string with uniformly distributed viscous damping can be described by [3]

\[
m \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} = H \frac{\partial^2 w}{\partial x^2}
\]

(1)

where \(w(x,t)\) = transverse displacement, \(m\) = mass density per unit length, \(c\) = viscous coefficient per unit length, and \(H\) = pretension of the string. For a string of length \(L\), fixed at both ends, \(w(x,t)\) can be approximated by a finite degrees of freedom (DOF) system:

\[
w(x,t) \cong \sum_{n=1}^{N} \sin(\frac{n\pi x}{L}) u_n(t)
\]

(2)

where the sinusoidal spatial functions \(\sin(n\pi x/L)\) are the normal modes for a string where \(c = 0\). Substituting \(w(x,t)\) into Equation 1 and rearranging yields

\[
\ddot{u}_n(t) + 2\zeta_n \omega_n \dot{u}_n + \omega_n^2 u_n = 0 \quad \text{where} \quad \omega_n = \sqrt{\frac{H}{mL}} \quad \text{and} \quad \zeta_n = \frac{c}{2m\omega_n}
\]

(3)

for \(n = 1, 2, \ldots, N\) (no sum on \(n\)). Equation 3 represents the equation of motion for the \(n\)th mode vibration of the string, \(\omega_n\) and \(\zeta_n\) respectively denote the corresponding circular natural frequency and damping ratio of the mode. It is to be noted that the equations for this \(N\)-DOF system are fully decoupled and each mode can be handled separately. Using the standard solution technique for a single DOF system, a general solution to Equation 3 is

\[
u_n(t) = A_n e^{-\zeta_n \omega_n t} \cos(\omega_{dn} t - \alpha_n)
\]

(4)

where \(A_n\) and \(\alpha_n\) are the amplitude and phase angle that are dependent on the initial conditions of the vibration, and

\[
\omega_{dn} = \omega_n \sqrt{1 - \zeta_n^2}
\]

(5)

is the damped natural frequency [4].

**Determination of Damping Ratios**

The damping ratio, or the fraction of critical damping, \(\zeta_n\), can be estimated experimentally. In the logarithmic decrement method, the damping ratio is found by measuring the amplitude of two consecutive peaks of damped free vibration and computing their ratio [4]. It can be shown that the ratio between the two consecutive peaks of the vibration is given by the following expression:
\[
\frac{u_n(t)}{u_n(t + T_{dn})} = \exp(\zeta_n \omega_n T_{dn}) = \exp\left(-\frac{2\pi \zeta_n}{\sqrt{1 - \zeta_n^2}}\right)
\]  \hspace{1cm} (6)

where \( T_{dn} = \frac{2\pi}{\omega_{dn}} \) is the damped natural period of the \( n \)th mode. Selecting two consecutive peaks \( u_i \) and \( u_{i+1} \) and taking the natural logarithm of Equation 6, one finds the expression for the logarithmic decrement defined by

\[
\delta \equiv \ln\left(\frac{u_i}{u_{i+1}}\right) = \frac{2\pi \zeta_n}{\sqrt{1 - \zeta_n^2}}
\]  \hspace{1cm} (7)

For lightly damped systems (\( \zeta_n < 0.2 \)), Equation 7 can be simplified to \( \delta \equiv 2\pi \zeta_n \). This simplification is valid for inherent damping ratios of most stay cables, which are almost always below 0.01. From this simplification and Equation 7, the damping ratio can be obtained by

\[
\zeta_n \approx \left(\frac{1}{2\pi}\right) \ln\left(\frac{u_i}{u_{i+1}}\right)
\]  \hspace{1cm} (8)

Equation 8 is valid for both displacement and acceleration decay curves for lightly damped systems.

**EXPERIMENTAL PROGRAM**

**The Test Bridge**

The Penobscot Narrows Bridge is a recently constructed cable-stayed bridge over the Penobscot River near Bucksport in Maine, USA. The new bridge is 646 m long with a main span of 354 m. Prior to completion of construction, the U.S. Federal Highway Administration (FHWA) was requested to test the stay cables and determine their natural frequencies and damping ratios, both before and after the installation of external dampers. The overview of the bridge is shown in Figure 1, and shown in the inset is a detailed view of cables supported by a pylon. Arrangement of stay cables is shown schematically in Figure 2, and the cables tested are labeled in the figure.

The bridge has two 136-m tall pylons with four fans of cables supporting the bridge deck. The fans on the outer sides are essentially symmetrical, as are the two inner fans supporting the main span of the bridge deck. The cables in the inner fans are longer than their counterparts in the outer fans. The fans are labeled A, B, C, and D running from the west to east side of the bridge. The cables are numbered from 1 to 20 in each fan, with 20 representing the longest. In this paper, a cable is referenced by its number, followed by its fan designation letter; i.e., Cable 19A. The cables consist of multiple steel strands loosely encased in a high density polyethylene (HDPE) pipe. The strands are threaded through a “cheese block” or neoprene spacer inside the HDPE pipe near the attachment point for external dampers. The strands are continuous through the pylon and are individually supported within a special steel saddle for each stay. The HDPE pipe has a constant diameter of 41-cm for all cables, despite the varying number of steel strands for each cable, and has a double-helix spiral bead on the surface.
Figure 1: The Penobscot Narrows Bridge in Maine, U.S.A.

Figure 2: Arrangement of stay cables tested

**EXPERIMENTAL SETUP AND PROCEDURES**

Dynamic testing was performed in two phases. The first phase took place during the latter stages of construction to establish the cable properties just prior to installation of external dampers. The second phase was conducted about nine months after the installation of dampers (shown in Figure 3) and subsequent opening of the bridge to traffic.

For Phase 1 testing, Cables 20A–12A were examined first, followed by Cables 20C–17C. Data was obtained using dual tri-axial accelerometers mounted at two separate locations on each cable, the first being positioned 19–21% up the length of the cable from the deck anchorage, and the second 3–4% up the cable. Data from accelerometers was recorded using a portable data acquisition system at a frequency of 100 Hz. Wind speed and direction were also recorded by the data acquisition system. The cables were excited manually in the vertical plane using a rope attached to the cable, while a spotter checked to make sure the proper amplitudes and modes
were achieved. When the cable reached a sufficient level of excitation, the rope was released, allowing the cable to freely oscillate and motion to decay. The data acquisition system began recording data before the excitation was started, and then continued until the decay subsided and only random wind-induced vibrations remained. Figure 4 shows the portable data acquisition system setup and an example of an accelerometer mounted on a stay cable during testing.

![Image](image.jpg)

**Figure 4:** The data acquisition system (left) and the accelerometer mounted on a stay cable (right)

Phase 2 testing of the cable-stays on the Penobscot Narrows Bridge was conducted nine months later. As in the first phase, the cable-stays were manually excited with a rope, while dual accelerometers measured the decay of the vibration using a portable data acquisition system. Due to more favorable weather conditions and shorter decay periods provided by the new dampers, it was possible to perform more than double the test runs completed during Phase 1. During Phase 2, the following cables from the four fans were tested: 20A–12A, 20C–14C, 20B–15B, and 20D–13D. The number of test runs performed for each cable varied between 7-10 runs, with the majority of cables undergoing eight or nine runs, which was an average of 1-2 more runs than in the first phase.

**ANALYSIS AND RESULTS**

**NATURAL MODAL FREQUENCIES**

To determine the natural modal frequencies of the cables, a spectral analysis was performed on the discrete time signal vector from each test. The power spectral density (PSD) was calculated using Welch’s modified periodogram method, resulting in a distribution of power per unit frequency spread over the Nyquist frequency domain. Figure 5 shows a sample acceleration-time record retrieved during Phase 1 testing from Cable 19A and the corresponding PSD distribution. Figure 6 shows the resulting 1st-mode natural frequencies of tested cables compared with the theoretical frequencies, determined from $f_n = \omega_n / 2\pi$ where $\omega_n$ is defined in Equation 3.

The natural frequencies of cables in Fan A, as shown in Figure 6, indicate their steady variation with the cable sequence, or equivalently, with the cable length. Frequency is a function of cable length, tension, and mass density per unit length. According to the design data, the ranges of these properties (length, tension, and mass density) are 34%, 5%, and 6%, respectively.
Therefore, the variation of frequencies is primarily due to differences in cable length. Overall, the results from the theory and experiment agree quite well, with a maximum variation of 8.5%.

The frequencies obtained from Phase 2 for cables in fans A and C could also be compared to the frequencies found in Phase 1. On average, the frequencies match within 0%–3%, although the frequencies on some of the shorter cables in the higher modes differed as much as 8%. In general, the spectral densities obtained from the Phase 2 testing produced clearer results than Phase 1.

**INHERENT DAMPING RATIOS**

After natural frequencies are identified, damping ratios of the cables were calculated. A measured time-domain waveform for each test run is a complex combination of numerous modes of vibration, and a band-pass filtering technique was used to decompose the signal into different modes. The modal band-pass frequencies were determined such that the power contribution from each mode to the PSD curve is sufficiently represented by a portion of the curve bracketed by the band-pass frequencies. A fourth-order elliptic filter was applied to the signal twice in order to completely suppress the unwanted noise outside the band while effectively preserving the signal within the cutoff frequencies. An example of the filter’s effect on the PSD plot of a data series from Cable 19A during Phase 1 testing is shown in Figure 7(a).
Once a band-pass filter has been applied, the reconstructed time series for the mode resembled a more consistent logarithmic decay. In addition, a time filter was established to eliminate the data associated with the manual excitation at the beginning of the run and also the random excitations prevalent after the decay had subsided. Figure 7(b) shows the reconstructed, band-pass filtered time series, which is subsequently truncated using the time filter and the resulting plot is shown in Figure 7(c). Once a logarithmic decay has been revealed, the damping ratio can then be extracted using Equation 8. Since the damping ratio is found from the ratio between two distinct, consecutive peaks, and that the ratio varies throughout the run as the peaks vary, a regression line was fitted to the data to minimize random errors. An average damping ratio was then calculated from this best-fit line. A sample plot of positive peaks and overlaid best-fit line is shown in Figure 7(d). To determine the effectiveness of a best-fit line, the coefficient of correlation was noted among the peak data points. In general, the correlation coefficient throughout the runs was quite high, averaging over 0.990.

![Figure 7](image)

**Figure 7**: Illustration of damping ratio determination for Cable 19A; (a) power spectral density curve after band-pass filtering with respect to the 1st mode, (b) band-pass filtered time series, (c) truncated time series, and (d) best-fit line of the natural log of the peaks

After the best-fit line was established, the damping ratio could then be determined for each cable. Due to the small number of available data sets for each cable and the fact that the population mean and variance are both unknown, the Student-t test was used to find a 90% confidence interval on the mean. This statistical process was performed for every cable for both the first and second modes, and the results are shown in Figure 8. Damping ratios calculated for the cables showed varying degrees of consistency over repeated runs.

There were several measured instances where damping ratio appeared to vary over the time of decay, which can be translated into the dependence of damping ratio on the amplitude of vibration. In general, the damping ratio decreased with time, or equivalently, with vibration.
amplitude. When the cable is vibrating at large amplitudes, it is believed that the energy is dissipated at a higher rate, resulting in larger damping ratio. This could be due to nonlinear or non-viscous type physical interactions between the steel strands and the HDPE pipe. Despite the apparent existence of some nonlinear effects, damping ratios could still be consistently estimated on an average basis throughout the entire decay history. Attention was paid to the possible effect of aerodynamic damping on calculated damping ratios. While there appears to be a slight trend between wind speed and damping ratio, speed range was limited during testing and there were not enough data points to establish any statistically significant regression properties.

**DAMPING RATIO COMPARISONS**

The installation of dampers on the cable-stays had a noticeable difference on the damping ratio values, causing the damping ratios to increase by at least a factor of five, and sometimes as high as 15. In general, first mode damping ratios for Phase 1 testing were between 0.10–0.39%, where in Phase 2 they ranged between 1.3–2.5%.

The damping ratios obtained during Phase 2 testing were generally more consistent than their Phase 1 counterparts, but still had some discrepancies that need to be addressed. Since two accelerometers are used during testing to provide redundancy in the results, mainly for the possibility of one being mounted near the proximity of a node at higher frequency, it was found that the two boxes rarely produced identical damping ratio data during the Phase 2 testing. For the majority of cases, the damping ratio obtained from Box 2 was higher, for some cables up to 30% higher. For first mode data, 28 of 30 cables had higher damping ratios from Box 2 data, while for the second mode this ratio fell to 21 of 30 cables. Since the accelerometer in Box 2 was mounted on the cable end closer to the bridge deck and the damper, it experienced higher levels of damping than the accelerometer located closer to the cable’s mid-span.

There is a greater correlation in Phase 2 data between cable length and damping ratio. As the cable length decreases, the damping ratio almost always increases. There are a few exceptions visible within the plots for each fan of cables, but there is definitely a stronger correlation than in similar plots from Phase 1. Conversely, there is no correlation between cable length and the size of the confidence interval. The Student-t test gives a range of values from which there is 90% confidence that the mean of the damping ratio is located within. For Phase 2 testing, this range varied between 0.05 to 0.40 for the damping ratio, comparable to 2.5–16% of the mean value, and it varied in no pattern related to cable length or size. These correlations and lack thereof held true for both the first and second modes.
Another obvious result of the addition of dampers was the length of the decay period was shortened by a great deal. For first mode decay, the period lasted on average 28.5 seconds for Box 1 and 20.6 seconds for Box 2, where the period was measured as the longest amount of time a best-fit line could overlay the data points with a correlation greater than 0.9900. Some periods were measured as high as 50.2 seconds. For second mode decay, the periods were much shorter, averaging 16.3 seconds for Box 1 and 15.4 seconds for Box 2.

**SCRUTON NUMBER ANALYSIS**

Another widely used mass-damping parameter indicating the level of cable damping with respect to vibration mitigation is the Scruton number defined by

\[ S = \frac{\zeta m}{\rho D^2} \]  

(9)

where \( \zeta \) = first-mode damping ratio, \( m \) = mass density per unit length of cable, \( \rho \) = mass density of air, and \( D \) = diameter of the cable pipe [5]. The Scruton number is frequently used in developing a criterion for controlling rain/wind-induced vibration of stay cables. For instance, based on Irwin’s suggestion [6], the PTI committee on cable stayed bridges has suggested that rain/wind vibrations of stay cables can be avoided if the Scruton number is kept at a value of 10 or higher [7]. Additionally, a reduced Scruton number of five has been suggested if the cable has an aerodynamic surface treatment [8]. The Scruton numbers shown in Figure 9 are calculated according to Equation 9 and using the 1st-mode damping ratios presented in Figure 8.

![Figure 9: Scruton Number Comparisons for Fan A](image)

It is apparent that the Scruton numbers calculated from the damping values obtained during Phase 1 testing are all equal to or less than two, far below the desired value of ten, indicating that the cable system under consideration was potentially vulnerable to rain/wind-induced (and perhaps wind-induced) vibrations. Even if a reduced Scruton number was used to account for the aerodynamic surface treatment, this would still be the case. Based on these results, it is confirmed that an appropriate vibration mitigation measure, such as external viscous dampers, had to be incorporated into the cable system.
After the installation of viscous dampers, the Scruton values calculated from the Phase 2 damping ratios improved to a range between 7–11, above the reduced Scruton target value of five. The dampers installed on the cable stays should provide sufficient protection towards controlling large-amplitude cable vibrations.

CONCLUSIONS

An experimental program was conducted to investigate the natural vibration frequencies and inherent damping ratios of stay cables of the Penobscot Narrows cable-stayed bridge. Measured frequencies compared well with calculated values. Damping ratios of the tested cables varied 0.10–0.39% for the first mode and 0.17–0.46% for the second mode. These ranges increased after the installation of dampers to 1.22–2.21% for the first mode and 1.41–2.48% for the second mode. Analysis indicated that measured damping ratios depend on the magnitude of vibration, suggesting the possibility of nonlinear or non-viscous type behavior of the cables. Final Scruton values, which are used as a criterion to determine effective cable vibration mitigation, ranged from 7–11, which is above the target value of five for cables with aerodynamic surface treatments.

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REFERENCES


